HW 4 – Due in Dropbox, 5pm Monday.

The actual Mid-Term will look nearly identical to this in form.

This practice exam is a slightly-modified version of the Midterm given in ECE147B in 2011.

Name SOLUTIONS

ECE 147B

Practice Midterm Exam

Due 5pm, Feb. 13, 2011

- You are allowed one, single-side 8 ½ x 11 sheet of self-prepared notes.
- Use **must** submit this "Cheat Sheet" with your exam; it will be returned with your graded exam, as is.
- **No calculators, laptops, or other computing devices are allowed.**

____/25	**Problem 1:**	**Discrete-time controller analysis**
____/15	**Problem 2:**	**Tustin transformation**
____/20	**Problem 3:**	**Corresponding s- and z-plane poles**
____/20	**Problem 4:**	**ZOH effects on a plant, P(s)**
____/20	**Problem 5:**	**State space (continuous time) equations**
____/100	**TOTAL**	

Midterm ID number.

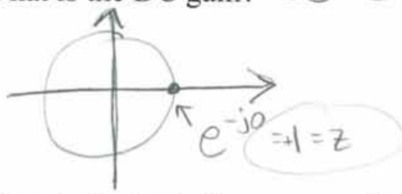
Below are two matched random numbers. Tear off the non-taped number and keep it.

We will post exam grades and statistics using these numbers instead of student names, to hide identities.

(Taped side)	**<u>TEAR AND KEEP TAB BELOW</u>**

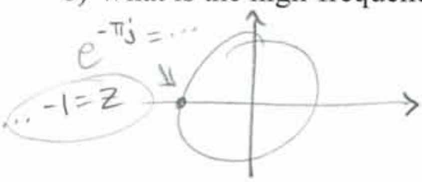
1) For this discrete-time transfer fn: $C(z) = \frac{z-.96}{z-.99}$, with sampling time T=.01 sec.

2 points a) What is the DC gain? $\omega=0$



$\frac{1-.96}{1-.99} = 4$

2 points b) What is the high-frequency gain? $\omega_{Nyquist} = \frac{\pi}{T}$, $\theta = \omega T = \pi$, is "highest" non-aliasing freq.



$\frac{-1-.96}{-1-.99} = \frac{-1.96}{-1.99} \approx .98 \approx 1$

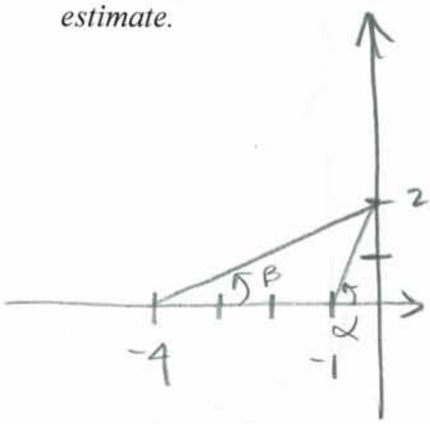
(Alt soln, $z=\infty$, (IVT) is also "OK" here, $\frac{\infty}{\infty}=1$)

5 points c) At *approximately* what frequency does C(z) have the greatest deviation in phase (i.e., away from 0 degrees phase, on a Bode plot)?

$z \to e^{sT}$, & for "small" sT, $s \approx \frac{z-1}{T}$.

$$\left.\begin{array}{l} s_1 \approx \frac{-.04}{.01} = -4 \ \frac{rad}{s} \\ s_2 \approx \frac{-.01}{.01} = -1 \ rad/s \end{array}\right\} \omega_{peak} = \sqrt{4 * 1} = 2 \ rad/s$$

8 points d) Estimate the peak phase shift C(z); that is, the phase shift for a sinusoidal input at the frequency calculated in part c. *You may with to use the tan function in the Appendix to estimate.*



