MODULE-8 SOLUTION

1 Problem 1

In this problem, we will try to improve the sample design on the web by picking (dominant) poles of the observer and controller closer to the origin. For the observer, now we pick poles at $[0, 0.2]$ (instead of $[0.1, 0.6]$), which gives the observer matrix:

$$L = \begin{bmatrix} 1.7512 \\ 0.7326 \end{bmatrix}$$

Similarly, for the controller, we want the system become faster. However, the faster it is, the larger control gain we have. Given condition that not using much more control effort than used in the Module 7, we pick the close loop poles at [0, 0.3] (instead of [0.7 0.8]) (trying to keep the control signal less than 1), which gives:

$$K = \begin{bmatrix} 61.5125, 128.0208 \end{bmatrix}$$

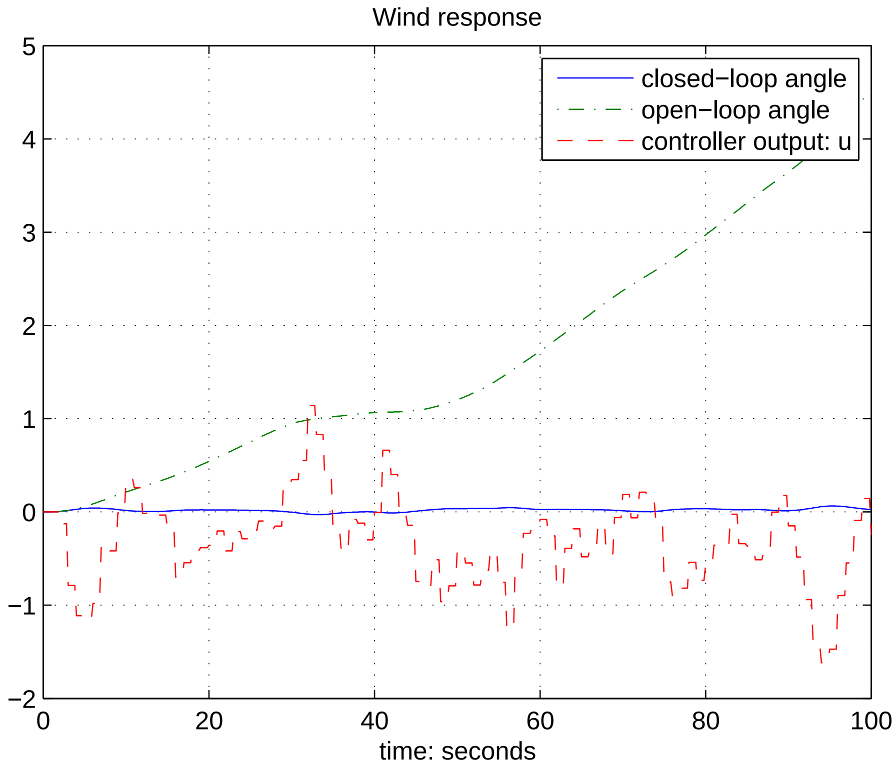


Figure 1: Control performance of modified observer and controller

We see that the system performance is better with not much more control effort.

2 Problem 2

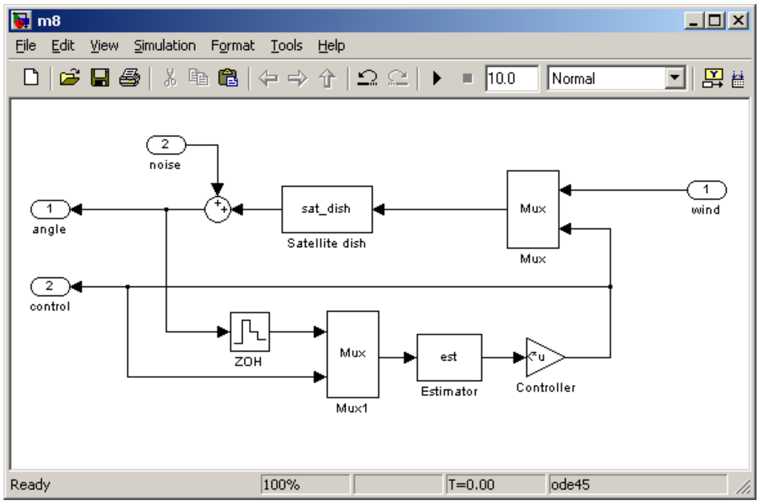


Figure 2: System with noise configuration

2.1 Full-order observer

First, try to use the above designed system with presence of noise:

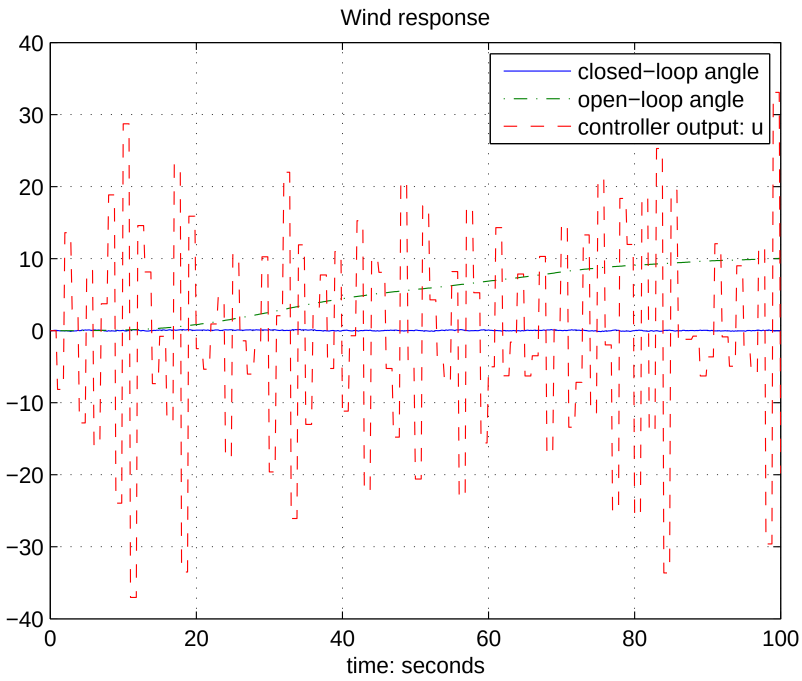


Figure 3: Previous design with presence of noise

It seems that the closed-loop angle is quickly stablized. However, the control effort is huge in comparision with previous case, so huge that the system cannot afford. To improve, we make the observer slower with poles at $[0.5, 0.6]$ and place the poles of the close loop system $[0.6, 0.7]$, which gives:

$$L = \begin{bmatrix} 0.8512 \\ 0.1625 \end{bmatrix} \text{ and } K = \begin{bmatrix} 12.3025, 60.5621 \end{bmatrix}$$

With this controller, system response:

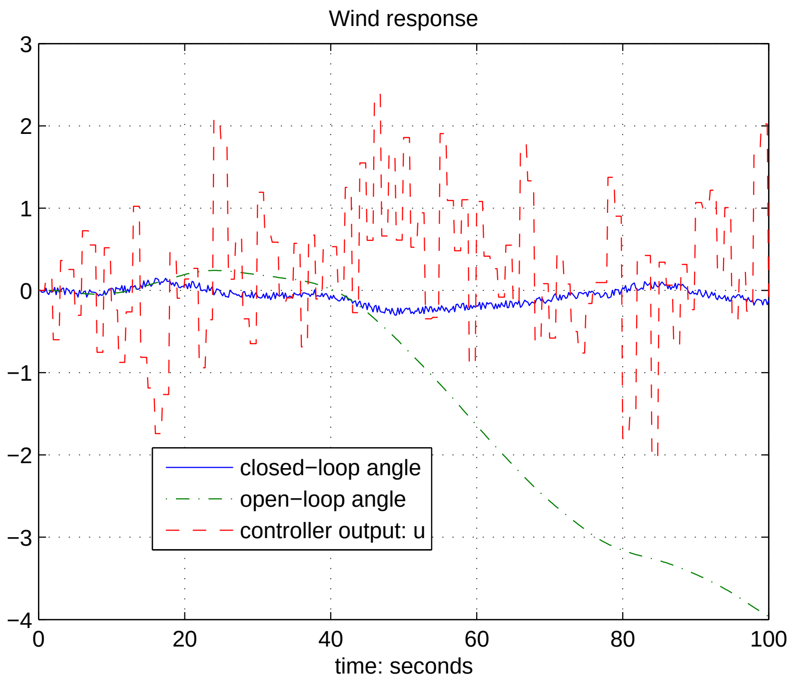


Figure 4: New design for full order case

2.2 Reduced-order observer

First, let check the performance of the sample design under noise. This design has pole for observer at 0.5 and poles for closed loop system at $[0.7, 0.8]$. We can see that noise make the control effert become larger.

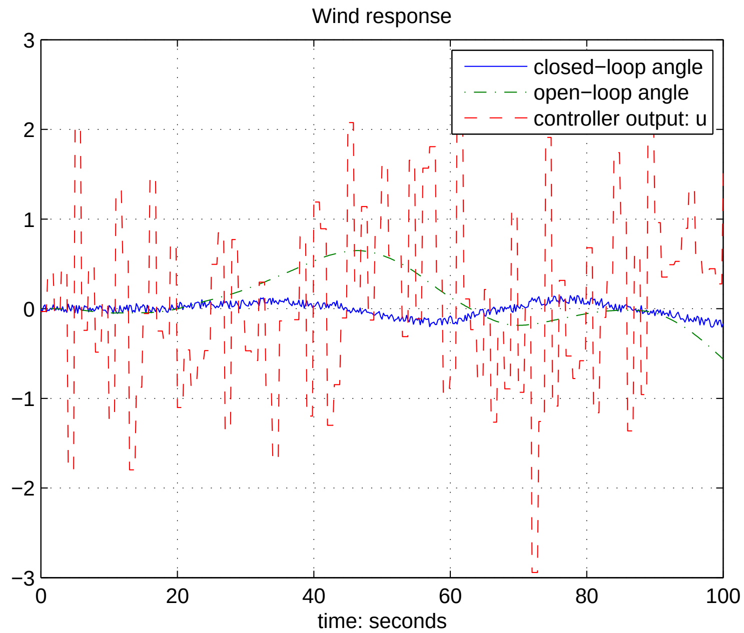


Figure 5: Sample Design under noise

To reduce the effect of noise, we make a observer little slower by picking its poles 0.65 (instead of 0.5), and the closed loop system poles at [0.6, 0.7]. Now the system response:

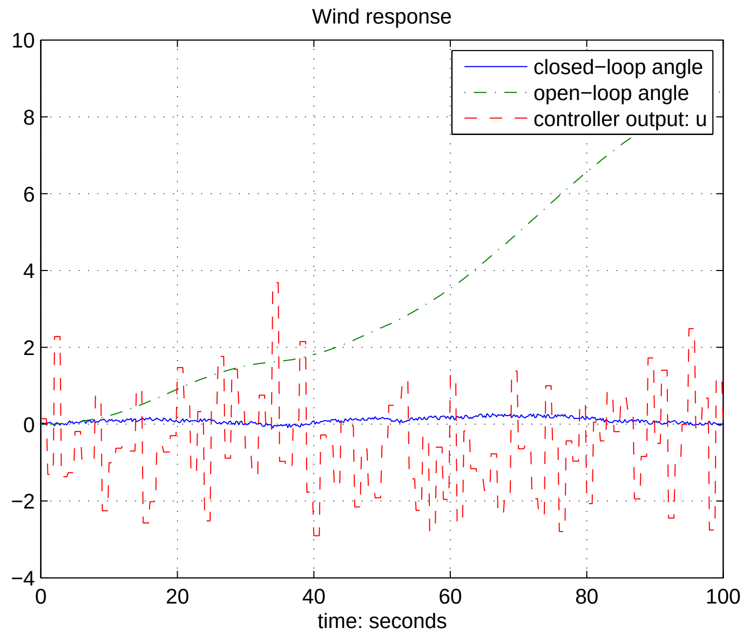
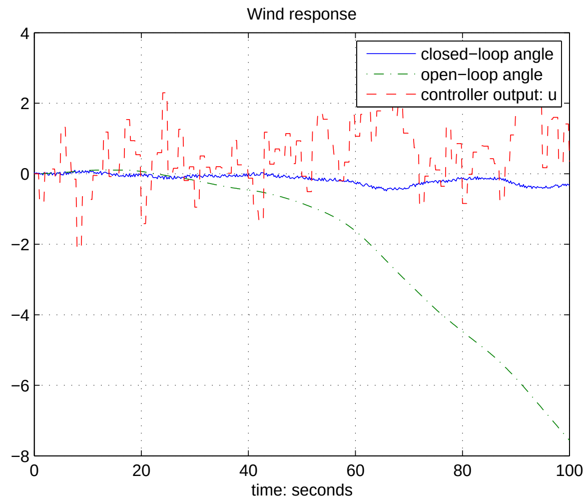


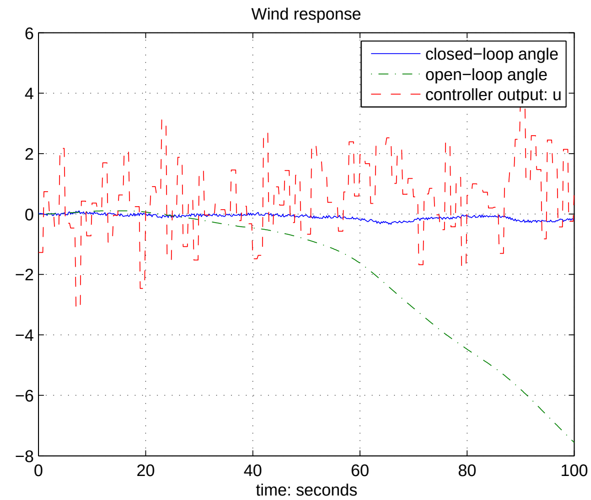
Figure 6: New design for reduce order case

3 Problem 3

In order to compare full order with reduce order designs, we simulate both desing with diffrent sample times: $T = 1, 2, 3$ s):

