# Practice Midterm Problems

1) Plot each of the following requested time responses. Carefully label the output at $k$=0, $k$=1, and $k$=2, and carefully indicate the final value (if bounded) as $k$ approaches infinity.

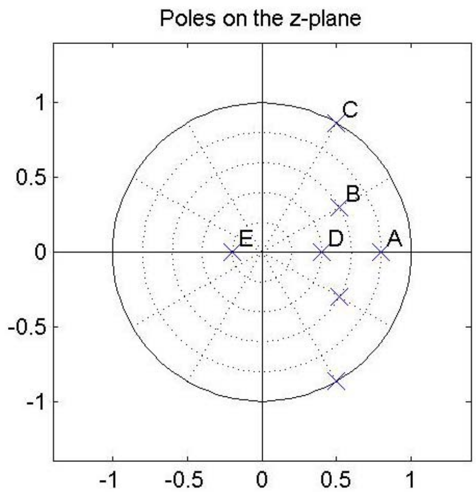
a) $\frac{Y(z)}{X(z)} = \frac{z}{z-.6}$ Sketch the **unit impulse** response.

b) $\frac{Y(z)}{X(z)} = \frac{z}{z-.6}$ Sketch the **unit step** response.

c) $\frac{Y(z)}{X(z)} = \frac{1.2}{z+.2}$ Sketch the **unit impulse** response.

d) $\frac{Y(z)}{X(z)} = \frac{1.2}{z+.2}$ Sketch the **unit step** response.

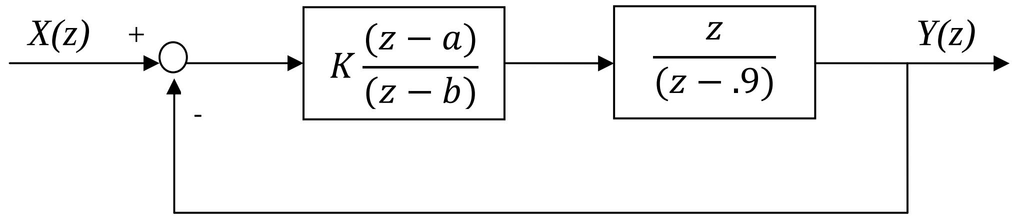
2) a-d) For pole locations A through D on the z-plane below, find the pole positions on the s-plane that correspond. Simply express each pole (or pole pair) **symbolically**, in terms of the sampling time, T, and do **not** solve any challenging math expressions numerically.




*Hint: By "correspond", we mean that the discrete-time system output over time could be generated by taking samples from a "corresponding" continuous-time output, i.e., that* $z = e^{sT}$ *relates the DT and CT poles.*

**EXTRA CREDIT:** e) Why would attempting to plot a single corresponding continuous-time pole location for case "E" be more problematic? (Recall that Problem 1 parts c and d correspond to pole location "E" in this problem; your time response solutions may provide some intuition.)

3) a) Provide values for $K$, $a$, and $b$ that will produce **deadbeat control** for:




b) Your solution should not be unique. Increase your value of $K$ by a factor of 10 and solve for corresponding values of $a$ and $b$ that produce deadbeat control. Compare the DC gain of the two closed-loop systems; does this result make sense, intuitively? (If you do not have an answer for part **a)**, what would your intuition be?)

4) Convert the continuous-time transfer fn given below, *C(s)*, to a discrete-time transfer fn, *C(z)*, using the **pole-zero matching** method. Keep the sampling time, *T*, as a variable in the expression you derive. ***(For study, repeat using other methods discussed in class...)***

$$C(s) = 40\left(\frac{s+10}{s+2}\right)$$

*Note: For advanced practice, then plug in T=.005, and solve by hand, using this approximation:*

$$e^{\delta} \approx 1+\delta \quad \textit{for} \quad |\delta| \ll 1$$

*For example,*

$$e^{-.03} = 0.9704 \approx 1 - .03$$

5) In this problem you will derive a **state space** representation and state feedback controller.

Assume that: $$P(s) = \frac{1000}{(s^2+12s+100)(s+100)}$$

is part of the familiar configuration below, with a sample time of T=0.0002 seconds.

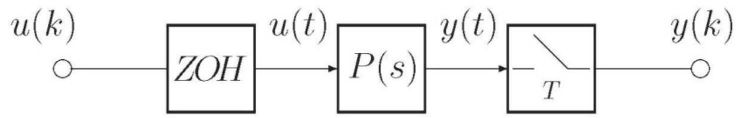



**Figure P5.** The standard sampled-data configuration.

The corresponding discrete-time transfer fn from *u(k)* to *y(k)* is:

$$P(z) = \frac{10^{-9} \cdot (1.3259z^2 + 5.2740z + 1.3111)}{z^3 - 2.9778\,z^2 + 2.9556z - 0.9778}$$

a) To represent the discrete-time dynamics in P(z) in state-space format, how many states are required in the state vector $\boldsymbol{x}$?

b) Derive matrices A, B, C, and D to represent this discrete-time dynamic system. Recall:

$$\boldsymbol{x}_{k+1} = A\boldsymbol{x}_k + Bu_k$$
$$y_k = C\boldsymbol{x}_k + Du_k$$

*Note: There are multiple possible solutions to part b), any one of which is acceptable.*

Now, assume the standard feedback regulator configuration discussed in class, where:

$$u = -BK$$

c) Solve for the vector of gains, K, that will result in deadbeat control for your state space representation.