$\angle C(z=2) = \beta - \alpha$

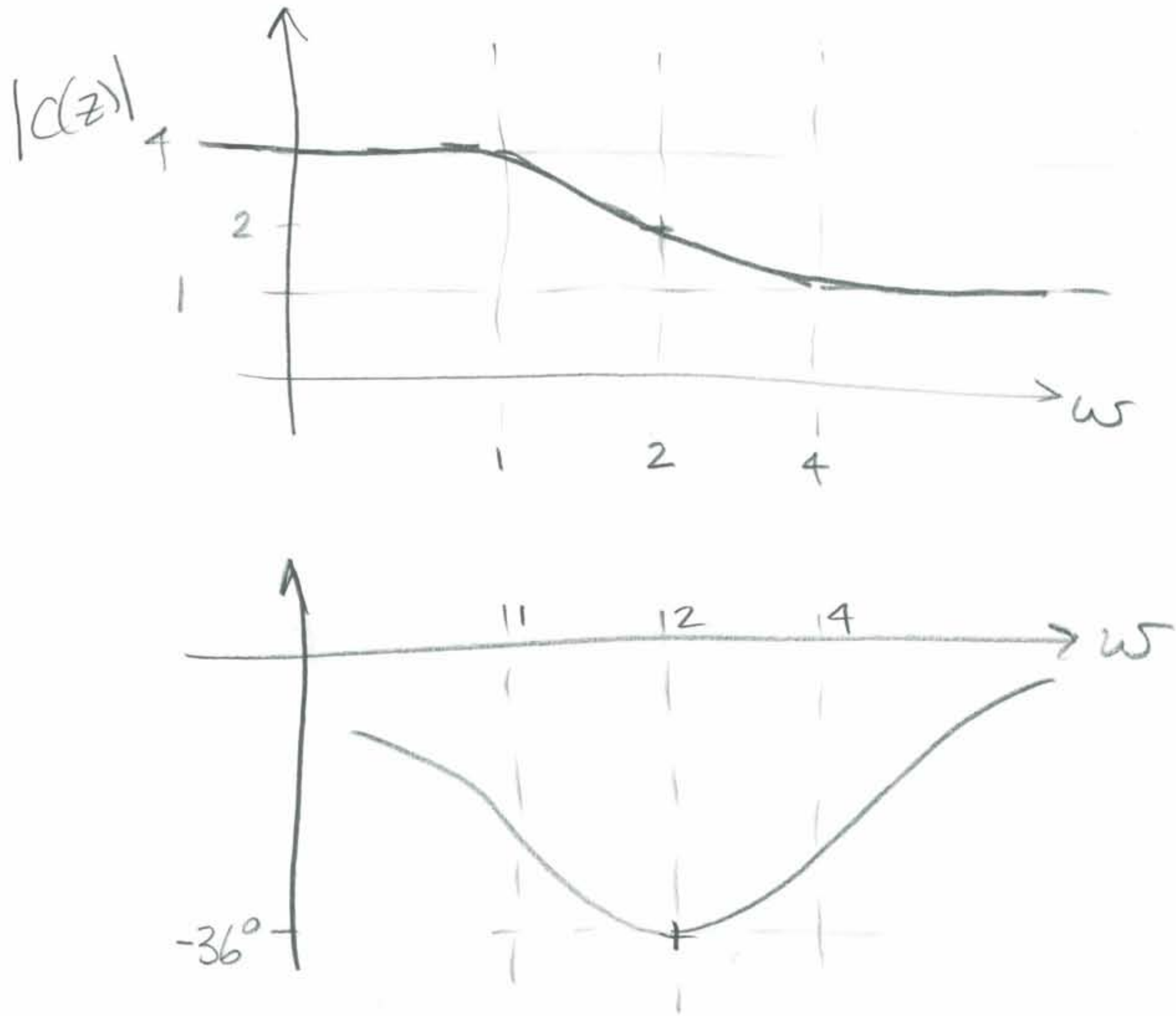
$\boxed{\text{due to zero @ 4 rad/s}}$ $\boxed{\text{due to pole @ 1 rad/s}}$

$$\left.\begin{array}{l} \tan\beta = \frac{2}{4} = .5 \to \beta \approx 27^\circ \\ \tan\alpha = \frac{2}{1} = 2 \to \alpha \approx 63^\circ \end{array}\right\} \alpha+\beta = 90^\circ$$

$\beta - \alpha \approx 27^\circ - 63^\circ = -36^\circ$

e) [Problem 1-e is at the top of the next page…]

8 points e) Sketch an approximate Bode plot for C(z). Label important details with care. (Please use the blank space below to do this.)



