HW 4 – Due in Dropbox, 5pm Monday.

The actual Mid-Term will look nearly identical to this in form.

This practice exam is a slightly-modified version of the Midterm given in ECE147B in 2011.

Name ______________________________

ECE 147B

Practice Midterm Exam

Due 5pm, Feb. 13, 2011

- You are allowed one, single-side 8 ½ x 11 sheet of self-prepared notes.
- Use **must** submit this "Cheat Sheet" with your exam; it will be returned with your graded exam, as is.
- **No calculators, laptops, or other computing devices are allowed.**

____/25 **Problem 1: Discrete-time controller analysis**

____/15 **Problem 2: Tustin transformation**

____/20 **Problem 3: Corresponding s- and z-plane poles**

____/20 **Problem 4: ZOH effects on a plant, P(s)**

____/20 **Problem 5: State space (continuous time) equations**

____/100 **TOTAL**

Midterm ID number.

Below are two matched random numbers. Tear off the non-taped number and keep it.

We will post exam grades and statistics using these numbers instead of student names, to hide identities.

(Taped side)	**TEAR AND KEEP TAB BELOW**

1) For this discrete-time transfer fn: $C(z) = \frac{z-.96}{z-.99}$, with sampling time T=.01 sec.

2 points a) What is the DC gain?

2 points b) What is the high-frequency gain?

5 points c) At *approximately* what frequency does C(z) have the greatest deviation in phase (i.e., away from 0 degrees phase, on a Bode plot)?

8 points d) Estimate the peak phase shift C(z); that is, the phase shift for a sinusoidal input at the frequency calculated in part c. *You may with to use the tan function in the Appendix to estimate.*

e) [Problem 1-e is at the top of the next page…]

8 points e) Sketch an approximate Bode plot for $C(z)$. Label important details with care. (Please use the blank space below to do this.)

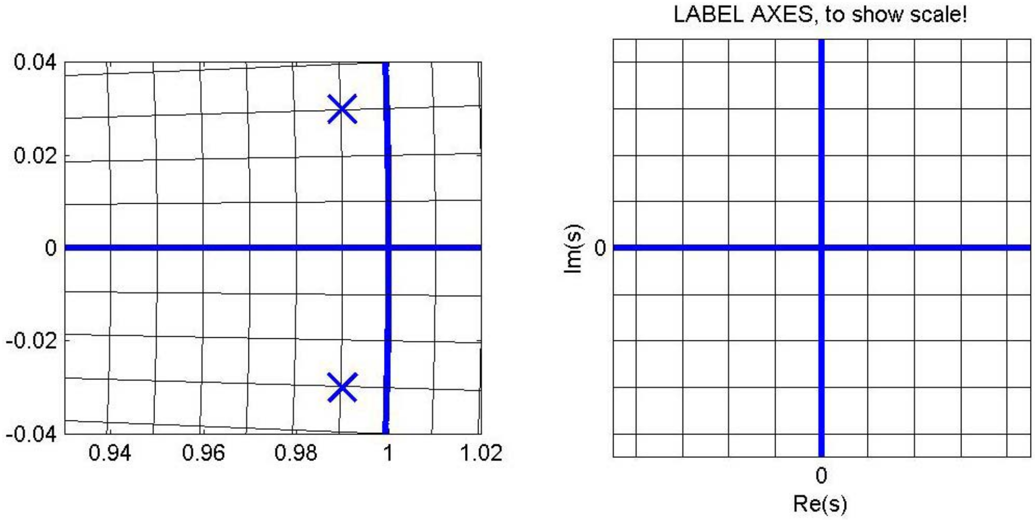
2) You are given the following continuous-time controller:

$$C(s) = 10\frac{(s+20)}{(s+100)}$$

10 points a) Use the Tustin transformation (aka bilinear transformation, or trapezoid rule) to convert C(s) to a discrete-time controller, C(z). Leave the sampling time, T, as a variable.

5 points b) Now, assume T=.001 seconds. Solve numerically for C(z).

3) The figure at left below shows poles (zoomed in on the unit circle) on the z plane for
$$P(z) = \frac{K}{(z-.99+.03j)(z-.99-.03j)} = \frac{K}{z^2-1.98z+.981}$$, where T=.001 seconds



5 points

a) Plot ***approximate*** corresponding pole locations on the empty Cartesian axes at the right above, being carefully to label enough of the grid marks on the x and y axes to show scale. (You may or may not find it helpful to complete parts b and/or c first…)

3 points

b) Calculate the corresponding locations for poles on the s plane using the forward Euler integration approximation. (That is: $s = \frac{z-1}{T}$, which can also be written as $z = Ts + 1$.) [This should be simple, if you use the factored representation of poles of P(z), top left.]