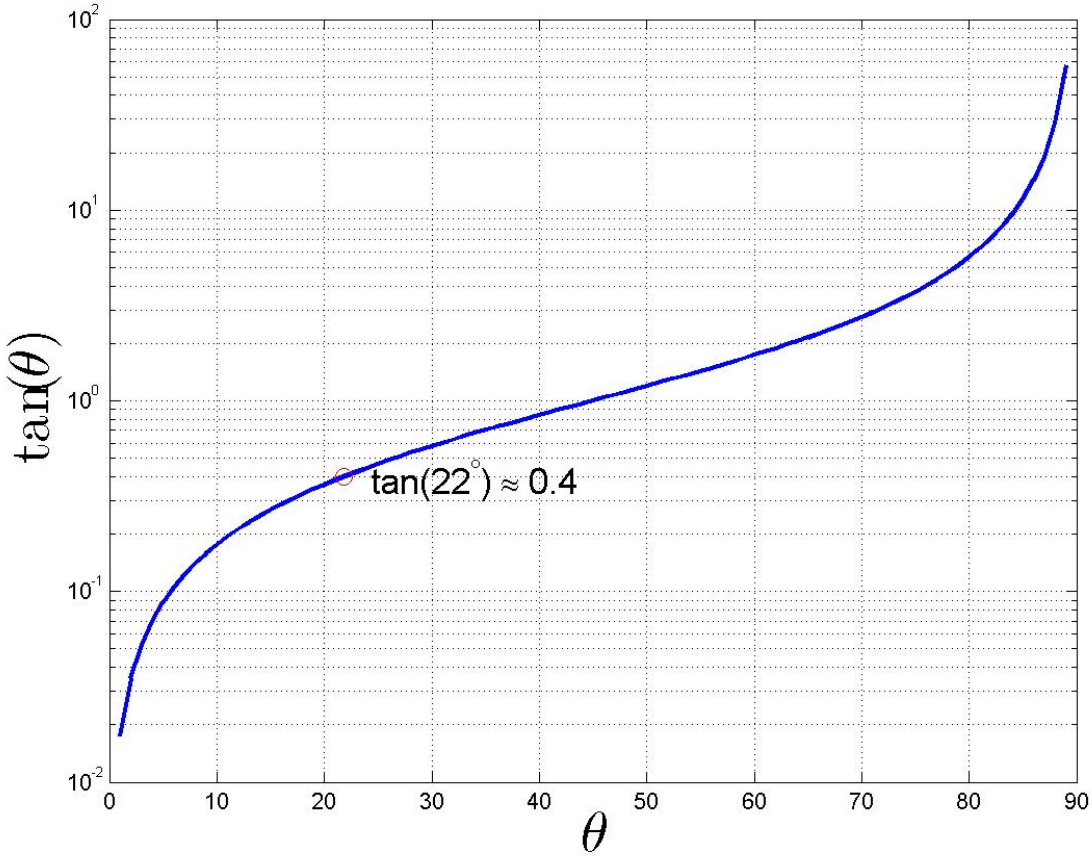
6) For the continuous-time plant, $P(s) = \frac{1}{s^2+10s+100}$,

a) What is the phase (in degrees) of the frequency response (Bode plot) at 100 rad/s?
*Note: Use the plot on the following page to solve for the tan and arctan functions required in calculating this by hand. Only estimate the phase to within about 1 degree.*

Now, assume P(s) is part of the sampled-data system configuration shown in Figure P5 (in Problem 5), and that the sample time is $T$=0.01 seconds.
b) What is the phase of P(z) at 100 rad/s?
*Hints: What is 100 rad/s in Hz? What fraction of a full cycle does a "half-sample delay" represent for the given value of $T$? What additional phase lag is this, in degrees?*

# Appendix: The tangent function.

*Note: This Appendix is likely to appear in the actual Midterm, too, so* ***learn to use it now*** *for Problem 6a, and expect similar problems involving DT transfer functions, too.*




The examples and figure below illustrate how to use semilog plot above:

- The complex number 5+2j has phase: $\text{atan}\left(\frac{2}{5}\right) \approx 22^\circ$.
- The complex number 5-2j has phase: $\text{atan}\left(\frac{-2}{5}\right) \approx -22^\circ$.
- The complex number -5-2j has phase: $\text{atan}\left(\frac{-2}{-5}\right) \approx -180^\circ + 22^\circ = -158^\circ$.