2) You are given the following continuous-time controller:

$$C(s) = 10\frac{(s+20)}{(s+100)}$$

10 points a) Use the Tustin transformation (aka bilinear transformation, or trapezoid rule) to convert C(s) to a discrete-time controller, C(z). Leave the sampling time, T, as a variable.

$$S \rightarrow \frac{2(z-1)}{T(z+1)} \Rightarrow C(z) = 10\,\frac{2(z-1)+20T(z+1)}{2(z-1)+100T(z+1)}$$

$$= 10\,\frac{(1+10T)z+(10T-1)}{(1+50T)z+(50T-1)}$$

5 points b) Now, assume T=.001 seconds. Solve numerically for C(z).

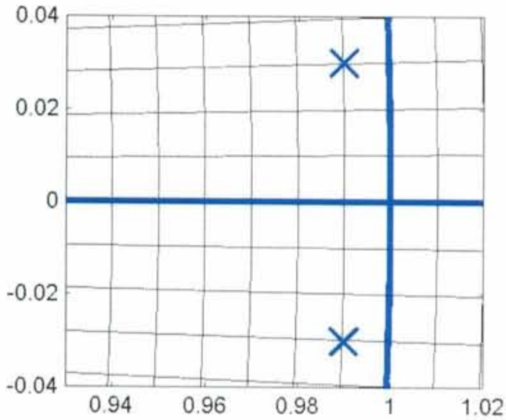
$$C(z) = \frac{(1+.01)z+(.01-1)}{(1+.05)z+(.05-1)} * 10$$

$$= 10\,\frac{(1.01)z-.99}{(1.05)z-.95}$$

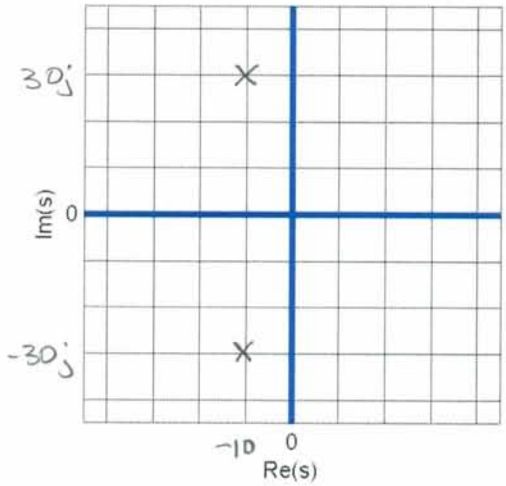
$$= \left(\frac{101}{105}\right)\cdot\left(\frac{z-\left(\frac{.99}{1.01}\right)}{z-\left(\frac{.95}{1.05}\right)}\right)$$

3) The figure at left below shows poles (zoomed in on the unit circle) on the z plane for

$$P(z) = \frac{K}{(z-.99+.03j)(z-.99-.03j)} = \frac{K}{z^2-1.98z+.981}$$, where T=.001 seconds



LABEL AXES, to show scale!



5 points a) Plot ***approximate*** corresponding pole locations on the empty Cartesian axes at the right above, being carefully to label enough of the grid marks on the x and y axes to show scale. (You may or may not find it helpful to complete parts b and/or c first…)

3 points b) Calculate the corresponding locations for poles on the s plane using the forward Euler integration approximation. (That is: $s = \frac{z-1}{T}$, which can also be written as $z = Ts + 1$.) [This should be simple, if you use the factored representation of poles of P(z), top left.]

$$z_1 = (1-.01)+.03j \rightarrow s_1 = \frac{(1-.01)+.03j-1}{.001} = -10+30j$$

$$z_2 = (1-.01)-.03j \rightarrow s_2 = \frac{(1-.01)-.03j-1}{.001} = -10-30j$$

3 points c) Demonstrate that the corresponding locations for poles on the s plane using **<u>one</u>** other method (e.g., backward Euler, Tustin **<u>or</u>** matched) are in "**approximately**" the same location.

