ECE 147 B sol'n HWK 2

$T = 1$ sec

(a) $\boxed{A-1-II}$: 1/2 cycle => 1 sec => 20 samples to get 1 full cycle.

$\therefore \theta = \frac{2\pi}{20} = \frac{\pi}{10}$. corresponds to p-z map II. Bode 1 hard to tell what gain is but larger resonant peak corresponds w/ bigger overshoot

$\boxed{B-3-IV}$: Slow rise time & single pole behavior in B => 1 pole close to $z=1$ => p-z IV

F.V = 10 => Gain = 20dB @ low freq & -20 dB/decade => Bode 3

$\boxed{C-2-I}$: Fast rise time w/out overshoot => conclude 2 real pole system in p-z map I. $\because$ real poles will have ~~shape of~~ Bode like 2

$\boxed{D-4-III}$: 1/2 cycle is 4 samples => $\theta = \frac{\pi}{4}$ in p-z map => map III. Gain a little less than 20dB in 4, F.V. = 8

(b) A: $p = re^{\pm j\frac{\pi}{10}}$, $r^k e^{jk\frac{\pi}{10}} = \overset{(10-6)}{4}$; $r^{k+10} e^{j(k\frac{\pi}{10}+\pi)} = 4-6 = -2$

$$\Rightarrow \frac{r^{k+10} e^{j(k\frac{\pi}{10}+\pi)}}{r^k e^{jk\frac{\pi}{10}}} = \frac{-2}{4} = -\frac{1}{2}$$

$$\Rightarrow r^{10} = \frac{1}{2} \Rightarrow r = 0.933$$

$$\frac{Kz}{\left(z-0.933e^{j\frac{\pi}{10}}\right)\left(z-0.933e^{-j\frac{\pi}{10}}\right)} ; \text{FV} \quad \frac{K}{1-2(0.933)\left(\cos\frac{\pi}{10}\right)+0.933^2} = 6 \Rightarrow K = \cancel{.0269}\ .5749$$

$$A \doteq \cancel{\frac{0.0269z}{z^2 - \cancel{1.8066z} + 0.8705}} \quad 1.7747z$$

$$A \doteq \frac{0.5749 \ \cancel{0.0269z}}{z^2 - 1.7747z + 0.8705}$$

B) $\tau \approx 0.5 \Rightarrow$ pole in $s \approx -2 \Rightarrow$ pole in z $e^{-2(.1)} = .8187$

$$\frac{K}{z - .8187} \Rightarrow \text{F.V. } 10 = \frac{K}{1 - .8187} \Rightarrow K = 1.813$$

$$\frac{1.813}{z - .8187}$$

(C) $\dfrac{K(z+1.5)}{z(z-0.5)}$; FV $= 10 = \dfrac{K(1+1.5)}{1(1-0.5)}$

$$\frac{10 \cdot \frac{1}{2}}{2.5} = K \Rightarrow 2 = K$$

$$\frac{2(z+1.5)}{z^2 - 0.5z}$$

(D) poles: $z = 0.5 \pm j0.5 \Rightarrow 0.5\sqrt{2}\, e^{\pm j\frac{\pi}{4}}$

$\dfrac{K}{z^2 - z + 0.5}$; FV $= 8 = \dfrac{K}{1 - 1 + 0.5} \Rightarrow K = 4$

$$\frac{4}{z^2 - 1z + 0.5}$$

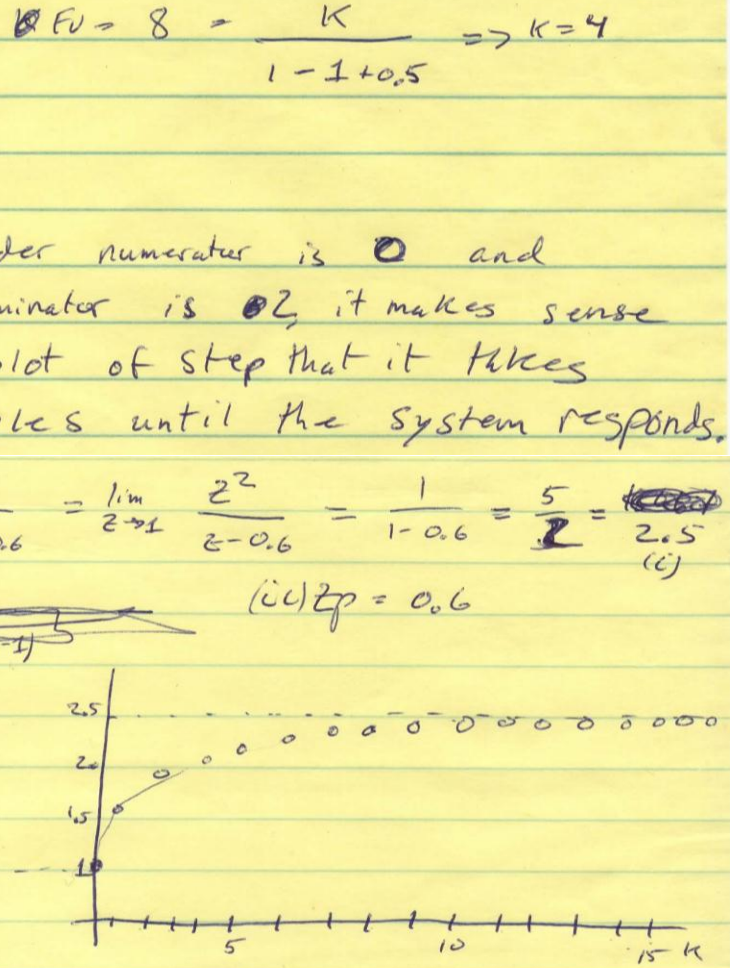
Note: order numerator is 0 and denominator is 2, it makes sense in the plot of step that it takes 2 samples until the system responds.

