Name ______________________________

ECE 147B

Final Exam

Mar. 18, 2011

- You are allowed two, single-side 8 ½ x 11 sheets of self-prepared notes.
- Use **must** submit these "Cheat Sheets" with your exam; they will later be returned, unmarked. **PUT YOUR NAME ON YOUR CHEAT SHEET!**
- **No calculators, laptops, or other computing devices are allowed.**

____ / 16 Problem 1: G(s) vs G(z) (via ZOH) Bode analysis.

____ / 20 Problem 2: C(z) controller design.

____ / 16 Problem 3: P(z) time response.

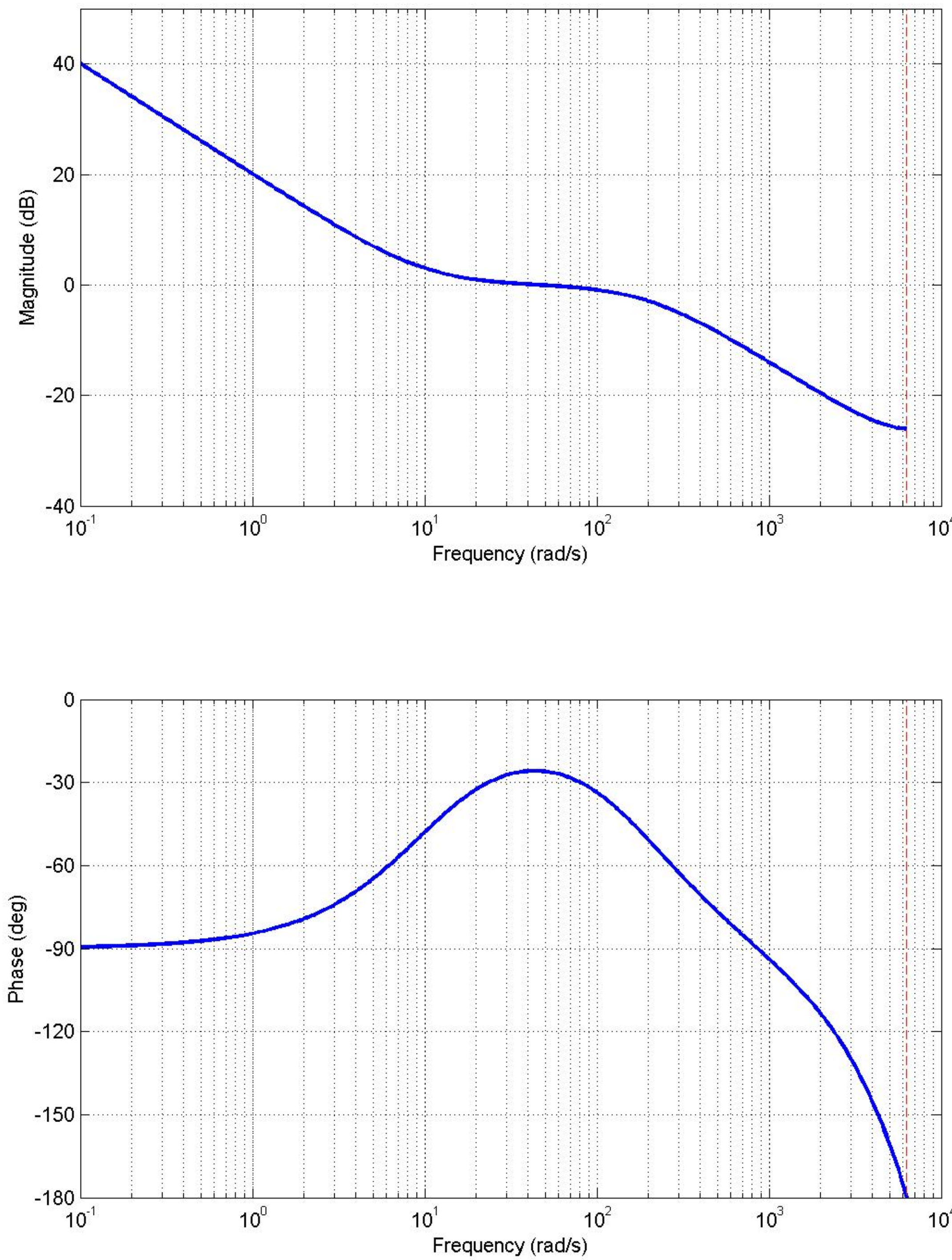
____ / 18 Problem 4: C(z) controller analysis.