BW Euler: $s \rightarrow \frac{z-1}{zT}$. For $|z| \approx 1$, $s \approx \frac{z-1}{1 \cdot T} = \frac{z-1}{T}$

which is what we used above

Matched: $z = e^{sT}$ $\left(\frac{1}{T}\log(z) = s\right)$, w/ $z \approx 1$, $\log(z) \approx z-1$, which yields $s = \frac{1}{T}(z-1)$, which is again what was used in b)

9 points 3-d) Below, sketch (roughly) a unit step response for P(z). Do not worry about capturing the discrete nature of the plot! Instead, focus on calculating *approximate* values for the following three values, which MUST BE LABELED CLEARLY on your plot:

i) Final value (Written in terms of K.)
ii) t_{peak} (Time of maximum overshoot.)
iii) Maximum percent overshoot

$S = -10 \pm 30j$

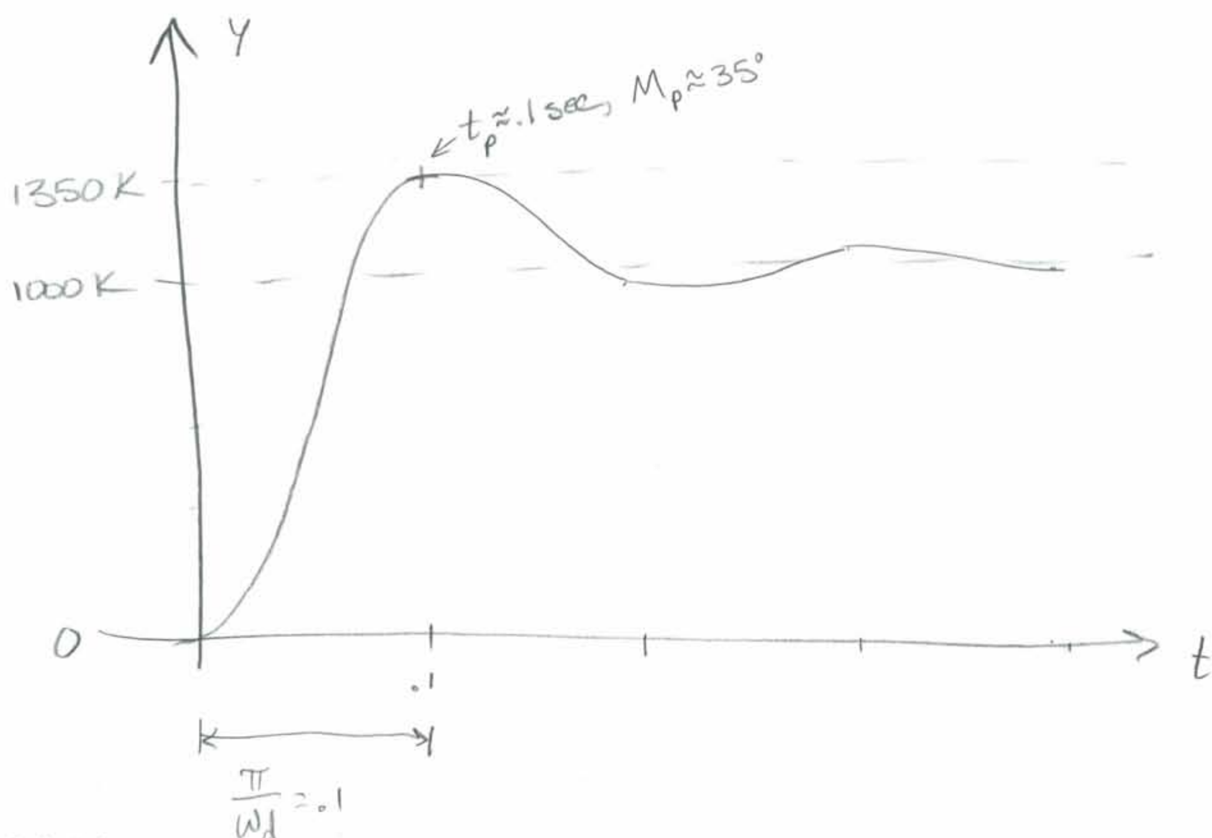
$\sigma = 10$ $\quad \omega_d = 30$

$\tau = \frac{1}{\sigma} = .1$ sec

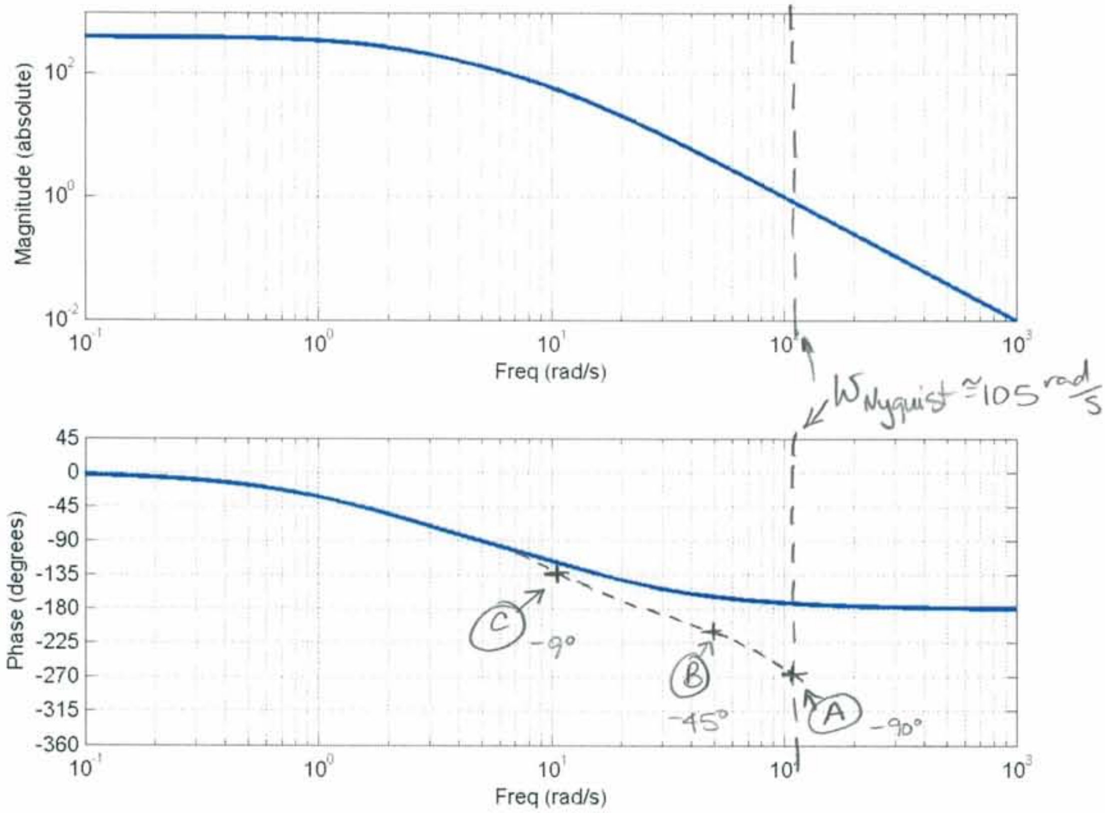
$t_{peak} = \frac{\pi}{\omega_d} \approx \frac{3.14}{30} \approx .1$ sec $\quad t_{peak}$

$M_p = e^{-t_{peak}/\tau} = e^{-.1/.1} \approx e^{-1} = \frac{1}{e} = \frac{1}{2.7} \approx .35$ (35%) $\quad M_p$

$F.V.: \frac{K}{1^2 - 1.98 + .981} = \frac{K}{1.981 - 1.98} = \frac{K}{.001} = 1000K$ $\quad$ F.V.