(2) (a)(i) $\lim_{z \to 1} (z-1) \dfrac{z}{(z-1)} \cdot \dfrac{z}{z-0.6} = \lim_{z \to 1} \dfrac{z^2}{z-0.6} = \dfrac{1}{1-0.6} = \dfrac{5}{2} = 2.5$ (i)

$\cancel{(b) \lim_{z \to 1} (z-1) \frac{z}{(z-1)} \frac{(z)}{(z-0.6)(z-1)}}$

(ii) $z_p = 0.6$

(iii) $y_k = x_k + 0.6\, y_{k-1}$



(b) (i) $\lim_{z \to 1} (z-1) \dfrac{z}{z-1} \dfrac{z}{(z-0.6)(z-1)} \longrightarrow \infty$

(ii) $z_p = 0.6, 1$

$$\frac{z}{(z-0.6)(z-1)} = \frac{z}{z^2 - 1.6z + 0.6} \Rightarrow y_k = x_{k-1} + 1.6\, y_{k-1} - 0.6\, y_{k-2}$$

2 (c) (i) $\lim_{z\to 1} (z-1)\frac{z}{(z-1)}\frac{z}{z^2-0z+(1-2\cos\frac{\pi}{5}+1)} = \frac{1^2}{1^2-0.618} = 2.618$

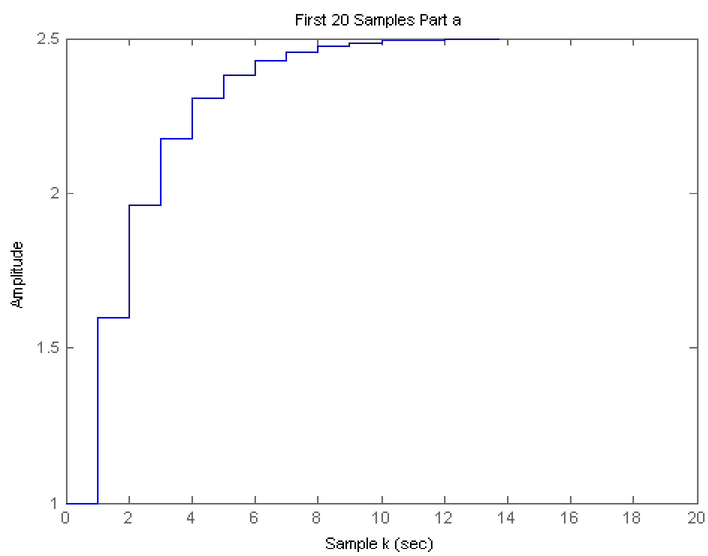
(ii) $z_p^2 = 0.618 \Rightarrow z_p = \pm 0.78615$

