Homework 7

Due: **FRIDAY**, March 9, 2012, **at 5 pm**, in the homework boxes outside Harold Frank Hall 3120.

Problem 1. State space control. For the open-loop DT system below:

$$A_d = \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} \quad , \quad B_d = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Solve ***by hand*** for the control gain vector, **K**, that results in the characteristic equation below:

$$z^2 - 1.2z + 0.8 = 0$$

Problem 2. (FPW means: Franklin, Powell and Workman.) FPW 8.6

Problem 3. FPW 8.12

Problem 4. FPW 8.22

Problem 5. For the DT system below (please note **correction to numerator**!):

$$\boldsymbol{G(z)} = \frac{\boldsymbol{0.045z}}{\boldsymbol{z^3 - 2z^2 + 1.95z - .95}} \ , \text{ with T=0.1 seconds}$$

a) Use MATLAB to generate a Nyquist plot. Use "axis([-2 .5 -1 1])" to zoom in and then "axis equal" to make the axes 1:1 in scale. Would the closed-loop system G(z)/(1+G(z)) be stable?

b) Use your Nyquist plot to show/calculate the approximate range of gains and range of phases for which the system is stable. (Label your plot as appropriate.)

Below are the problems from FPW, for anyone who does not own the text.

8.6 For the open-loop system

$$G(s) = \frac{y(s)}{u(s)} = \frac{1}{s^2}$$

preceded by a ZOH and a sample rate of 20 Hz,

(a) Find the feedback gain **K** so that the control poles have an equivalent s-plane ω_n = 10 rad/sec and ζ = 0.7.

(b) Find the estimator gain $\mathbf{L}_p$ so that the estimator poles have an equivalent s-plane ω_n = 20 rad/sec and ζ = 0.7.

(c) Determine the discrete transfer function of the compensation.

(d) Design a lead compensation using transform techniques so that the equivalent s-plane natural frequency $\omega_n \cong$ 10 rad/sec and $\zeta \cong$ 0.7. Use either root locus or frequency response.

(e) Compare the compensation transfer functions from (c) and (d) and discuss the differences.

8.12 For the open-loop system

$$\mathbf{\Phi} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{\Gamma} = \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix},$$

check the observability for:

(a) $\mathbf{H} = [0 \quad 1]$.

(b) $\mathbf{H} = [1 \quad 0]$.

(c) Rationalize your results to (a) and (b), stating why the observability or lack of it occurred.

8.22 Determine the state-command input structure for a feedback system with the Type 0 plant

$$G(s) = \frac{10}{s^2 + 0.18\,s + 9}.$$

(a) Convert the system to discrete state-space form with $T = 0.1$ sec.

(b) Find $\mathbf{K}$ to obtain equivalent s-plane control poles at $\zeta = 0.5$ and $\omega_n = 4$ rad/sec.

(c) Find $\mathbf{L}_p$ to obtain equivalent s-plane estimator poles at $\zeta = 0.5$ and $\omega_n = 8$ rad/sec.

(d) Determine N_u and $\mathbf{N}_x$, then sketch a block diagram specifying the controller equations including the reference input.

(e) Do you expect there will be a steady-state error for this system for a step input?

(f) Plot the step response and confirm your answer to (e).