

Homework 1

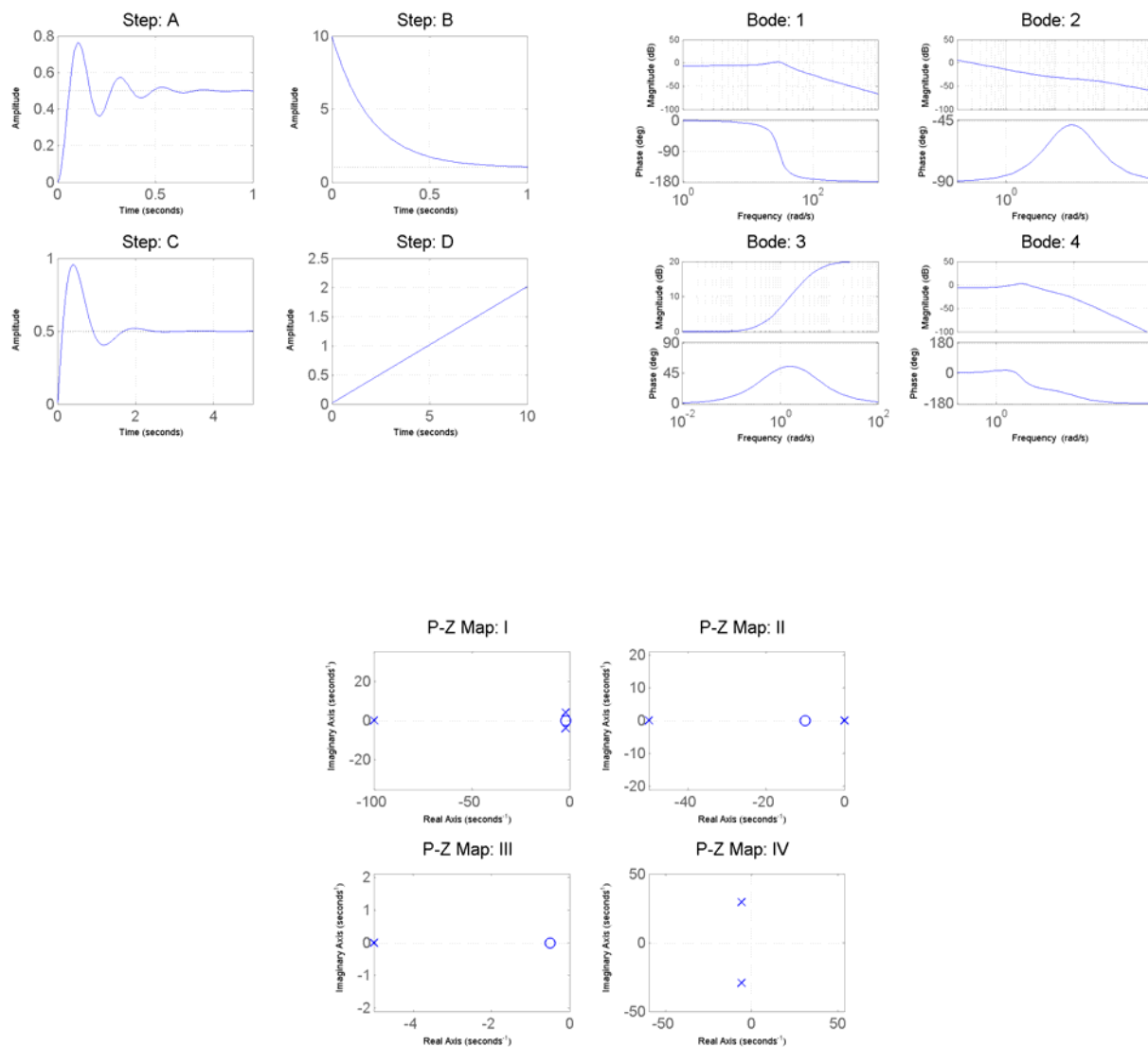
Due: January 20, 2011, at 5 pm, in the homework boxes outside Harold Frank Hall 3120.

Problem 1. Continuous-time dynamics.

Below are 4 step responses (a,b,c,d), 4 Bode plots (1,2,3,4), and 4 pole-zero plots (I,II,III,IV).

a) Match these up. (i.e., list the 4 correctly-matched triplets. e.g., if A, 1, and I match, write “A-1-I”.)

b) Estimate the continuous-time transfer function for each system.