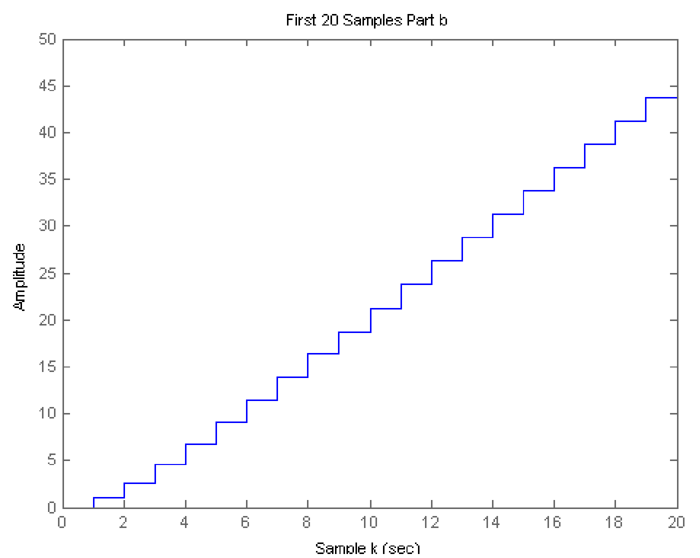
(iii) see attached plot

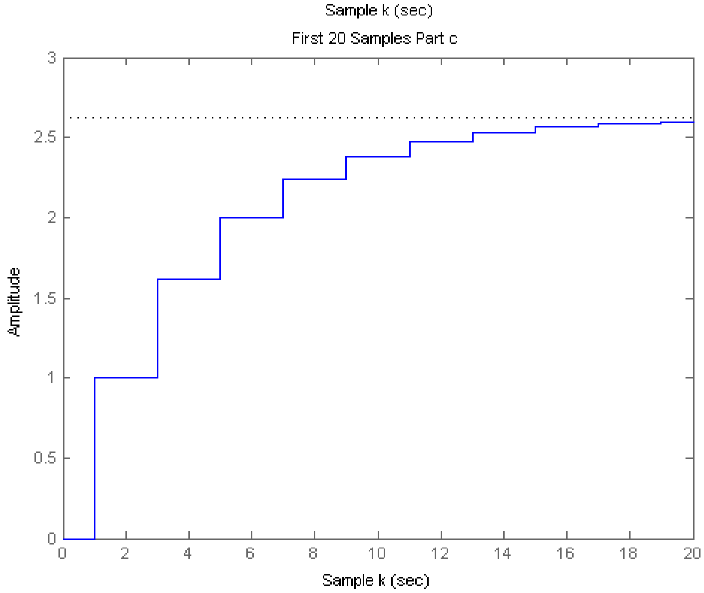
(d) (i) $\lim_{z\to 1} (z-1)\frac{z}{(z-1)}\frac{z}{z^2-\phi z+(1.1-2.2\cos\frac{\pi}{5})} = \frac{1^2}{1^2-.6798} = 3.1234$

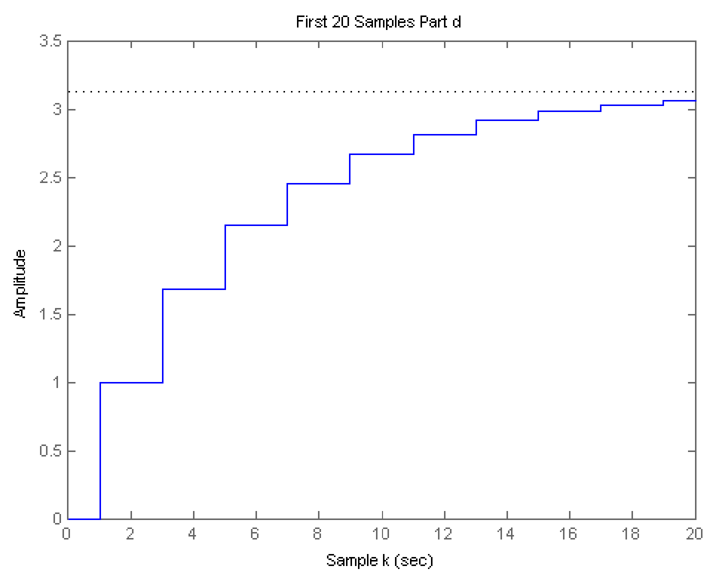
(ii) $z_p^2 = 0.6798 \Rightarrow z_p = \pm 0.8245$

(iii) see attached plot









(2) (e) When there is a pole at $z = 1$, using the DT Final Value theorem, the limit tends to ∞ ∵ There is a $(z-1)$ term still in the denominator.

Slope is derivative of T.F. so in D.T.

$$\lim_{z \to 1} \frac{2}{T} \frac{z-1}{z+1} X(z) = \lim_{z \to 1} \frac{(z-1)}{T} X(z)$$

This yields slope wrt time, ie $\frac{d}{dt}$.
To find it wrt sample #k use $\lim_{z \to 1} (z-1) X(z)$

(f) If $\exists$ a pole, z_p, $\ni |z_p| = 1$, $z_p \neq 1$, ie. z_p lays somewhere on the unit circle, then the system is marginally stable and oscillates at the frequency $w_0 \ni$ $z_p = e^{j w_0 T}$. The FVT yields average value of response

(g) If a pole lies outside the unit circle, the system is unstable. In this case the Final Value Theorem will not be valid

(3) (a) (i) $\frac{K_a z}{z - 0.9}$ $z_p = 0.9$ (ii) $10 = \lim_{z \to 1} \frac{(z-1)z}{z-1} \frac{K_a z}{z-0.9} = \lim_{z \to 1} \frac{K_a z^2}{z - 0.9}$