4) Below is a Bode plot for some system, P(s), which is part of the sampled-data configuration discussed in class, where a zero-order hold precedes the input and the output are sampled, and where the sampling time is T=.03 seconds.



5 points a) What is the Nyquist frequency, $\omega_{Nyquist}$, for the system?

$$\omega_{Ny} = \frac{\pi}{T} \approx \frac{3.14}{.03} \approx 105 \ \frac{rad}{s}$$

6 points b) Write a general expression giving the additional lead or lag due to a **half-sample delay** at frequency $\omega = K\omega_{Nyquist}$, where $0<K<1$? (Leave K as a variable is your answer. Parts a and b have nothing to do with the particular Bode plot shown above.)

@ ω_{Ny}, the delay is $\frac{1}{2}$ sample, & we have 2 samples per cycle...
This means delay is $\frac{1}{4}$ cycle, or $-90°$ (@ ω_{Ny}).

At some lower frequency, delay results in phase shift of $-K \cdot 90°$

9 points c) On the same axes where phase for P(s) is plotted above, sketch a shifted version to represent the phase of P(z). You may assume the phase is simply shifted by a half-sample delay, as in part b. **Draw only the phase; not gain.** Label these 3 points "A", "B", "C":

A) Frequency and phase at $\omega_{Nyquist}$.
B) Frequency and phase at $\frac{1}{2}\omega_{Nyquist}$.
C) Frequency and phase at $\frac{1}{10}\omega_{Nyquist}$.

(see above...)

5) Recall the continuous time state space representation of a dynamic system, given below:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

In this problem, $\boldsymbol{x}$, $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, and $\boldsymbol{D}$ are defined as:

$$x = \begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix} \qquad A = \begin{bmatrix}0 & 1 \\ 12 & -1\end{bmatrix} \qquad B = \begin{bmatrix}0 \\ 2\end{bmatrix}$$

$$C = [3 \quad 0] \qquad D = [0]$$

4 points a) Write a differential equation relating u to θ.

$\ddot{\theta} = 12\theta - \dot{\theta} + 2u$

$\ddot{\theta} + \dot{\theta} - 12\theta = 2u$

4 points b) What is the transfer function from u to y? (**Note this is to output, y**; not to θ…)

$y = 3\theta, \therefore \frac{Y}{u} = 3\frac{\theta}{u}; \frac{\theta(s)}{U(s)} = \frac{2}{s^2+s-12}; \therefore \frac{Y(s)}{U(s)} = \frac{6}{s^2+s-12}$

6 points c) Write out the characteristic equation from $det\{sI - A\} = 0$. (Hint this should match the denominator of the transfer function…) What are the poles of the system?

$\det(sI - A) = \det\left(\begin{bmatrix}s & 0 \\ 0 & s\end{bmatrix} - \begin{bmatrix}0 & 1 \\ 12 & -1\end{bmatrix}\right) = \det\begin{pmatrix}s & -1 \\ -12 & (s+1)\end{pmatrix} = 0$

$s(s+1) - 12 = 0 \rightarrow s^2 + s - 12 = 0$

Poles: $s = -\frac{1}{2} \pm \frac{1}{2}\sqrt{1+48}$

$= -\frac{1}{2} \pm \frac{1}{2}\cdot 7$

$s_1 = -4, s_2 = 3$

2 points d) Is the open-loop system stable? (Why?)

No. Pole @ $s = +3$ is in RHP.

4 points e) What is its final value to a step response?

Undefined! This is a TRICK QUESTION, since unstable!