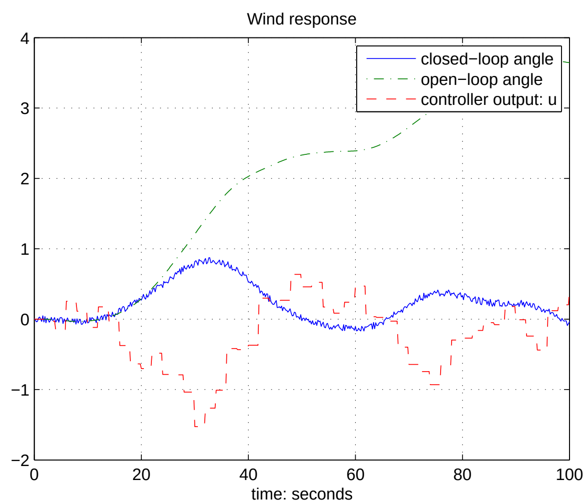


(a) Using full order observer

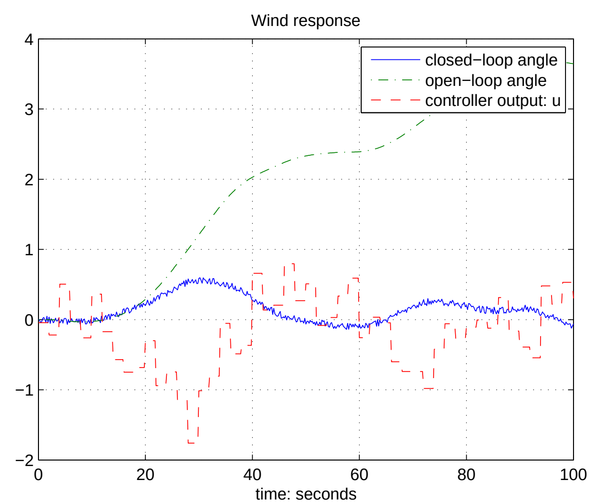


(b) Using reduced order observer

Figure 7: Sample time $T = 1s$

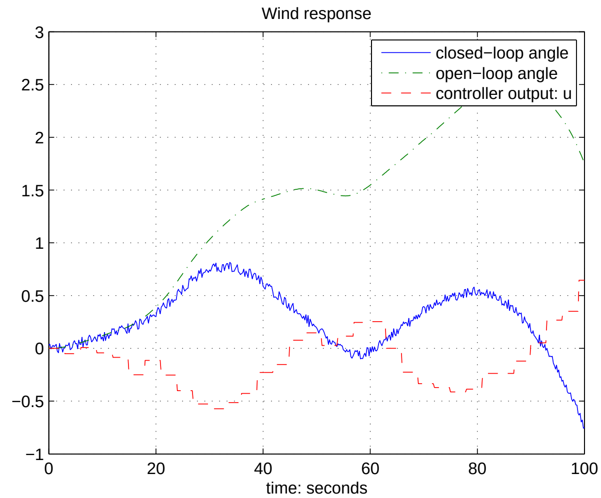


(a) Using full order observer

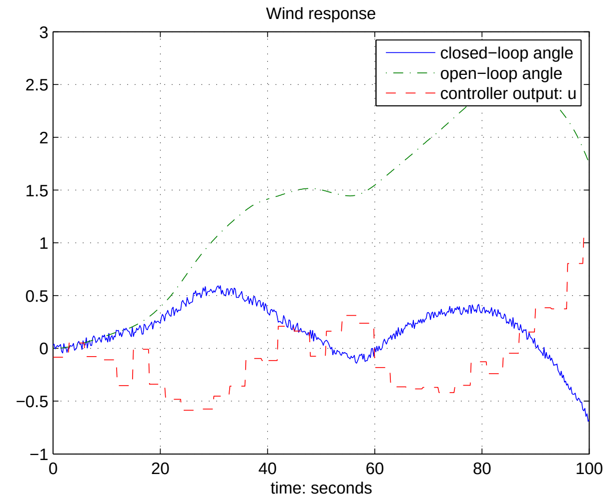


(b) Using reduced order observer

Figure 8: Sample time $T = 2s$



(a) Using full order observer



(b) Using reduced order observer

Figure 9: Sample time $T = 3s$

From above simulations, we can see that using reduced order observer give us better performance (Note that in 2 cases, we use have the same controller matrix).

4 Problem 4

Now we have to deal with the tracking problem in which we have to find N_x and N_u matrices.

$$\left\{ \begin{array}{c} C_d N_x = I \\ (A_d - I)N_x + B_d N_u = 0 \end{array} \right. \Rightarrow \left\{ \begin{array}{lcl} N_x & = & [1,0]^T \\ N_u & = & 0 \end{array} \right. \Rightarrow \bar{N} = N_u + KN_x = KN_x$$

Now using the control design in problem 3 for this structure:

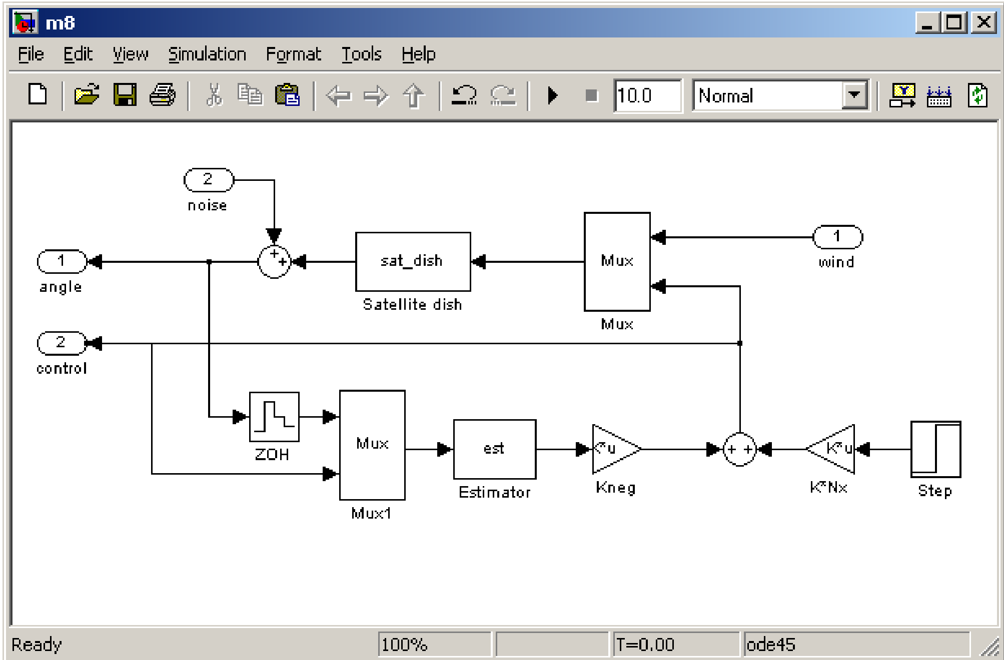


Figure 10: Reference tracking with full order estimate

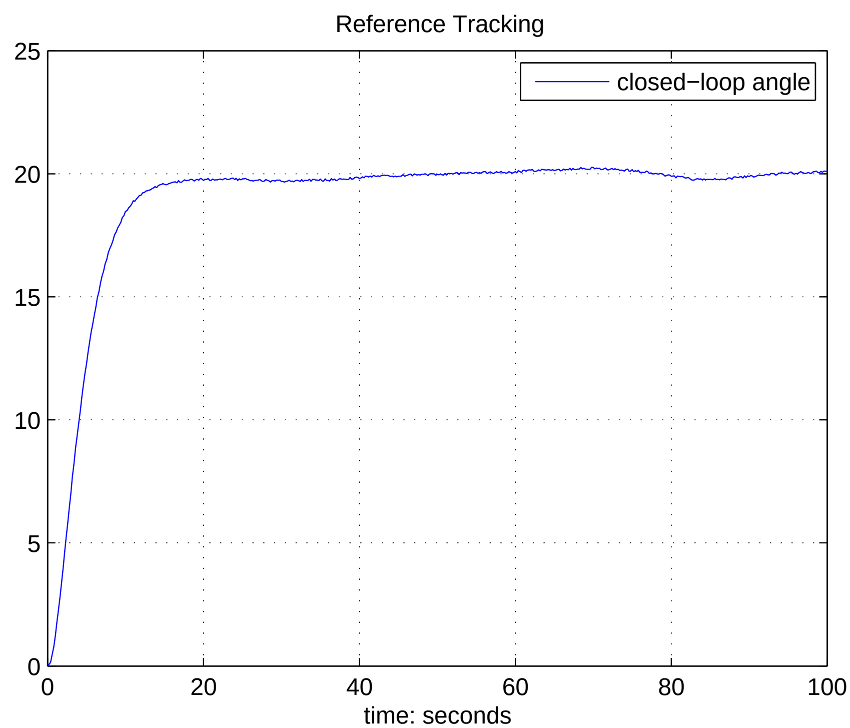
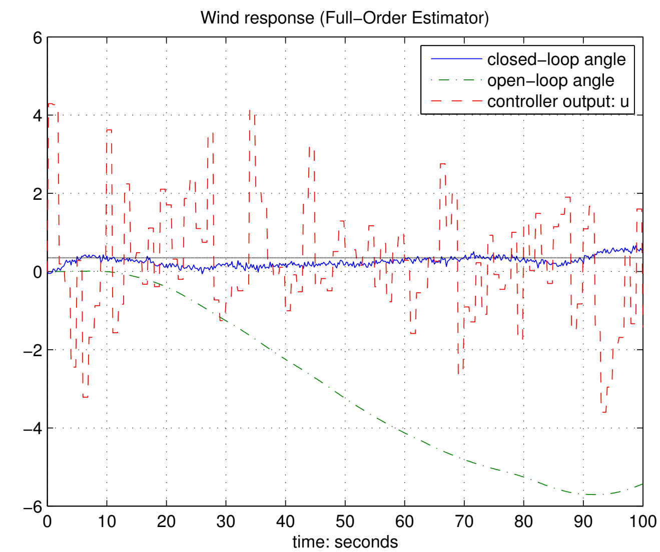