$\Rightarrow K_a = 10 * 0.1 = \boxed{1 = K_a}$

(iii) $y_{k+1} = x_{k+1} + 0.9 y_k \Rightarrow y_k = x_k + 0.9 y_{k-1}$

K	-1	0	1	2	3	4	5
X	0	1	1	1	1	1	1
Y	0	1	1.9	2.71	3.4390	4.095	4.6856

Plot is of $\frac{1}{z - 0.9}$

(iv) $s = \frac{\ln(0.9)}{T} = -1.0536$

(3) (b) $\lim_{z \to 1} \frac{K_b z}{z + 0.9} (z-1) \cdot \frac{z}{z-1} = \lim_{z \to 1} \frac{K_b z^2}{z + 0.9} = \frac{K_b}{1.9} \Rightarrow K_b = 10 \cdot 1.9$

$\boxed{K_b = 19}$ (ii)

(i) $z_p = -0.9 = 0.9 e^{j\pi}$

(iii) $y_k = 19 x_k - 0.9 y_{k-1}$

K	-1	0	1	2	3	4	5
X	0	1	1	1	1	1	1
Y	0	19	1.9	17.3	3.44	15.9	4.69

(iv) $sT = \ln(0.9) + \ln(e^{j\pi})$
$\Rightarrow s = -1.0536 \pm j31.416$

(c) (i) $\frac{K_c z}{z^2-0.81}$ $\Rightarrow$ $z_p = \pm 0.9$

(ii) $\lim_{z\to 1} (z-1)\frac{z}{z-1}\frac{K_c z}{z^2-0.81} = \lim_{z\to 1}\frac{K_c z^2}{z^2-0.81} \Rightarrow \frac{K_c}{0.19} = 10 \Rightarrow K_c = 1.9$

(iii) $y_K = 1.9\, x_{K-1} + 0.81\, y_{K-2}$

K	-2	-1	0	1	2	3	4	5	6
x	0	0	1	1	1	1	1	1	1
y	0	0	0	1.9	1.9	3.439	3.439	4.6856	4.6856

(iv) from (a) (iv) $z_p = 0.9 \Rightarrow s = -1.0536$

$s_{2,3} = -1.0536 \pm j31.416$

(d) (i) $\frac{K_D z}{z^2+0.81}$; $z^2+0.81=0$ $\Rightarrow z^2 = -0.81$ $\Rightarrow z_p = \pm j\, 0.9$

(ii) $\lim_{z\to 1} (z-1)\frac{z}{z-1}\frac{K_D z}{z^2+0.81} = \frac{K_D 1^2}{1+0.81} \Rightarrow K_D = 10 * 1.81$