Problem 2. Stability for continuous-time (CT) systems.

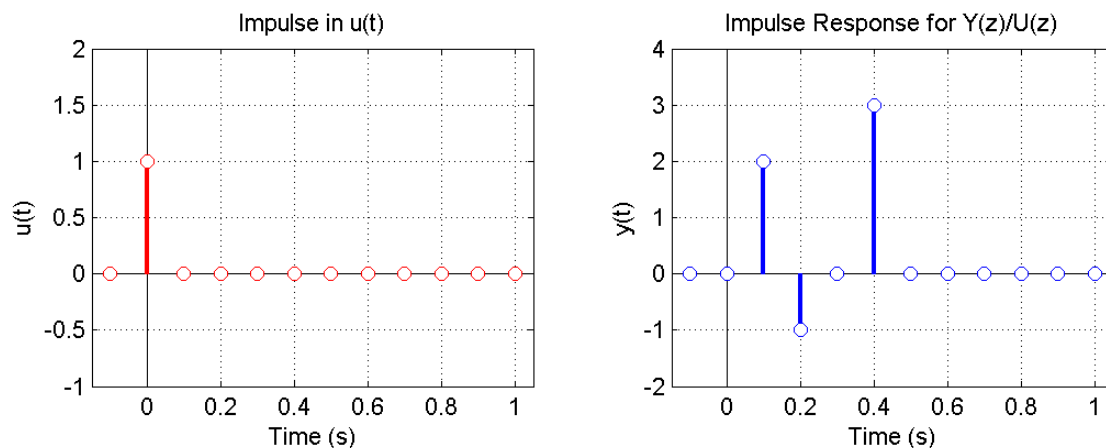
$$G(s) = \frac{-s + 5}{6s + 2}$$

Assume a unity-feedback configuration with a gain, K , as the (proportional) controller. For what values of K will the resulting closed-loop system be stable? (HINT: assume K might be negative or positive!) Justify your answer via each of the following two methods:

- Algebraically solve for the closed-loop transfer function, keeping K as a variable; then reason (as appropriate) about when the poles would all be stable.
- Sketch the Nyquist (frequency response) plot for the system; then reason (as appropriate) about when the poles would all be stable. *Hints:* 1. Please feel encouraged to exploit the “nyquist” function in MATLAB to get started, but for your submitted homework, please just *sketch and carefully label the plot by hand*. 2. What is the nyquist plot for “ $-G(s)$ ”?

Problem 3. Discrete-time transfer functions.

Below at left is an impulse in $u(t)$. At right is the response of a discrete-time system, $P(z)=Y(z)/U(z)$, to this impulse, plotted as discrete values over time.



- What is the sampling time, and what is the discrete-time transfer function, $P(z) = \frac{Y(z)}{U(z)}$? (Assume $y(t)$ continues to be 0 for all $t > 0.4$ sec in the plot at right, above...)
- By hand, plot the **step response** of $P(z)$.

Problem 4. Numerical integration approximations.

Consider the following continuous-time lag controller:

$$C(s) = \frac{(s + 50)}{(s + 2)}$$

- By hand, calculate the change (i.e., magnitude of increase or decrease, as appropriate) in DC gain this controller would provide to the loop transmission, $C(s)P(s)$.
- By hand, calculate the maximum phase change i.e., magnitude of increase or decrease, as appropriate) that occurs due to this controller, and at what frequency does this occur? (We will call this frequency ω_a .)
- Recall from lecture that i) Forward Euler, ii) Backward Euler, and iii) Tustin (trapezoid rule) approximations results in the following mappings between the differential operator, s , and the difference equation operator, z , respectively:

$$\text{i) } s \leftrightarrow \frac{z-1}{T}$$

$$\text{ii) } s \leftrightarrow \frac{z-1}{Tz}$$

$$\text{iii) } s \leftrightarrow \frac{2(z-1)}{T(z+1)}$$

For the remainder of the problem, use a sample period, T , of 0.01 seconds.

- Calculate the forward difference approximation, $C_{\text{forward}}(z)$, for $C(s)$. What is its phase at $z_a = e^{j\omega_a T}$?
- Calculate the backward difference approx., $C_{\text{backward}}(z)$, for $C(s)$. What is its phase at $z_a = e^{j\omega_a T}$?
- Calculate the bilinear difference approximation, $C_{\text{bilinear}}(z)$, for $C(s)$. What is its phase at $z_a = e^{j\omega_a T}$?

