MODULE-8 SOLUTION

1 Problem 1

In this problem, we will try to improve the sample design on the web by picking (dominant) poles of the observer and controller closer to the origin. For the observer, now we pick poles at $[0, 0.2]$ (instead of $[0.1, 0.6]$), which gives the observer matrix:

$$L = \begin{bmatrix} 1.7512 \\ 0.7326 \end{bmatrix}$$

Similarly, for the controller, we want the system become faster. However, the faster it is, the larger control gain we have. Given condition that not using much more control effort than used in the Module 7, we pick the close loop poles at [0, 0.3] (instead of [0.7 0.8]) (trying to keep the control signal less than 1), which gives:

$$K = \begin{bmatrix} 61.5125, 128.0208 \end{bmatrix}$$

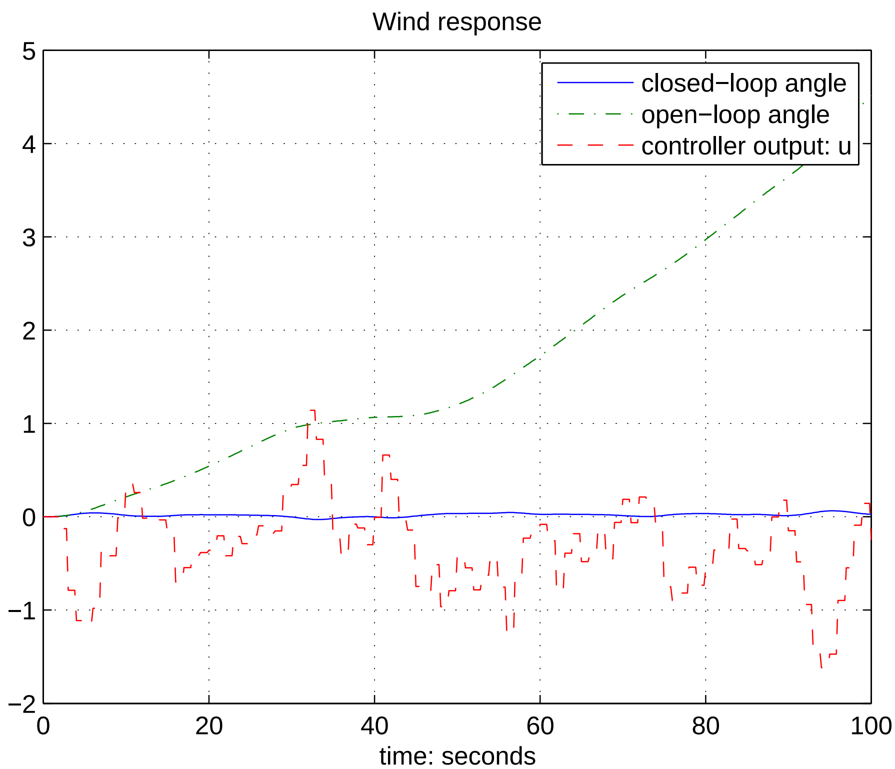


Figure 1: Control performance of modified observer and controller

We see that the system performance is better with not much more control effort.

2 Problem 2

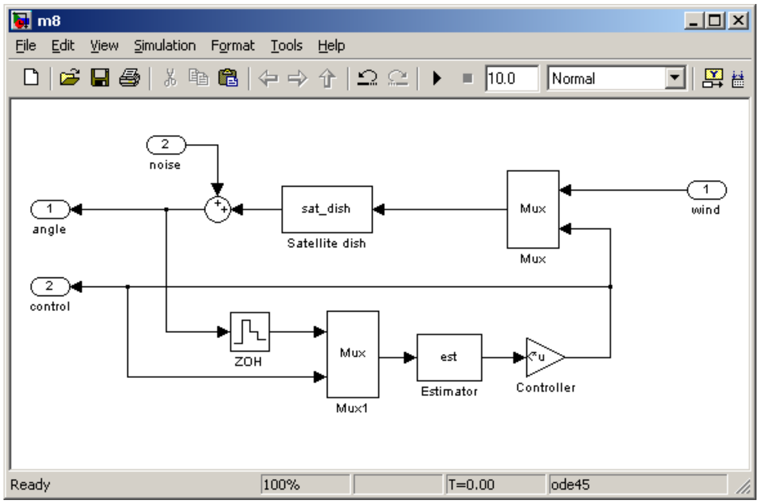


Figure 2: System with noise configuration

2.1 Full-order observer

First, try to use the above designed system with presence of noise:

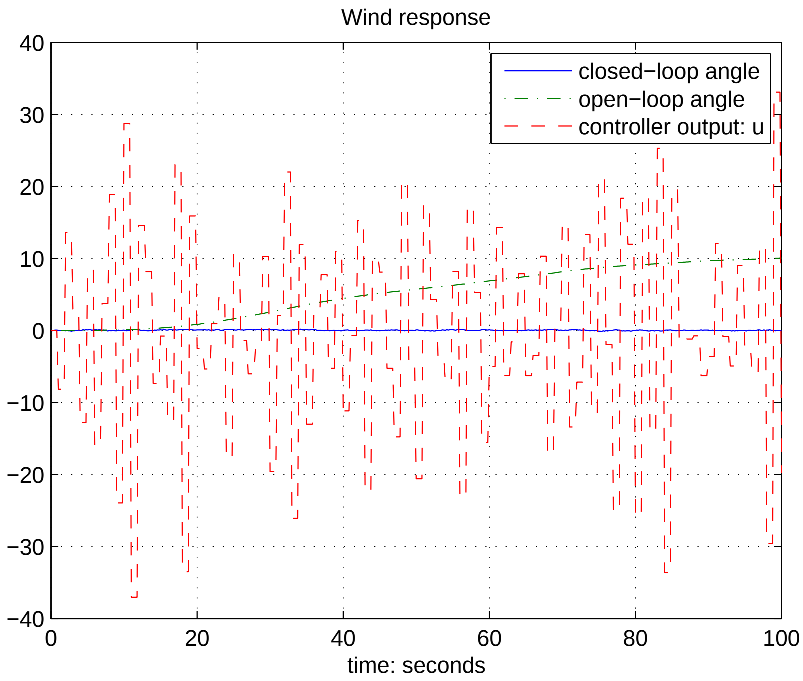


Figure 3: Previous design with presence of noise

It seems that the closed-loop angle is quickly stablized. However, the control effort is huge in comparision with previous case, so huge that the system cannot afford. To improve, we make the observer slower with poles at $[0.5, 0.6]$ and place the poles of the close loop system $[0.6, 0.7]$, which gives:

$$L = \begin{bmatrix} 0.8512 \\ 0.1625 \end{bmatrix} \text{ and } K = \begin{bmatrix} 12.3025, 60.5621 \end{bmatrix}$$

With this controller, system response:

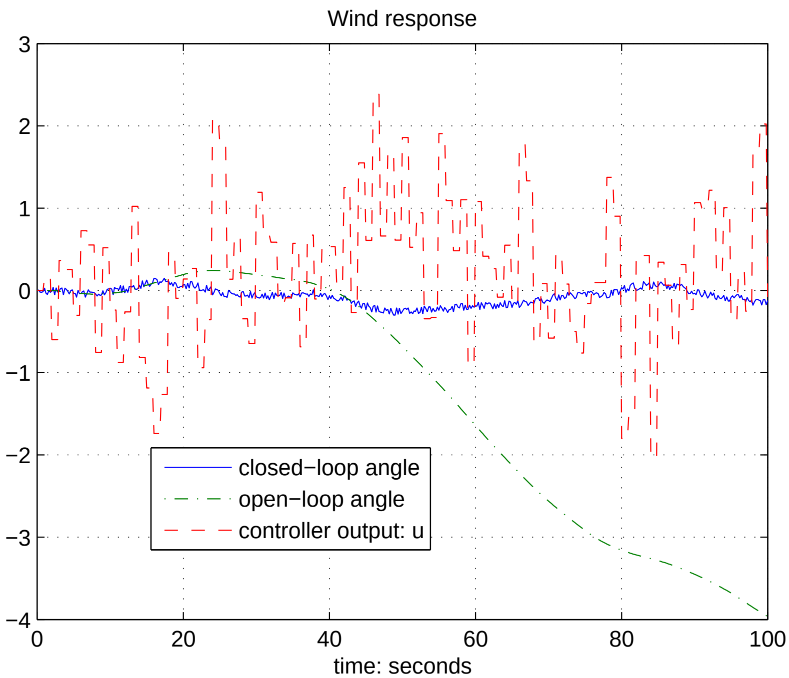


Figure 4: New design for full order case

2.2 Reduced-order observer

First, let check the performance of the sample design under noise. This design has pole for observer at 0.5 and poles for closed loop system at $[0.7, 0.8]$. We can see that noise make the control effert become larger.