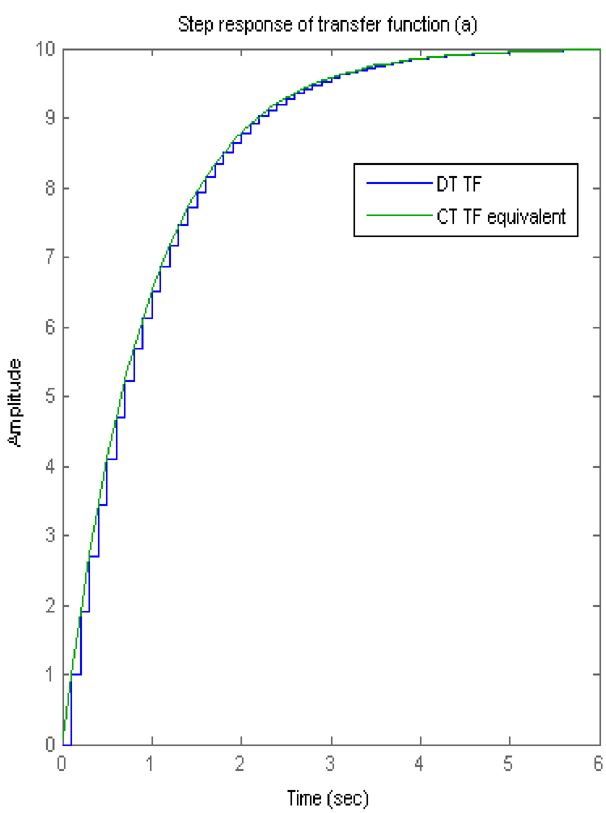
$K_D = 18.1$

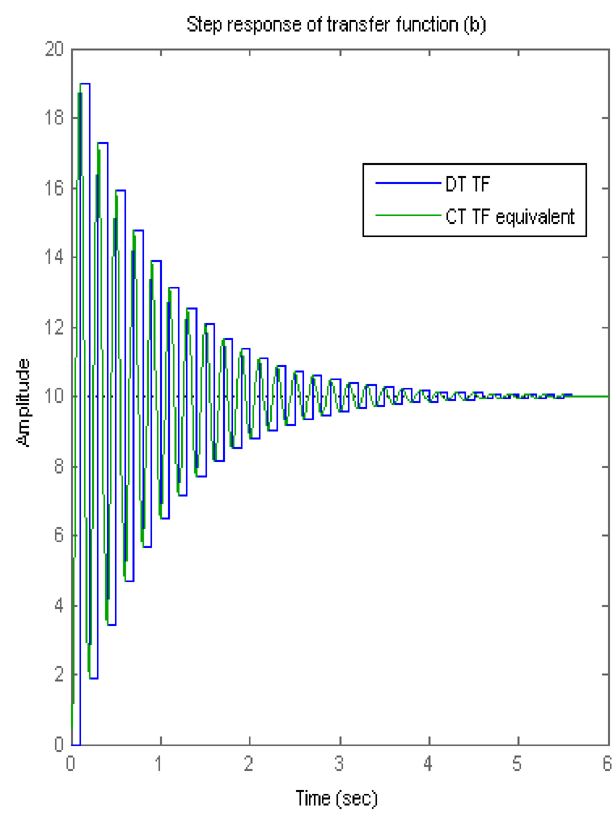
(iii) $y_K = 18.1\, x_{K-1} - 0.81\, y_{K-2}$

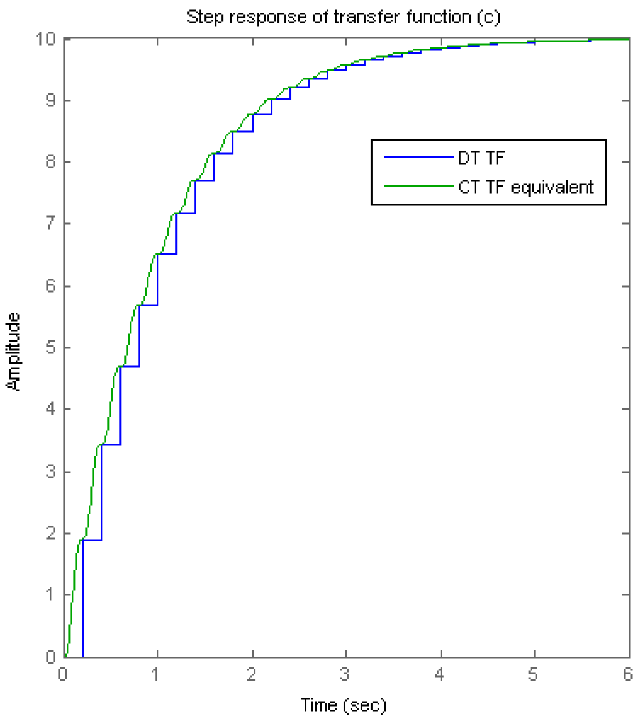
K	-2	-1	0	1	2	3	4	5	6
x	0	0	1	1	1	1	1	1	1
y	0	0	0	18.1	18.1	3.439	3.439	15.3144	15.3144

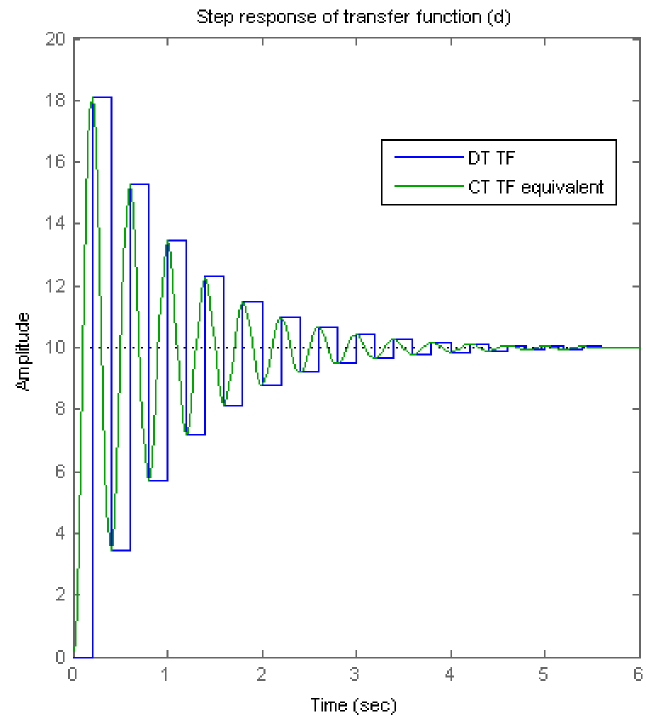
(iv) $z_p = 0.9\, e^{j\pm\frac{\pi}{2}}$