Extra Credit f) Solve for K_1 and K_2 for a feedback regulator of the form $\boxed{u = -BKx}$

u=-Kx (typo!)
det{A-BK}=0
(new poles)

to place the poles at $s = -10 \pm 10j$

$\dot{x} = Ax - BKx$
$= (A - BK)x$

char. eqn should be:

$s^2 + 20 + 200 = 0$

$10j$, -10, $\omega_n = \sqrt{2}\cdot 10$, $\omega_n^2 = 200$

$A - BK = A - \begin{bmatrix}0 \\ 2\end{bmatrix}[K_1 \quad K_2] = \begin{bmatrix}0 & 1 \\ 12-2K_1 & -1-2K_2\end{bmatrix}$

$-12 + 2K_1 = 200$

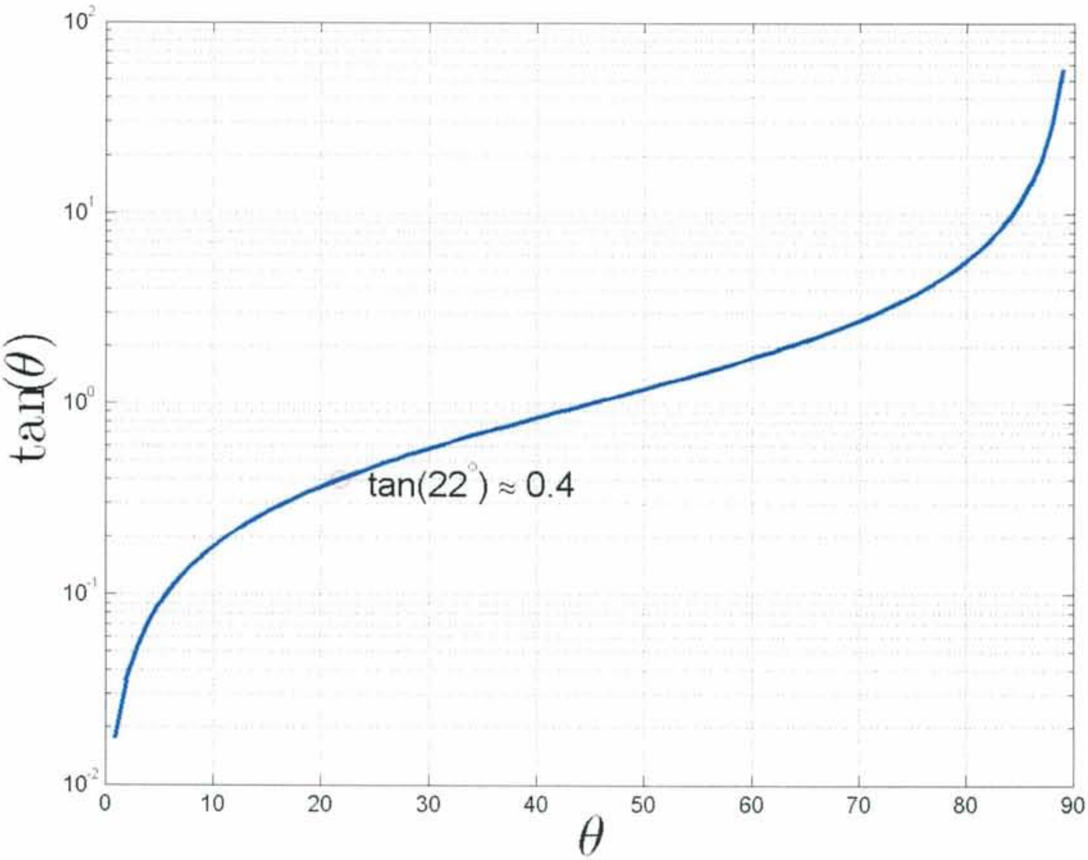
$K_1 = \frac{212}{2} = 106$

$1 + 2K_2 = 20$

$K_2 = \frac{19}{2} = 9.5$

Appendix: The tangent function.

Note: This Appendix is likely to appear in the actual Midterm, too, so ***learn to use it now*** *for problems involving phase calculations for both CT and DT transfer functions.*



The examples and figure below illustrate how to use semilog plot above:

- The complex number 5+2j has phase: $\text{atan}\left(\frac{2}{5}\right) \approx 22^\circ$.
- The complex number 5-2j has phase: $\text{atan}\left(\frac{-2}{5}\right) \approx -22^\circ$.
- The complex number -5-2j has phase: $\text{atan}\left(\frac{-2}{-5}\right) \approx -180^\circ + 22^\circ = -158^\circ$.