____ / 20 Problem 5: State space control and estimation.

____ / 20 Problem 6: State space block diagram analysis.

(110 total points possible.)

1) In this problem, a continuous-time plant, P(s), is in the standard ZOH (zero-order hold) configuration. Below is a Bode plot for the resulting discrete-time plant, P(z).



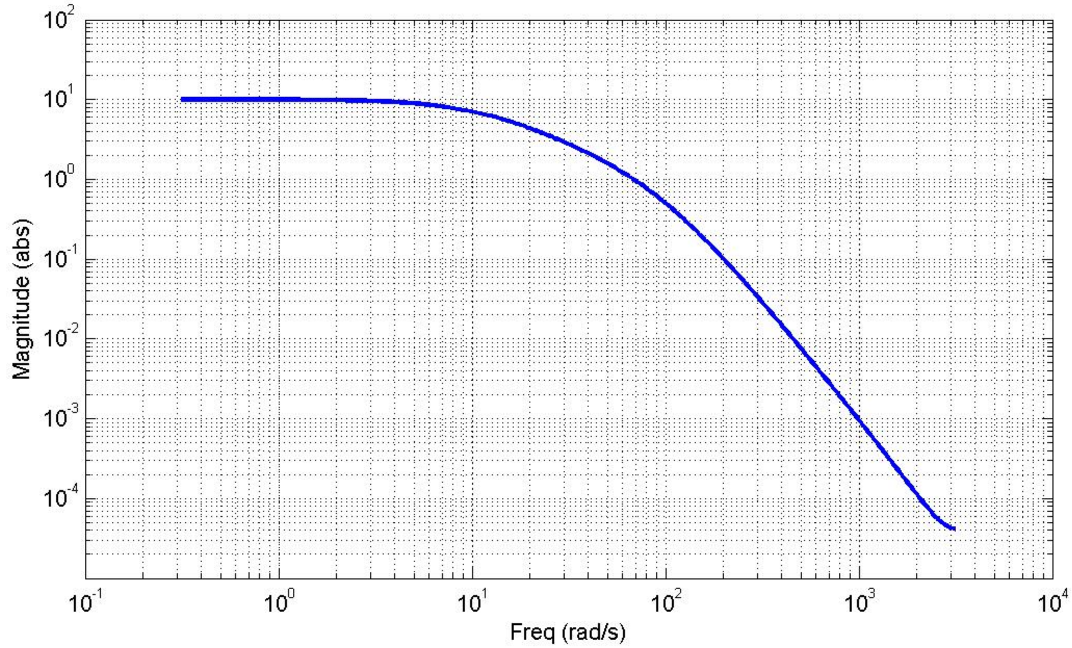
(Problem 1, continued…)