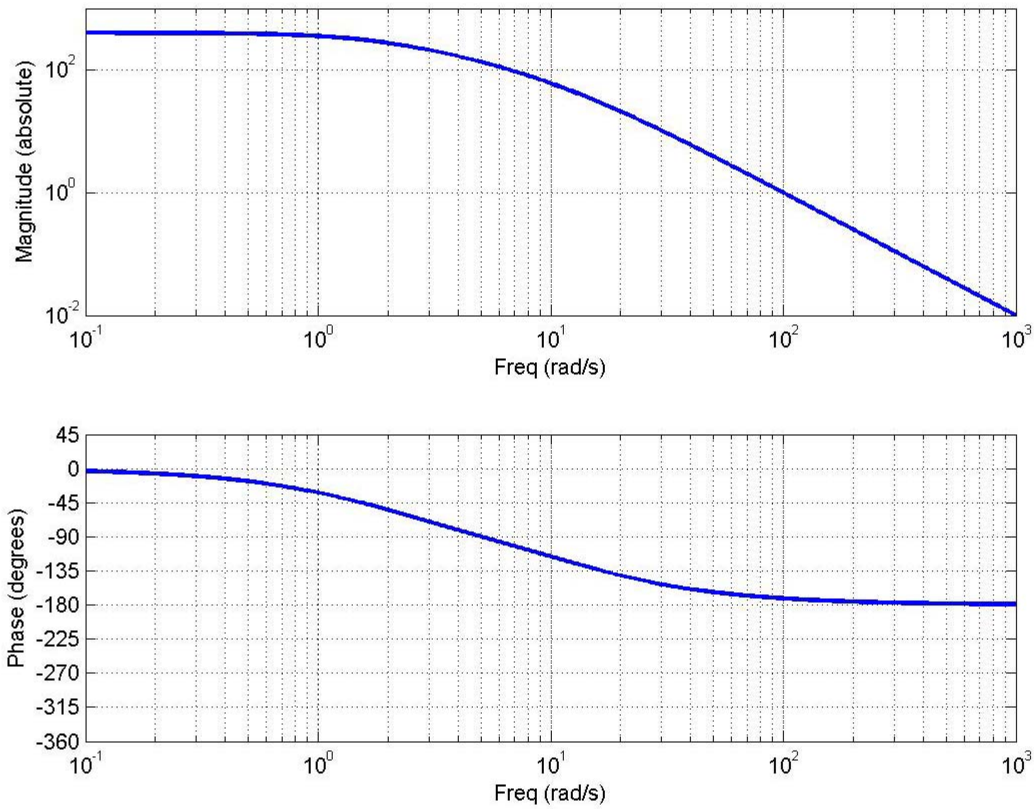
3 points

c) Demonstrate that the corresponding locations for poles on the s plane using **<u>one</u>** other method (e.g., backward Euler, Tustin **<u>or</u>** matched) are in "**approximately**" the same location.

9 points 3-d) Below, sketch (roughly) a unit step response for P(z). Do not worry about capturing the discrete nature of the plot! Instead, focus on calculating *approximate* values for the following three values, which MUST BE LABELED CLEARLY on your plot:

i) Final value (Written in terms of K.)
ii) t_{peak} (Time of maximum overshoot.)
iii) Maximum percent overshoot

4) Below is a Bode plot for some system, P(s), which is part of the sampled-data configuration discussed in class, where a zero-order hold precedes the input and the output are sampled, and where the sampling time is T=.03 seconds.



5 points

a) What is the Nyquist frequency, ω_{Nyquist}, for the system?

6 points

b) Write a general expression giving the additional lead or lag due to a **half-sample delay** at frequency $\omega = K\omega_{Nyquist}$, where 0<*K*<1 ? (Leave *K* as a variable is your answer. Parts a and b have nothing to do with the particular Bode plot shown above.)

9 points

c) On the same axes where phase for P(s) is plotted above, sketch a shifted version to represent the phase of P(z). You may assume the phase is simply shifted by a half-sample delay, as in part b. **Draw only the phase; not gain.** Label these 3 points "A", "B", "C":

A) Frequency and phase at ω_{Nyquist}.

B) Frequency and phase at $\frac{1}{2}\,\omega_{\text{Nyquist}}$.

C) Frequency and phase at $\frac{1}{10}\,\omega_{\text{Nyquist}}$.