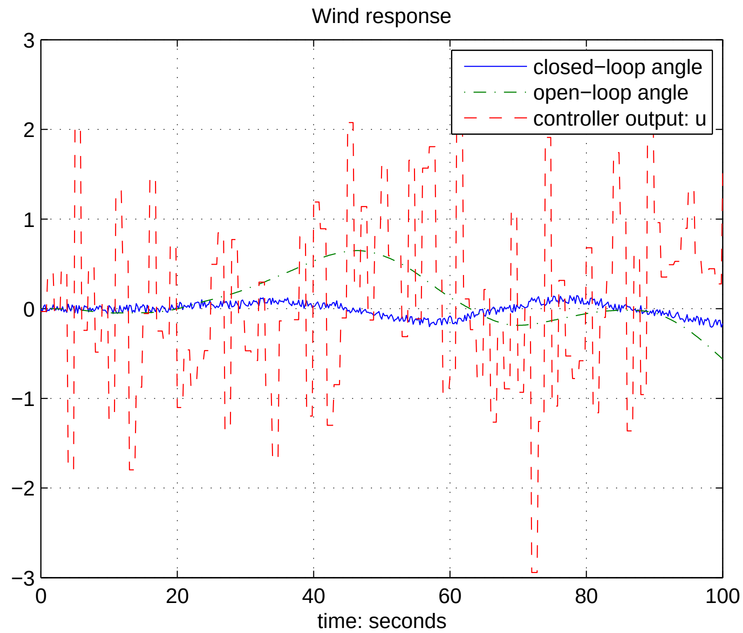


Figure 5: Sample Design under noise

To reduce the effect of noise, we make a observer little slower by picking its poles 0.65 (instead of 0.5), and the closed loop system poles at [0.6, 0.7]. Now the system response:

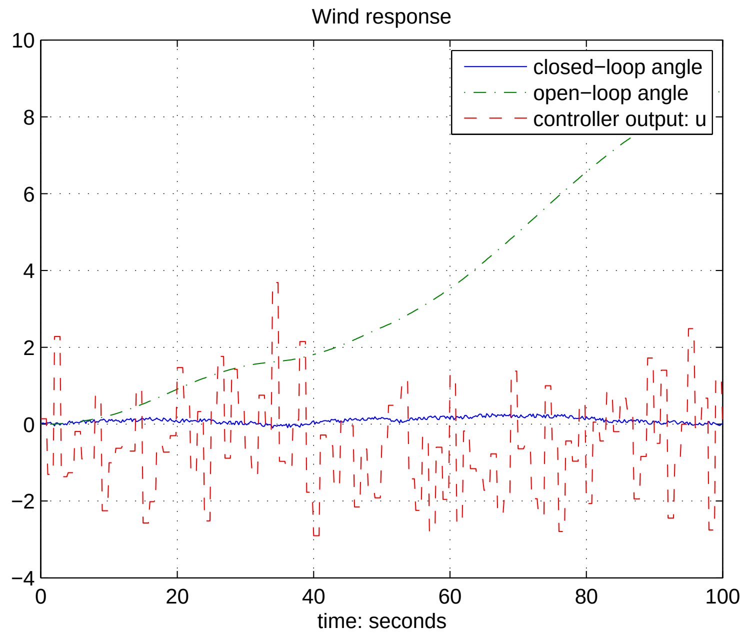
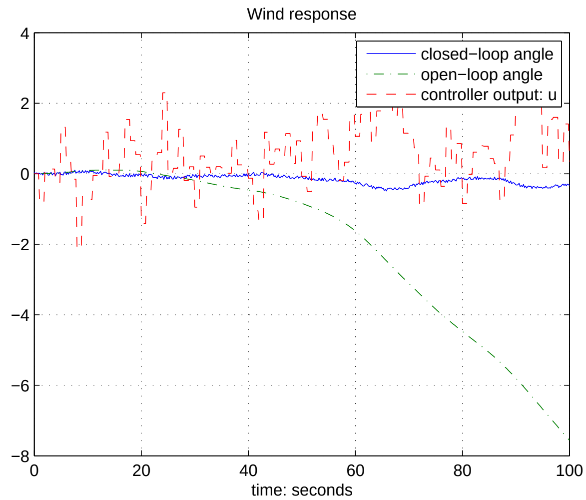