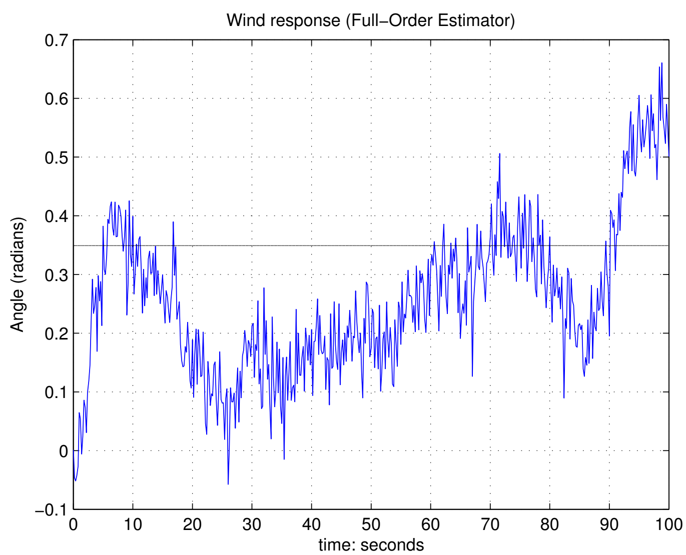
Figure 11: Reference tracking with full order estimate

Problem 4 - UPDATE

The output (angle) of the system is measured in radians, rather than degrees, so, for a 20 degree step input, we should be tracking a 0.3491 radian step input. Using the same pole locations (and K and L matrices) as in Problem 2 for the full-order estimator, we obtain the following results.



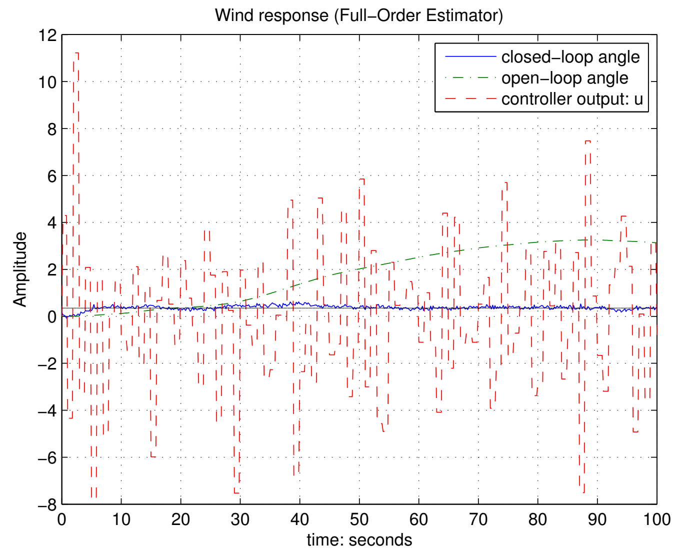
(a) Plot of the closed-loop angle, open-loop angle, and the control effort.



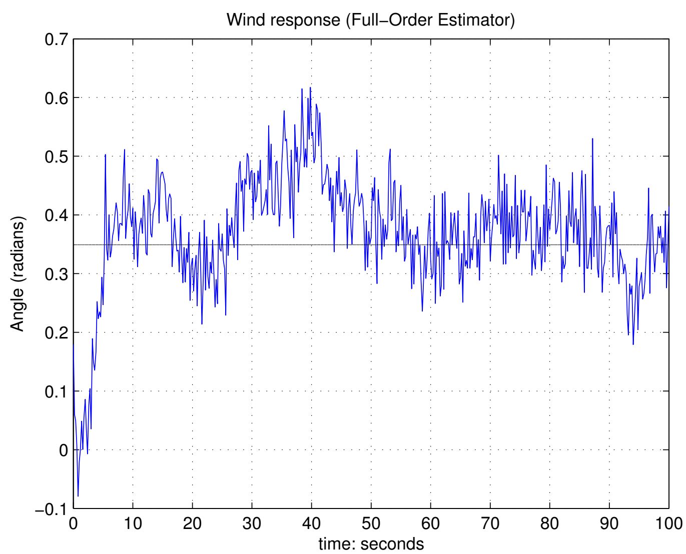
(b) Plot of the closed-loop angle only.

Figure 1: Response of system to a 0.3491 radian step input.

While the control effort is reasonable, the angle significantly varies about the desired output value (0.3491 radians). Note that, for the above simulation, the observer pole locations are at $z = 0.5$ and $z = 0.6$, while the closed-loop pole locations are at $z = 0.6$ and $z = 0.7$. The slowest observer pole, $z = 0.6$, is only about 1.5 times as as fast as the slowest closed-loop pole. We can move the observer poles closer to the origin to maintain a more appropriate distance between the observer poles and the closed-loop poles. This improves the response because the estimated states are converging faster to the actual states, allowing for more accurate feedback. Therefore, we will move the observer pole locations to $z = 0.2$ and $z = 0.3$. Now, the slowest observer pole, $z = 0.6$, is about three times as fast as the slowest closed-loop pole ($z = 0.2$ is about three times as fast as $z = 0.6$). Running the simulation again, we obtain the following results.



(a) Plot of the closed-loop angle, open-loop angle, and the control effort.



(b) Plot of the closed-loop angle only.

Figure 2: Response of system to a 0.3491 radian step input.

The control effort is much larger in this case due to the observer poles being closer to the origin, which requires a larger L matrix; however, the closed-loop response tracks the reference with more accuracy than with the previous design.