5) Recall the continuous time state space representation of a dynamic system, given below:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

In this problem, ***x***, ***A***, ***B***, ***C***, and ***D*** are defined as:

$$x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 1 \\ 12 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$C = [3 \quad 0] \qquad D = [0]$$

4 points a) Write a differential equation relating u to θ.

4 points b) What is the transfer function from u to y? (**Note this is to output, y**; not to θ…)

6 points c) Write out the characteristic equation from $det\{sI - A\} = 0$. (Hint this should match the denominator of the transfer function…) What are the poles of the system?

2 points d) Is the open-loop system stable? (Why?)

4 points e) What is its final value to a step response?

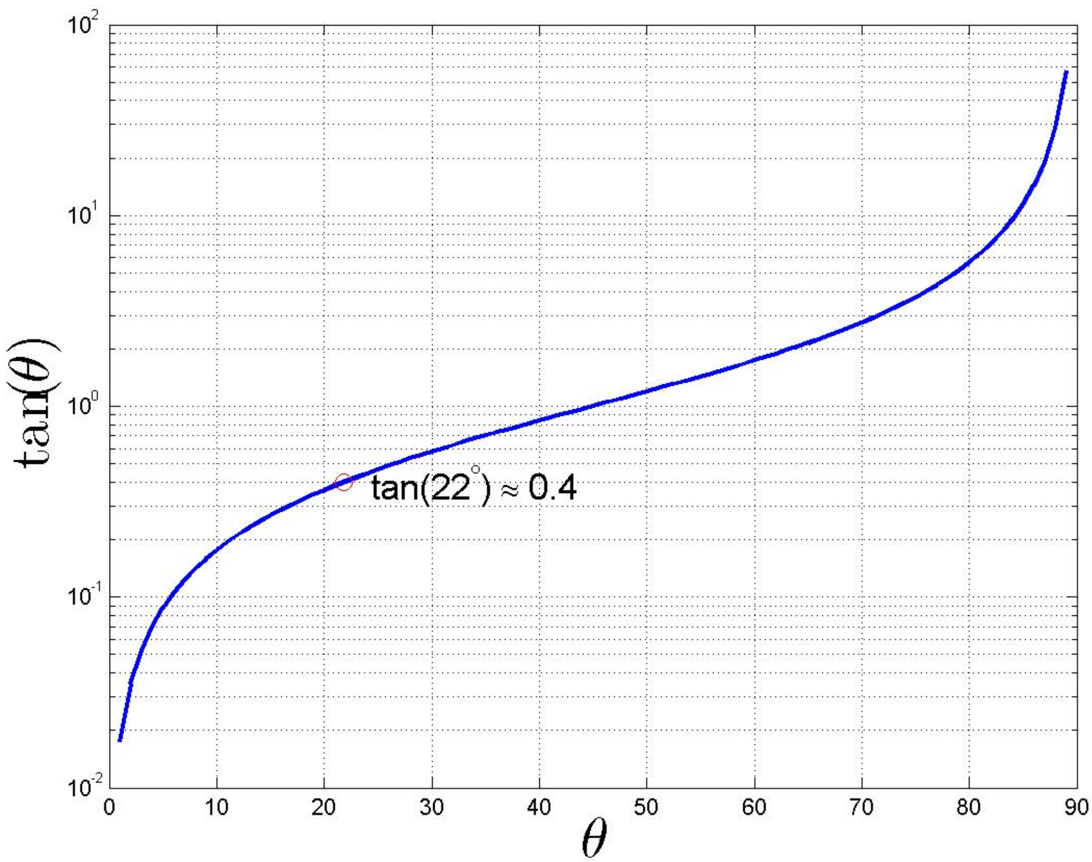
Extra Credit f) Solve for K_1 and K_2 for a feedback regulator of the form

u=-BKx

to place the poles at $s = -10 \pm 10j$

Appendix: The tangent function.

Note: This Appendix is likely to appear in the actual Midterm, too, so ***learn to use it now*** *for problems involving phase calculations for both CT and DT transfer functions.*



The examples and figure below illustrate how to use semilog plot above:

- The complex number 5+2j has phase: $\text{atan}\left(\frac{2}{5}\right) \approx 22^\circ$.
- The complex number 5-2j has phase: $\text{atan}\left(\frac{-2}{5}\right) \approx -22^\circ$.
- The complex number -5-2j has phase: $\text{atan}\left(\frac{-2}{-5}\right) \approx -180^\circ + 22^\circ = -158^\circ$.