$\Rightarrow s_p = -1.0536 \pm j\, 15.708$










```
% problem 3 hwk2 plot of transfer functions
% Chris Steward 1/31/2013

T = 0.1;
% part a
tfa = tf(1,[1 -0.9],T);
sa = log(0.9)/T;
Ka = 10*-sa;
tfact = tf(Ka,[1 -sa]);
% CT TF equivalent:
%   10.54
%-------------
%  s + 1.054
figure, subplot(1,2,1), step(tfa);
hold on, step(tfact); hold off,
legend('DT TF','CT TF equivalent', 'location', 'best')
title('Step response of transfer function (a)')

% part b
tfb = tf(19,[1 0.9],T);
sb = log(-0.9)/T;
denB = conv([1 -sb],[1 -conj(sb)]);
Kb = 10*abs(sb)^2;
tfbct = tf(Kb,denB);
% CT TF equivalent
%  9881
%-----------------------
% s^2 + 2.07 s + 988.1
subplot(1,2,2), step(tfb);
hold on, step(tfbct); hold off,
legend('DT TF','CT TF equivalent', 'location', 'best')
title('Step response of transfer function (b)')
```

```
tfc = tf(1.9,[1 0 -0.9^2],T);
% if you notice parts a and b the poles of this system in CT are a
% so our denominator CT tf is a product of these 3 pole locations
denC = conv([1 -sa],denB);
Kc = 10*-sa*abs(sb)^2;
tfcct = tf(Kc,denC);
% % CT TF equiv
%            1.041e004
% -------------------------------
% s^3 + 3.161 s^2 + 990.3 s + 1041
figure, subplot(1,2,1),step(tfc);
hold on, step(tfcct); hold off,
legend('DT TF','CT TF equivalent', 'location', 'best')
title('Step response of transfer function (c)')

tfd = tf(18.1,[1 0 +0.9^2],T);
sd1 = log(0.9*1j)/T;
denD = conv([1 -sd1],[1 -conj(sd1)]);
Kd = 10*abs(sd1)^2;
tfdct = tf(Kd,denD);
% CT TF Equiv
%        2479
% --------------------
% s^2 + 2.107 s + 247.9
subplot(1,2,2), step(tfd);
hold on, step(tfdct); hold off,
legend('DT TF','CT TF equivalent', 'location', 'best')
title('Step response of transfer function (d)')
```

(4)(a) $C(s) = \frac{s+20}{s+100}\Big|_{s=100j} = \frac{j100+20}{100(1+j)} = \frac{1+j5}{5(1+j)} = \frac{(1+j5)(1-j)}{5\cdot 2}$

$= \frac{6+j4}{10} = \sqrt{\frac{36+16}{100}}\, e^{j\phi}$

$\phi = \text{atan}\left(\frac{4}{6}\right)$

$= 0.5880\,(\text{rad}) = 33.6901^\circ$

$\boxed{= 0.7211\, e^{j33.6901\frac{\pi}{180}}}$

$\approx \text{sqrt}(0.52)$

(b) $C_b(z) = C(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}} = \frac{\frac{2}{T}\frac{z-1}{z+1}+20}{\frac{2}{T}\frac{z-1}{z+1}+100} = \frac{2(z-1)+20T(z+1)}{2(z-1)+100T(z+1)}$

$= \frac{(2+20T)z+(20T-2)}{(2+100T)z+(100T-2)}\Bigg|_{T=0.01} = \frac{2.2z-1.8}{3z-1}$

$C_b(e^{j100T}) = \cancel{\ldots}$

$\hookrightarrow C_b(e^{j}) = \frac{2.2e^{j}-1.8}{3e^{j}-1}\left(\frac{3e^{-j}-1}{3e^{-j}-1}\right) = \frac{4.2937+j2.692}{6.7582}$

$= \frac{\sqrt{4.2937^2+2.6927^2}}{6.7582}\, e^{j\phi_b} = \boxed{0.7499\, e^{j32.093\frac{\pi}{180}}}$

$\phi_b = \text{atan}\left(\frac{2.6927}{4}\right)$

$= .5601\,(\text{rad}) = 32.093^\circ$

(c) $C_c(z) = C(s)\Big|_{s=\alpha\frac{z-1}{z+1}} = \frac{\alpha(z-1)+20(z+1)}{\alpha(z-1)+100(z+1)}$; $\alpha = \frac{\omega_0}{\tan\left(\frac{\omega_0 T}{2}\right)} = \frac{100}{\tan\left(\frac{1}{2}\right)}$

$= 183.0488$

$= \frac{203.0488z - 163.0488}{283.0488z - 83.0488}$

$C_c(z)\Big|_{z=e^{j100\times.01}=e^{j}} = \frac{203.0488e^{j}-163.0488}{283.0488e^{+j}-83.0488}$

$= \cancel{\ldots}$

$= \frac{-53.3411+j170.8597}{69.8831+j238.1773} = \frac{36.9673\times10^3+j24.6448\times}{61.6121\times10^3}$

MATCHES PART(a) !! win!