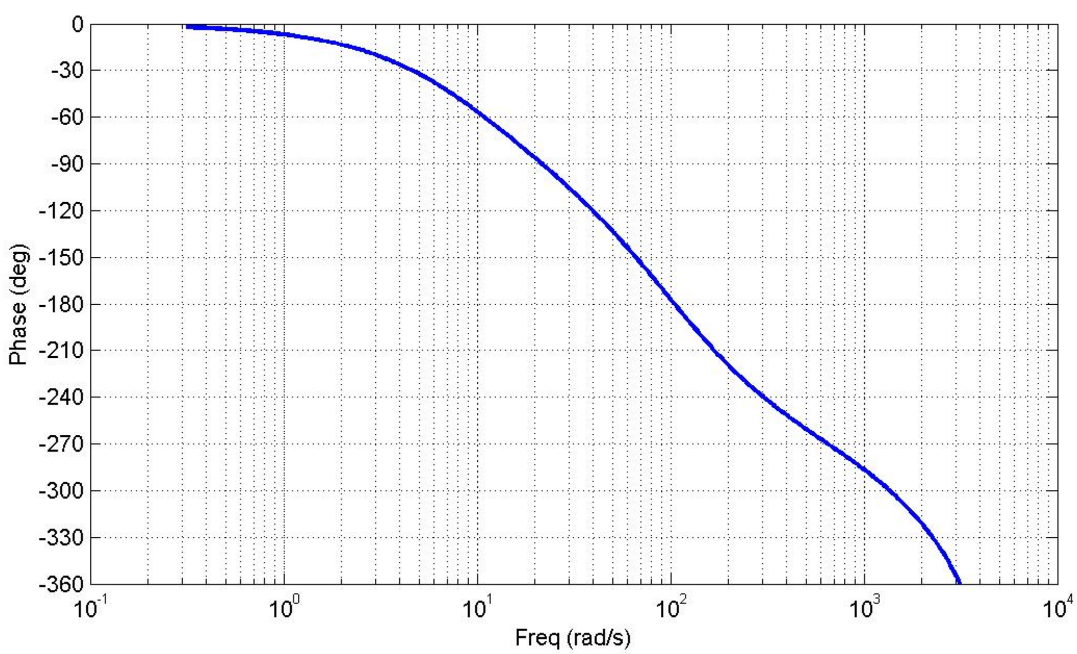
a) The thin, dashed line toward the right edge of the plot indicates the Nyquist frequency for the system. What is the sample time, T (approximately)?

b) Estimate the poles and zeros (s plane values) for the CONTINUOUS TIME system, G(s).

c) Estimate the poles and zeros (z plane values) for the actual DISCRETE-TIME system, G(z).

2) In this problem, you are asked to analyze proposed discrete-time controllers on a given plant. A Bode plot of the (open-loop) plant, G(z), is shown below. T=.001 seconds.





(Problem 2, continued…)

a) First, assume we plan to close the loop using a proportional controller with unity gain: *C(z)=1*. From the plot on the previous page, estimate the following. (You must also label the plot to illustrate each value, too!)

i) Crossover frequency :

ii) Gain margin :

iii) Phase margin :

b) Would the resulting closed loop system (from part a) be stable? (Why or why not?)

c) Suppose your lab partner proposes the following continuous-time controller:

$$C(s) = K\frac{(s+\frac{100}{2.5})}{(s+2.5\cdot 100)} = K\frac{(s+40)}{(s+250)}$$

What is the peak phase this CT controller adds, and at what frequency does this occur? (Hint: See Appendix B!)

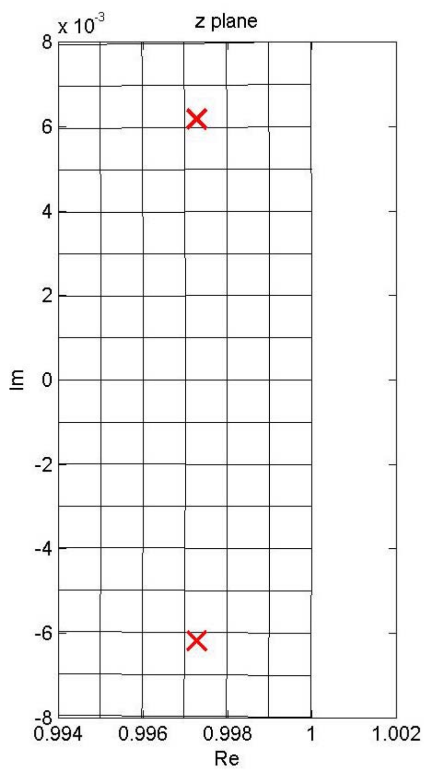
d) Design a DT controller, C(z), with a crossover frequency of 100 rad/s that provides same phase at crossover as the CT design in part c. Your controller should have the form:

$$C(z) = K\frac{(z-a)}{(z-b)} \qquad \text{(Recall T=.001 sec.)}$$

3) Discrete-time system time response analysis.

a) Given $P(z) = \frac{5z^2 - z + k}{z^2}$, solve for the value of k such that the steady state response to a unit step input is zero.

b) Assume P(z) has poles on the z-plane as shown below (plus a zero at the origin), and that the sampling time is T=.001 sec. Estimate t_{peak} and the percent overshoot for a unit step response. (Tip, if you cannot calculate these values, then instead estimate ω_n, ω_d, and ζ for a system with matching poles on the s plane…)