Use MATLAB to generate a Bode plot for the original $C(s)$ along with all 3 approximations on a single graph, using a frequency range from 0.1 to 1000 rad/s. Discuss the quality of each approximation.

Problem 5. DT control design via emulation.

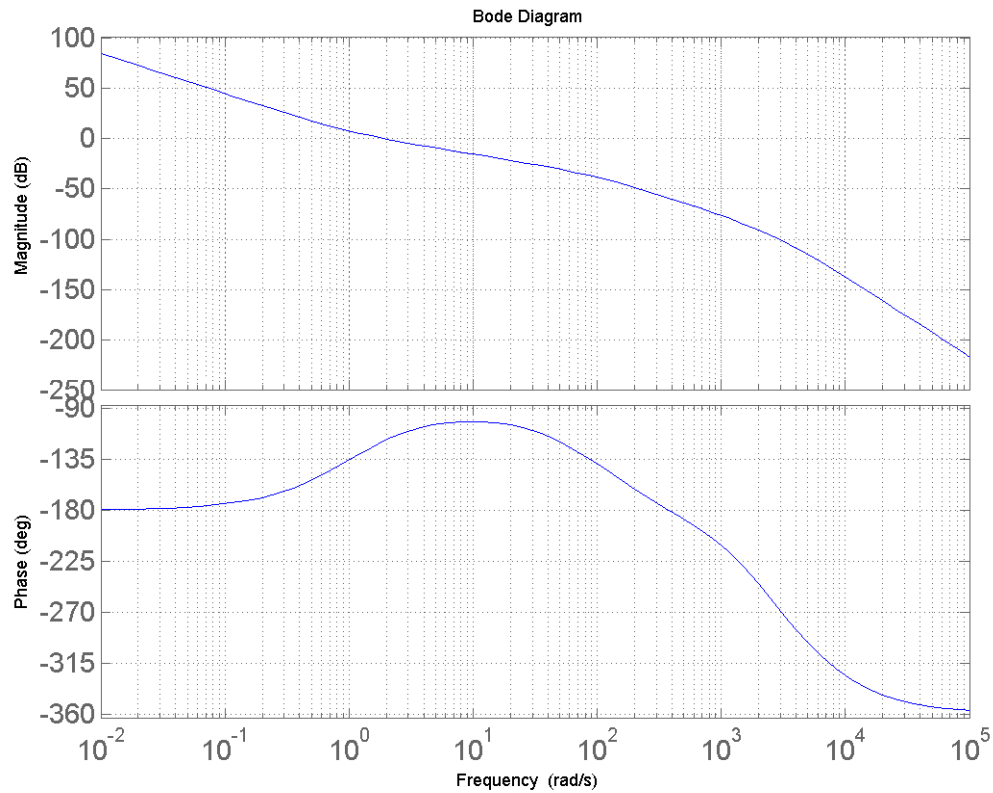
- Design a continuous-time lead compensator, $C(s)$, for the plant $P(s) = 20/s^2$ so that the overshoot, M_p , is between 10% and 20% and the peak time, t_p , is *approximately* 0.05 seconds (e.g., in the range 0.04 to 0.06 seconds). Include a MATLAB step response plot to verify your answer.
- What are the poles and zeros of the closed-loop system? What effect do the zeros have on overshoot, and why?

Repeat c) and d) for sampling rates of $T=.001$, $T=.01$, and $T=.02$ seconds.

- By hand, convert $P(s)$ to $P(z)$, assuming a zero-order hold, and convert $C(s)$ to $C(z)$, using the Tustin approximation. (Please use MATLAB to check your work..)
- Calculate the closed-loop transfer function for the discrete-time (DT) system, and generate a step response plot, using MATLAB. What are the peak time and percent overshoot for the DT system? **Put all 4 step responses (the CT step and all three DT sampling rate cases) on a single plot, clearly labeled.**

Problem 6. Gain and phase margins.

a) From the Bode plot below, estimate gain and phase margin. Also indicate the crossover frequency. (Assume there are no poles or zeros outside the frequency range shown.)



b) From the Nyquist plot below, estimate gain and phase margin. (At right is “zoom” view...)