$\boxed{= .721\, e^{j33.6901\frac{\pi}{180}}}$

④ (d) Case of $\alpha = \frac{\omega_0}{\tan\left(\frac{\omega_0 T}{2}\right)} = \frac{2}{T}$

$$\Rightarrow \tan\left(\frac{\omega_0 T}{2}\right) = \frac{\omega_0 T}{2} \Rightarrow \tan\left(\frac{\omega_0 T}{2}\right) - \frac{\omega_0 T}{2} = 0$$

$$\Rightarrow \omega_0 = 0$$

So at DC phase and magnitude should match, theoretically.

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⑤ (a) $G(s) = \frac{1}{s}$ (i), (ii) & (iii) attached code

Algebra for code

$$\frac{KG}{1+KG} = \frac{K}{s+K} \Rightarrow \text{pole @ } s = -K$$

$$\Rightarrow \text{inside contour for } |K| < 1$$

$\frac{1}{s}$ pole inside ~~contour~~ square, the plot is CCW

because $\underset{\text{closed loop inside}}{Z} = \underset{\text{CW circle}}{N} + \underset{\text{o.L. inside}}{P} = -1 + 1 = 0$

(b) $G(s) = \frac{1}{s(s-1.5)} \Rightarrow \frac{KG}{1+KG} = \frac{K}{s^2 - 1.5s + K}$

$$\Rightarrow \text{poles } s_{1,2} = \frac{1.5 \pm \sqrt{2.25 - 4K}}{2}$$

Case (1) $\sqrt{2.25 - 4K} < 0 \Rightarrow K > 0.5625$

$s_{1,2} = .75 \pm j\frac{\sqrt{4K - 2.25}}{2}$. Inside square $\Rightarrow \frac{\sqrt{4K-2.25}}{2} < 1$

$\Rightarrow 4K - 2.25 < 4$

$\Rightarrow K < 1.5625$

(5) (b) Case (2c) $K < 0.5625$; 2 real poles

$+\sqrt{\ }$ moves outside first

$.75 + \frac{\sqrt{2.25-4K}}{2} < 1 \Rightarrow \sqrt{2.25-4K} < .5$

$\Rightarrow 2.25 - 4K < .25$

$\Rightarrow 4K > 2$

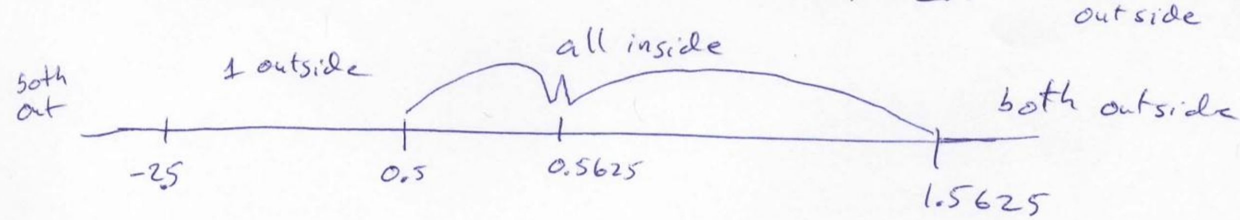
$\Rightarrow K > 0.5$

$\Rightarrow K < 0.5$] 1 pole outside

$\left| .75 - \frac{\sqrt{2.25-4K}}{2} \right| > 1 \Rightarrow \frac{\sqrt{2.25-4K}}{2} > 1.75$