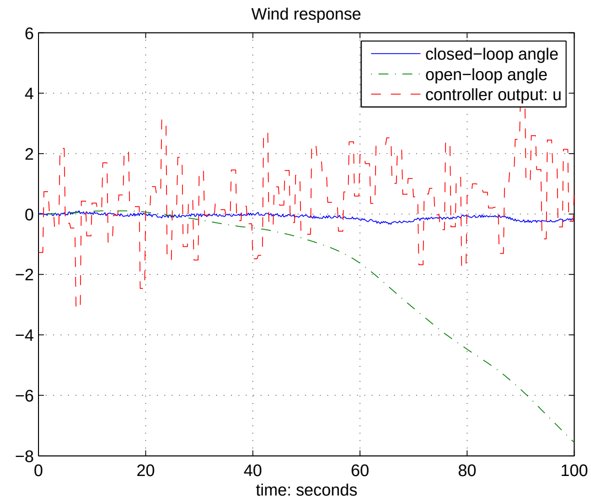
Figure 6: New design for reduce order case

3 Problem 3

In order to compare full order with reduce order designs, we simulate both desing with diffrent sample times: $T = 1, 2, 3$ s):

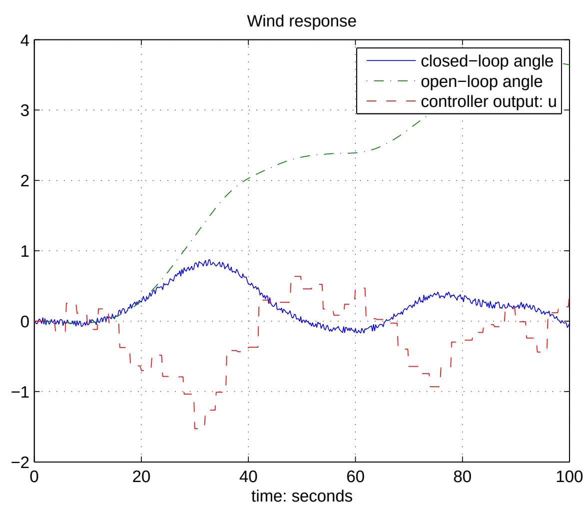


(a) Using full order observer

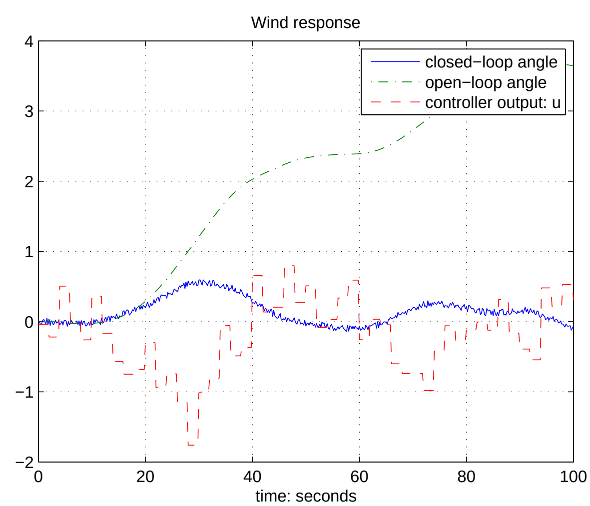


(b) Using reduced order observer

Figure 7: Sample time $T = 1s$

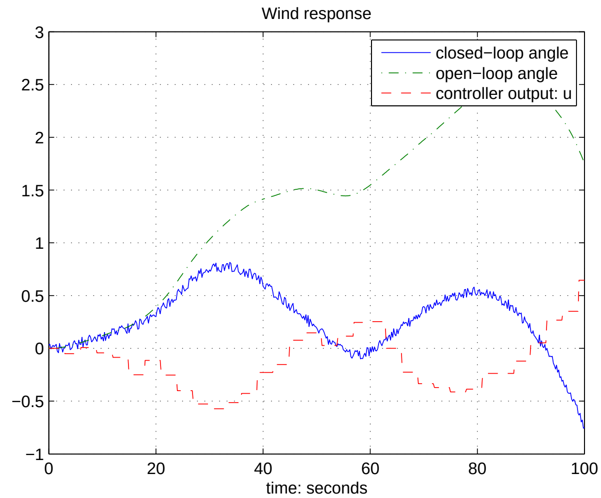


(a) Using full order observer

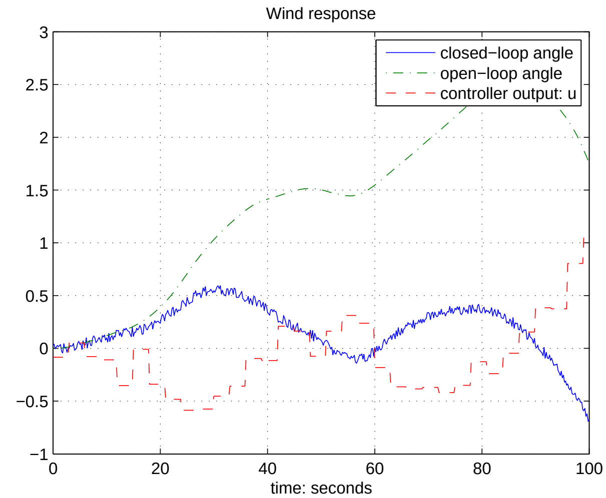


(b) Using reduced order observer

Figure 8: Sample time $T = 2s$



(a) Using full order observer



(b) Using reduced order observer

Figure 9: Sample time $T = 3s$

From above simulations, we can see that using reduced order observer give us better performance (Note that in 2 cases, we use have the same controller matrix).

4 Problem 4

Now we have to deal with the tracking problem in which we have to find N_x and N_u matrices.

$$\left\{ \begin{array}{c} C_d N_x = I \\ (A_d - I)N_x + B_d N_u = 0 \end{array} \right. \Rightarrow \left\{ \begin{array}{ccc} N_x & = & [1,0]^T \\ N_u & = & 0 \end{array} \right. \Rightarrow \bar{N} = N_u + KN_x = KN_x$$

Now using the control design in problem 3 for this structure:

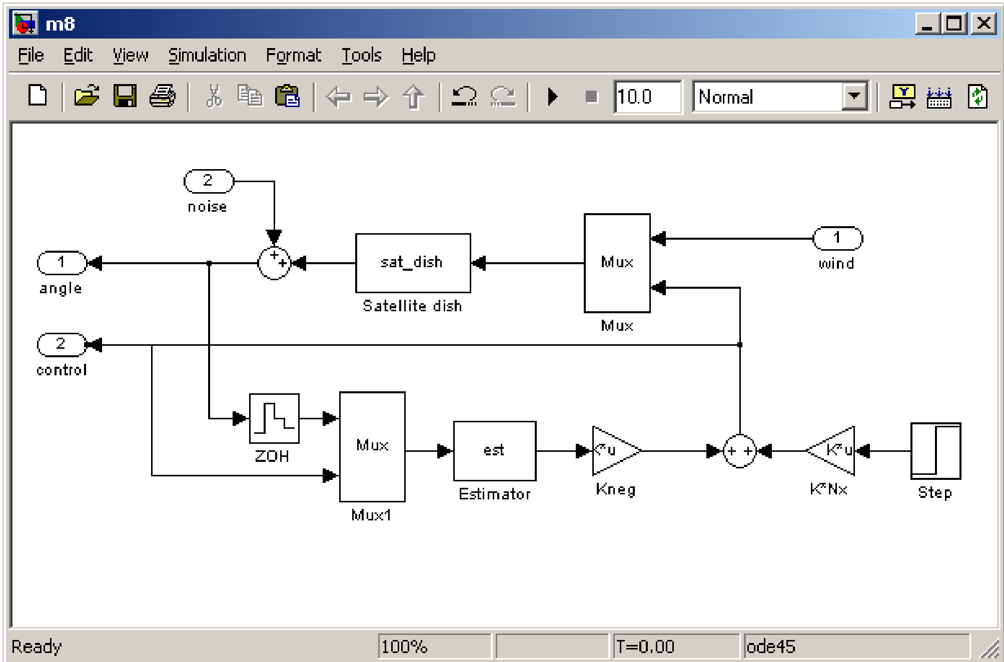


Figure 10: Reference tracking with full order estimate

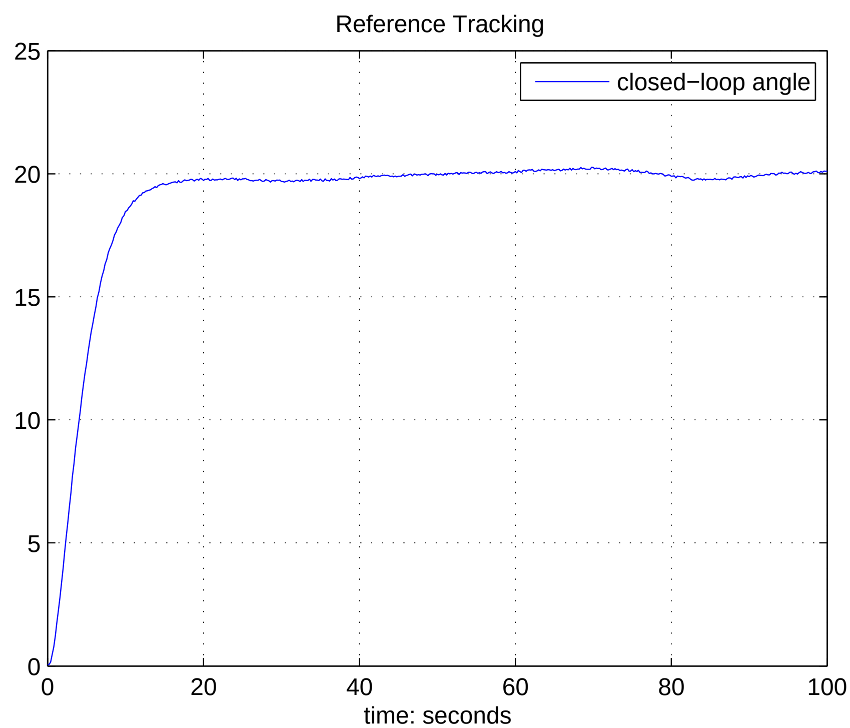


Figure 11: Reference tracking with full order estimate

5 Problem 5

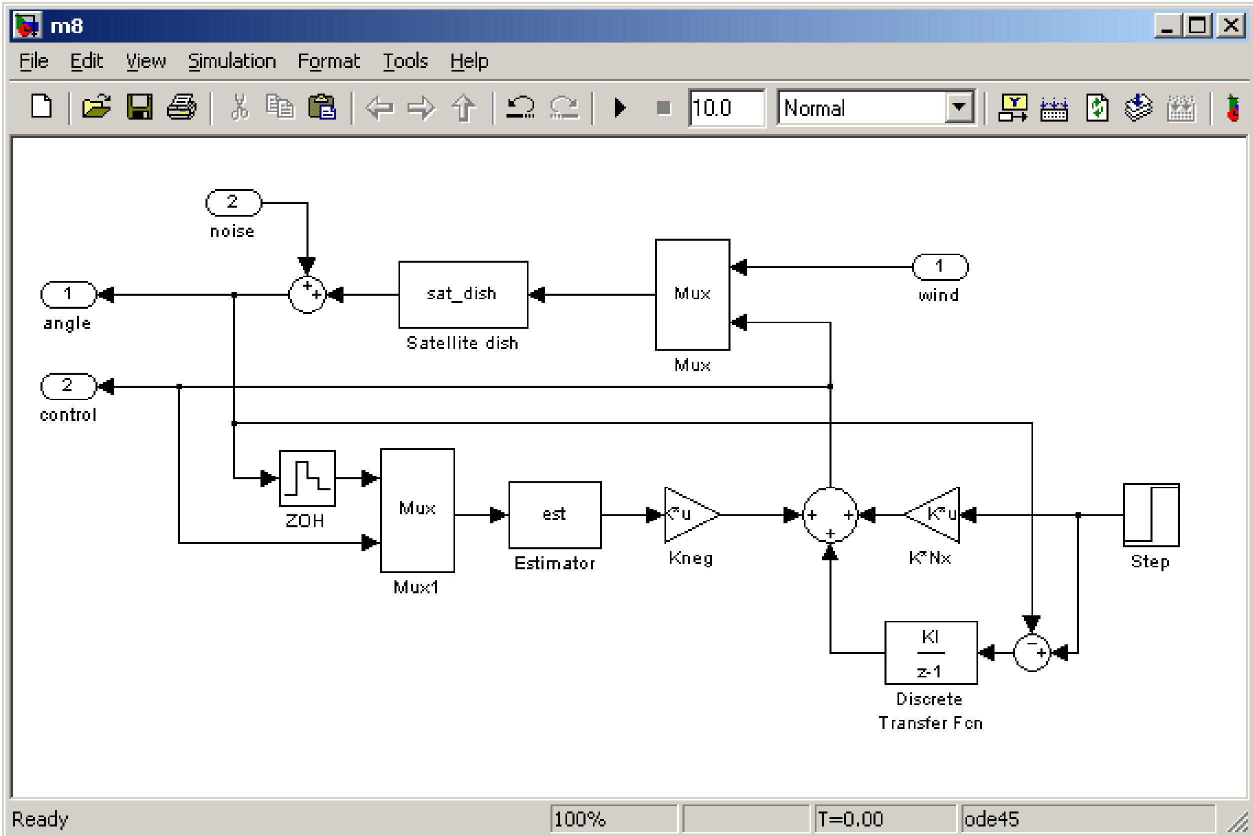


Figure 12: Reference tracking / Reduced order with integral controller

Now we still have: $\begin{cases} N_x &= [1,0]^T \\ N_u &= 0 \end{cases}$