4) For this discrete-time controller: $C(z) = \frac{z-.96}{z-.9975}$, with sample time T=.005 s.

a) At approximately what frequencies (rad/s) are the equivalent CT pole and zero?

b) What is the DC gain of C(z)?

c) Somewhere between the frequency of the equivalent CT pole and zero (from part a), the phase of C(z) has its greatest deviation from 0 degrees. Is the phase change positive or negative? At what frequency (rad/s) on a Bode plot does this occur? And what is (approximately) the peak deviation in phase for C(z)?

d) What type of controller is this, and what is its function? (i.e., in general, why would one use this type of controller?)

5) For a continuous-time state space system,

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

with the following A,B,C, and D matrices:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad A = \begin{bmatrix} -15 & 100 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = [0 \quad 2] \qquad D = [0]$$

a) Write a continuous-time transfer function from input U(s) to output Y(s).

b) Based on the A matrix, is this system stable? (What can you say about the poles…?)

c) Solve for the vector *K* for a state space controller, *u*=-*BK*, such that the resulting poles are the solution to the characteristic equation: $s^2 + 25s + 400 = 0$.

[..continues next page!]

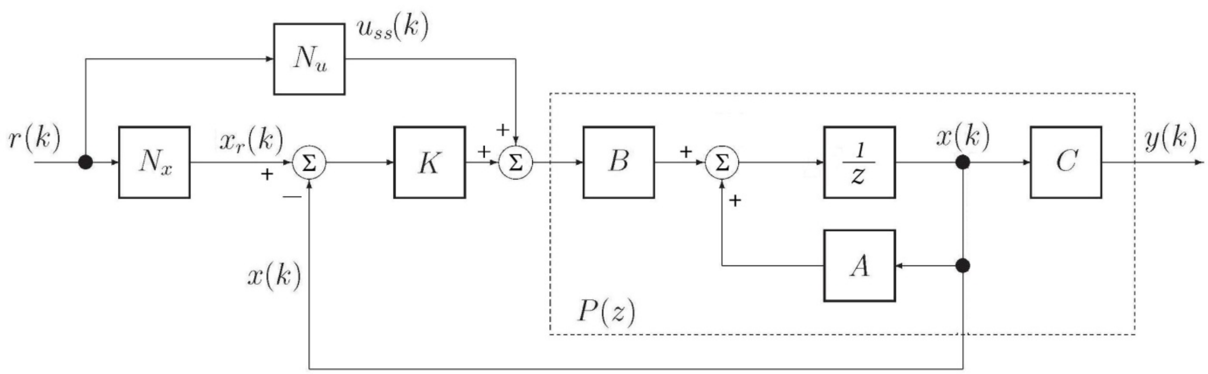
5-d) Assume you now wish to solve for a vector L for a state space estimator. Write out a MATLAB command that will place the estimator poles at pdes=[-30+30j, -30-30j].

e) EXTRA CREDIT: Assume all estimator gains in L are equal to 50 (where L is of appropriate size, from part d). The poles are solutions to what characteristic equation??

6) Below is a block diagram for a state space feedback configuration we have discussed in class this year. Assume the state space matrices have the following dimensionalities:

A (nxn) ; B (nx1) ; C (mxn) ; [D is just all zeros…]

Also assume *r(k)* and *y(k)* match one another in size. (i.e., that *r(k)* gives a desired set of references for the outputs, *y(k)*.)



a) What are the size and function of N_x?

b) What are the size and function of N_u?

c) Note that *r(k)*, the reference input, gives a set of desired outputs for *y(k)*. Write an expression that solves (generally) for N_x to scale a desired reference *r(k)* appropriately. (Do NOT assume any of the matrices are scalars here!) For full credit, show work deriving your answer.

d) Now assume that all the state space matrices are simply scalar values. Reduce the block diagram shown to a single input single output (SISO) transfer function from input *R(z)* to output *Y(z)*. (Hint: use block diagram algebra, or write algebraic equations by hand… More space provided on next page!)