5 Problem 5

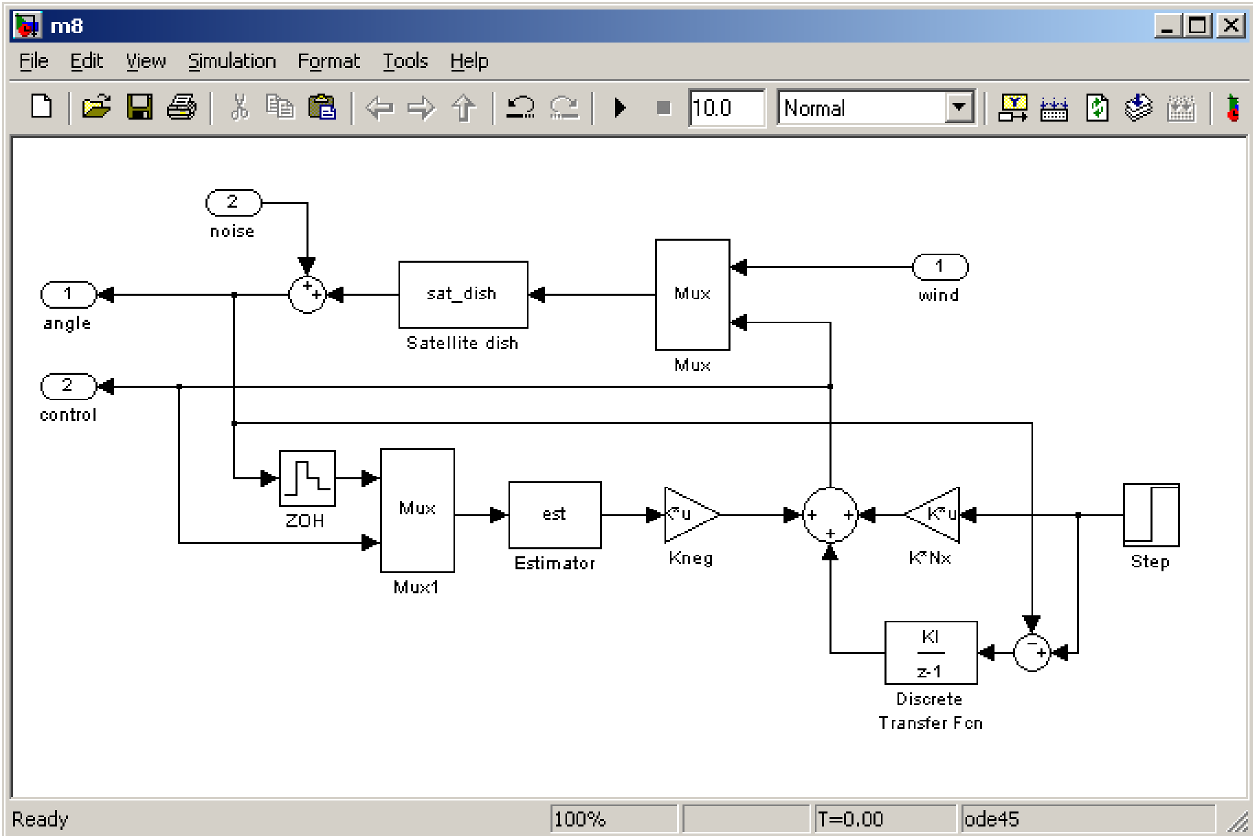


Figure 12: Reference tracking / Reduced order with integral controller

Now we still have: $\begin{cases} N_x &= [1, 0]^T \\ N_u &= 0 \end{cases}$

However, in order to find K_I, we need to calculate the augmented plant model:

$$\bar{A} = \begin{bmatrix} 1 & C_d \\ 0 & A_d \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0.9754 \\ 0 & 0 & 0.9512 \end{bmatrix} \text{ and } \bar{B} = \begin{bmatrix} 0 \\ B_{dTC} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.0049 \\ 0.0098 \end{bmatrix}$$

$$\text{Choose poles: } p = [0.5, 0.6, 0.7]$$

$$\bar{K} = place(\bar{A}, \bar{B}, p) = [6.1512, 45.0835, 95.2954]$$

$$\Rightarrow K_I = 6.1512 \text{ and } K = [45.0835, 95.2954]$$

Simulate this controller with above structure:

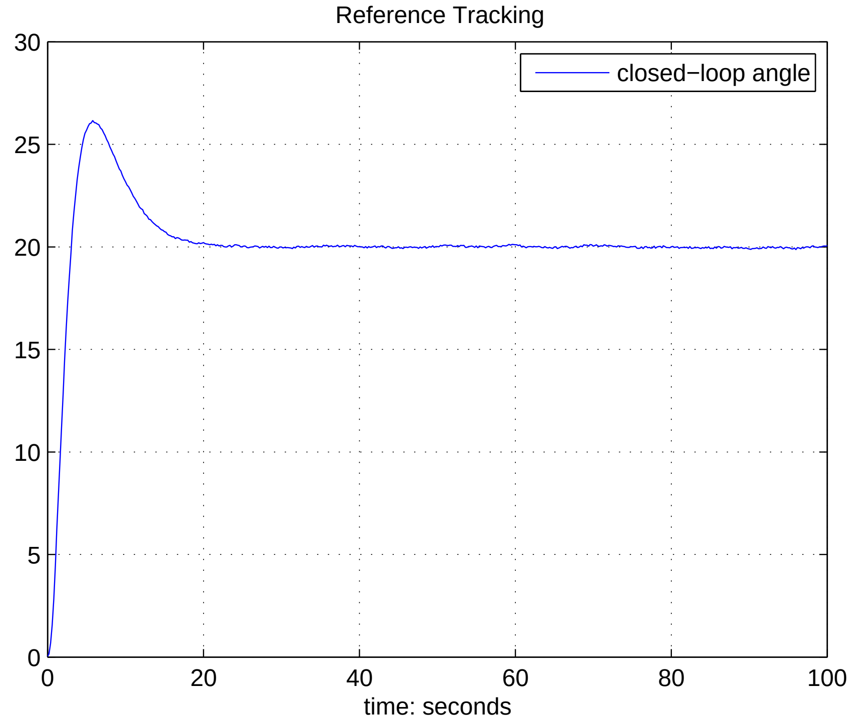
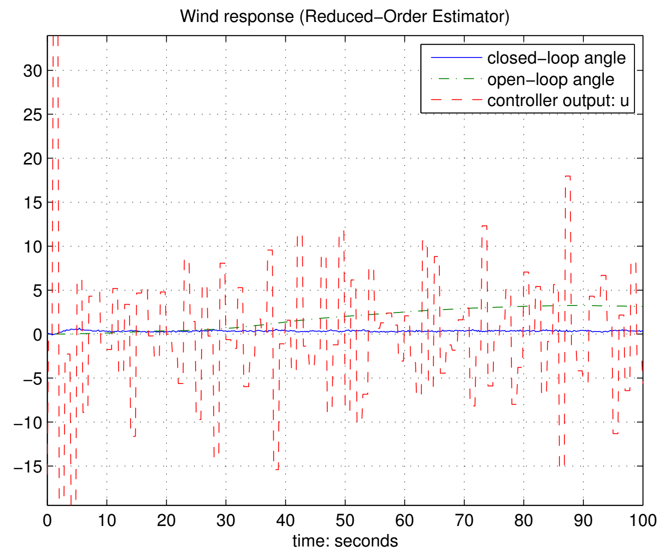


Figure 13: Reference tracking with full order estimate

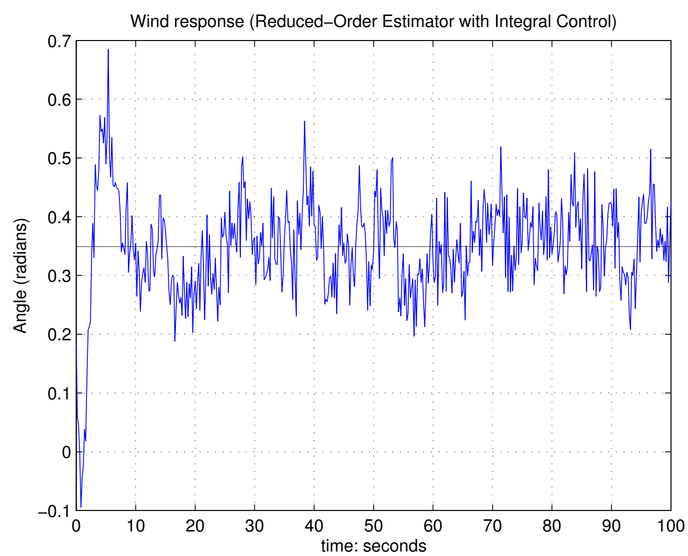
We see that with the additional integral controller, the system is much faster. But it also introduces overshoot.

Problem 5 - UPDATE

Using the same design as in the original solution to Problem 5, we obtain the following results for a reference of 0.3491 radians (20 degrees).



(a) Plot of the closed-loop angle, open-loop angle, and the control effort.



(b) Plot of the closed-loop angle only.

Figure 3: Response of system to a 0.3491 radian step input.

Again, the reduced-order estimator provides a better response than the full-order estimator in that the steady-state error is reduced; however, there is overshoot and the control effort is much larger than in the full-order estimator case.

Appendix

Note that x times faster implies that the pole location is three times farther into the left-half plane for a continuous-time system. For a discrete-time system, this corresponds to raising the pole location to the x power. This can be shown as follows:

Let s be a pole of the system $G(s)$. Then,

$$z = e^{sT}$$

is a pole of $G(z)$, where $G(z)$ is the pole-zero matched equivalent of $G(s)$ with sampling time T.

Let $\bar{s} = xs$. Then,

$$\bar{z} = e^{\bar{s}T} = e^{xsT} = \left(e^{sT}\right)^x = z^x$$

Therefore, x times faster corresponds to raising the pole location to the x power for a discrete system.