$\Rightarrow 2.25 - 4K > 12.25$

$\Rightarrow K < -2.5$ both poles outside



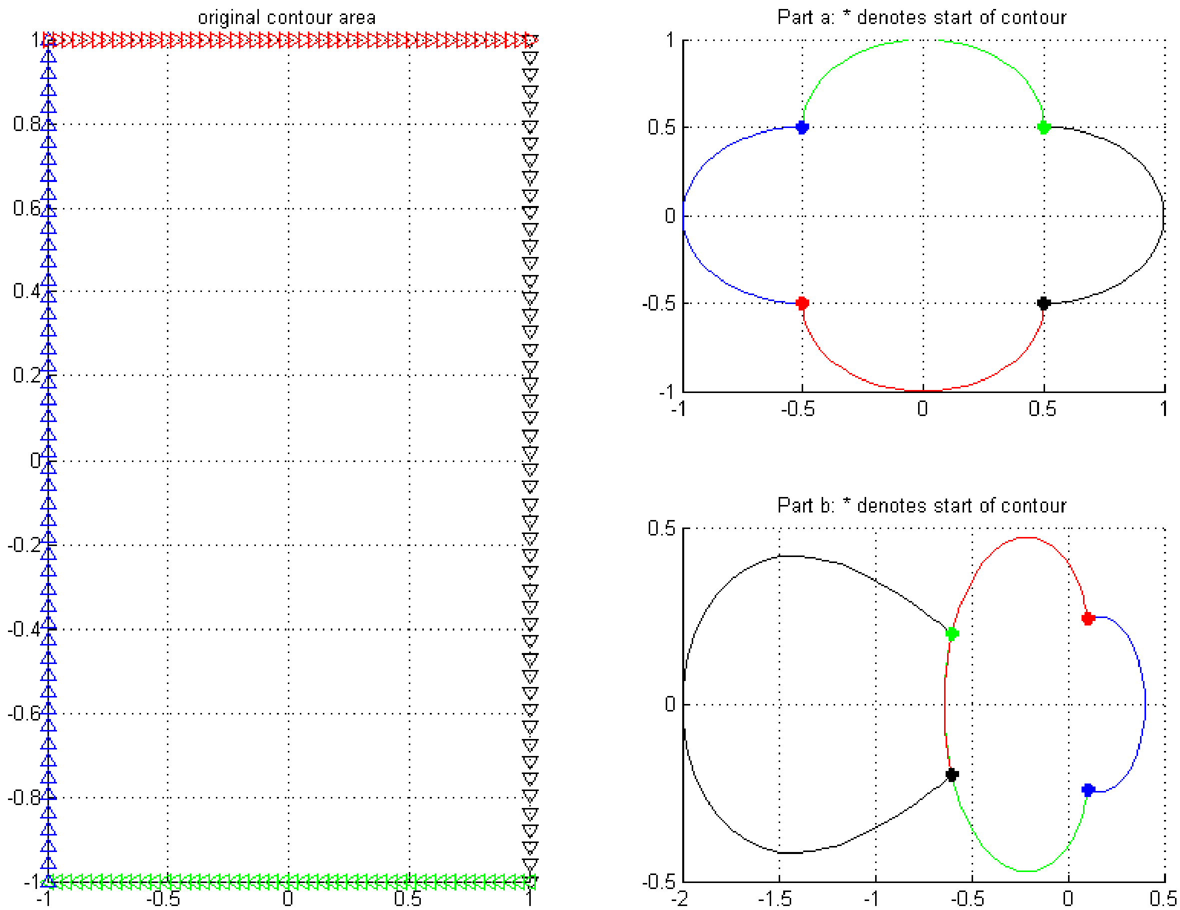
$\frac{1}{s(s-1.5)}$ 1 pole in & 1 pole outside square

$-0.64 < -\frac{1}{K} < 0.4 \Rightarrow$ CCW encirclement $\Rightarrow$ both outside

$Z = N + P = -1 + 1 = 0$

$-2 < -\frac{1}{K} < -0.64 \Rightarrow$ CW encirclement $\Rightarrow$ both inside

$Z = N + P = 1 + 1 = 2$




```
% Chris Steward
% Homework 2 Problem 5
% ECE 147B 1/31/2013

% create values for borders
x = linspace(-1,1,50);
% create the sides
right = -1i*x + 1;  % times -i to start contour from i -> -i
bot   = -1*x - 1i;  % -1 to start from 1
left  = 1i*x - 1;   %
top   = x + 1i;

% verify this is right
% colors go in k,g,b,r
figure, subplot(2,2,[1 3]), hold on, grid on
plot(right,'kv')
plot(bot, 'g<')
plot(left,'b^')
plot(top,'r>')
hold off,
title('original contour area')

% make first plot of part a
aright = 1./(right);
abot   = 1./bot;
aleft  = 1./left;
atop   = 1./top;
subplot(2,2,2), hold on,grid on
plot(aright,'k')
plot(abot, 'g')
plot(aleft,'b')
plot(atop,'r')
plot(aright(1),'k*','linewidth',3)
plot(abot(1), 'g*','linewidth',3)
plot(aleft(1),'b*','linewidth',3)
plot(atop(1),'r*','linewidth',3)
hold off,
title('Part a: * denotes start of contour')
```

```
% do part b
bright = 1./(right.*(right - 1.5));
bbot = 1./(bot.*(bot - 1.5));
bleft = 1./(left.*(left -1.5));
btop = 1./(top.*(top -1.5));
% make plot of b
subplot(2,2,4), hold on,grid on
plot(bright,'k')
plot(bbot, 'g')
plot(bleft,'b')
plot(btop,'r')
plot(bright(1),'k*','linewidth',3)
plot(bbot(1), 'g*','linewidth',3)
plot(bleft(1),'b*','linewidth',3)
plot(btop(1),'r*','linewidth',3)
hold off,
title('Part b: * denotes start of contour')
```