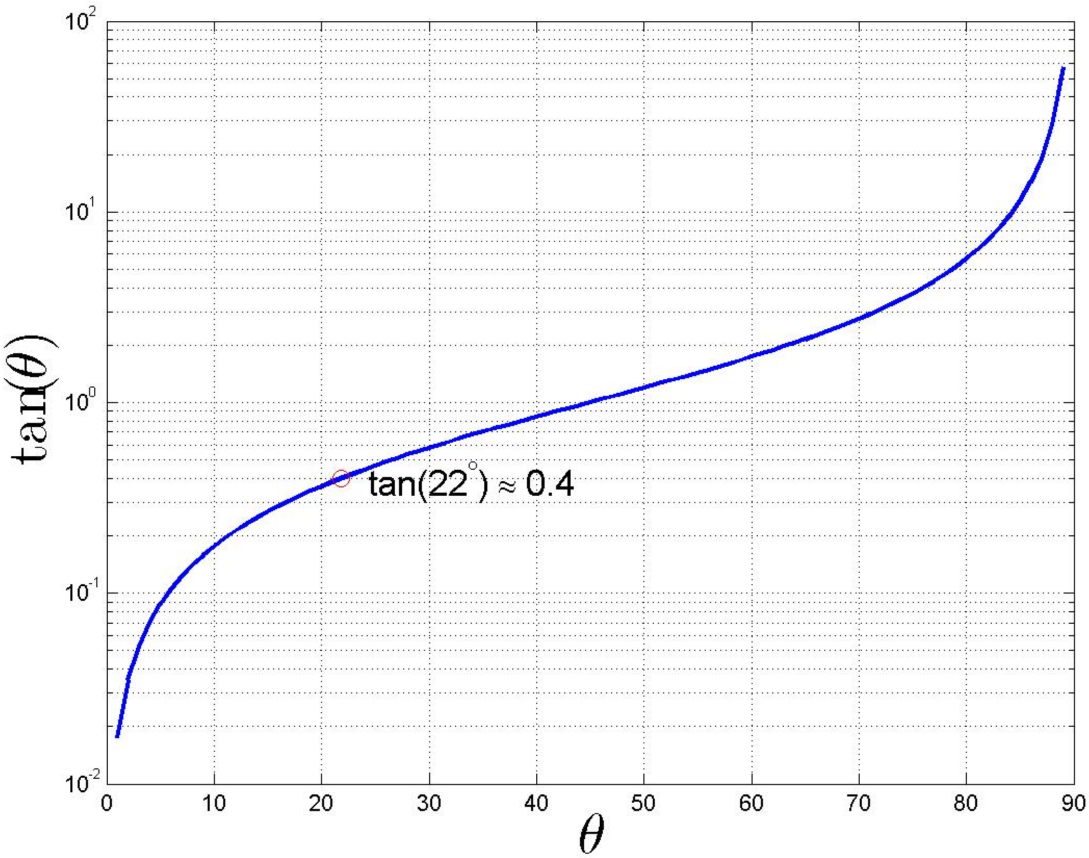
d) [more space provided below for block diagram reduction, etc…]

e) EXTRA CREDIT: Solve for N_u to obtain zero steady state error, in terms of the variables in your transfer function from part d.

End of exam. Have a great Spring Break!

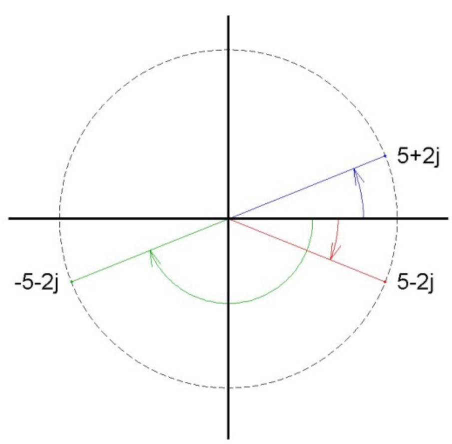
Appendix A: The tangent function.