However, in order to find K_I, we need to calculate the augmented plant model:

$$\bar{A} = \begin{bmatrix} 1 & C_d \\ 0 & A_d \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0.9754 \\ 0 & 0 & 0.9512 \end{bmatrix} \text{ and } \bar{B} = \begin{bmatrix} 0 \\ B_{dTC} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.0049 \\ 0.0098 \end{bmatrix}$$

$$\text{Choose poles: } p = [0.5, 0.6, 0.7]$$

$$\bar{K} = place(\bar{A}, \bar{B}, p) = [6.1512, 45.0835, 95.2954]$$

$$\Rightarrow K_I = 6.1512 \text{ and } K = [45.0835, 95.2954]$$

Simulate this controller with above structure:

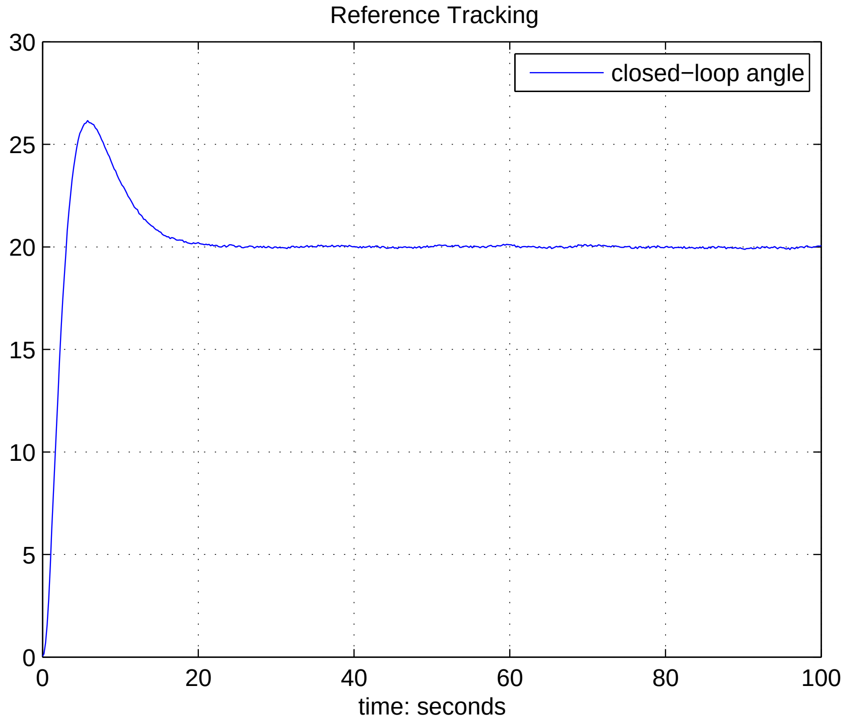


Figure 13: Reference tracking with full order estimate

We see that with the additional integral controller, the system is much faster. But it also introduces overshoot.