Note: This Appendix is likely to appear in the actual Final Exam, too, so ***learn to use it now****.*



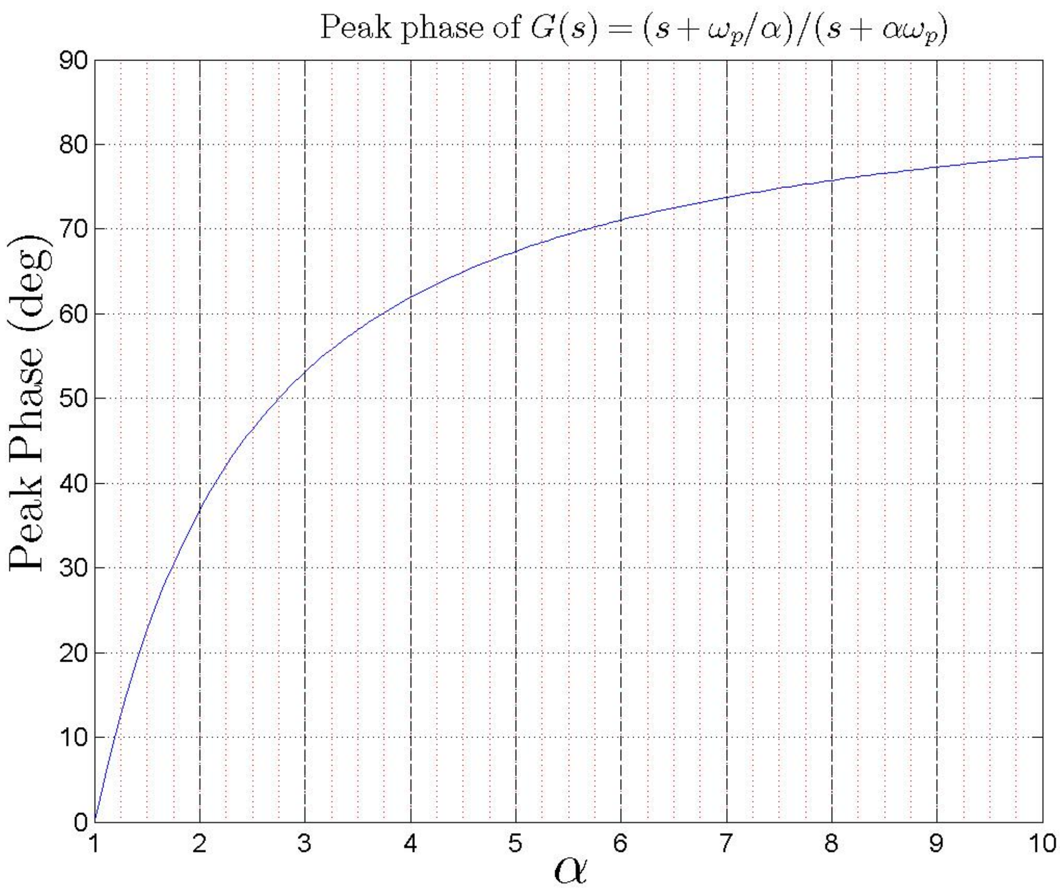
The examples and figure below illustrate how to use semilog plot above:

- The complex number 5+2j has phase: $\text{atan}\left(\frac{2}{5}\right) \approx 22^\circ$.
- The complex number 5-2j has phase: $\text{atan}\left(\frac{-2}{5}\right) \approx -22^\circ$.
- The complex number -5-2j has phase: $\text{atan}\left(\frac{-2}{-5}\right) \approx -180^\circ + 22^\circ = -158^\circ$.



Appendix B: Lead and Lag phase shift prediction.

The figure below is similar to Figure 2.17 in FPW (page 41), except that the definition of "alpha" below is different. (Below, it is the square root of the value in Fig. 2.17 – which makes design a bit easier, since you can avoid taking a square root by using the figure below…)



Example usage. Say we desire a continuous-time compensator to provide 30 degrees of phase. Find 30 degrees on the y axis, and this corresponds to alpha=1.75 (approximately). Say we wish this peak phase to occur at 500 rad/s, then:

$$G(s) = \frac{(s + \frac{500}{1.75})}{(s + 500 \cdot 1.75)} = \frac{(s + 500\frac{4}{7})}{(s + 500\frac{7}{4})} = \frac{(s + \frac{2000}{7})}{(s + \frac{3500}{4})} \approx \frac{(s + 300)}{(s + 900)}$$

For discrete-time controller design, this can then be converted using Tustin (or Tustin with pre-warp, as appropriate/required).