ECE 147B **Digital Control** Systems: Theory and Design Winter 2013

Note: Show Work! Copying is CHEATING!

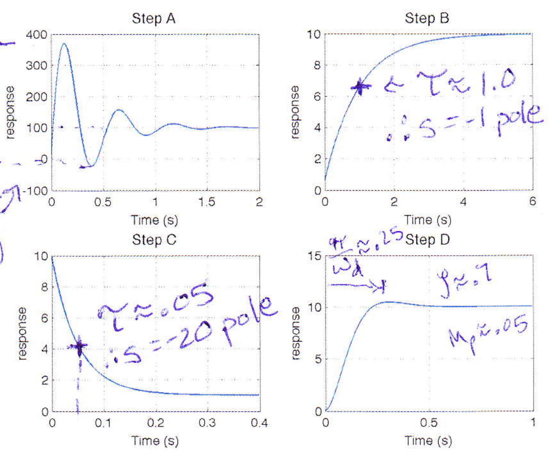
# Homework 1 — SOLUTIONS

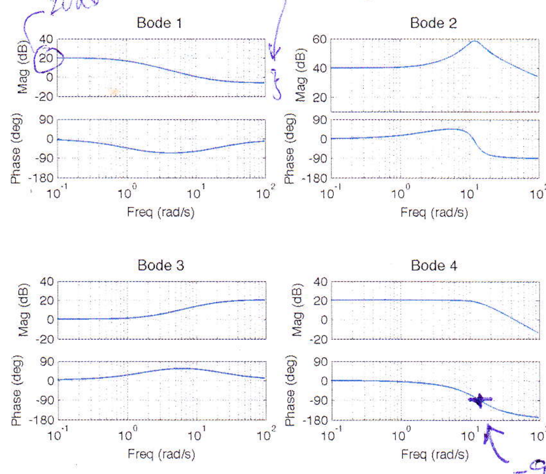
**Due:** January 18, 2013, **at 5 pm**, in the homework boxes outside Harold Frank Hall 3120A.

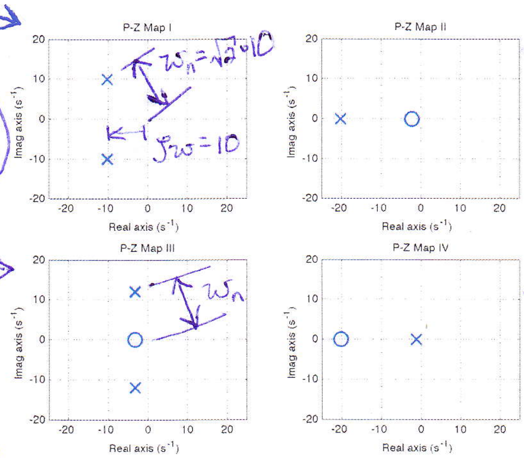
**Problem 1.** Continuous-time dynamics.
Below are 4 step responses (A,B,C,D), 4 Bode plots (1,2,3,4), and 4 pole-zero plots (I,II,III,IV).

a) Match these up. (i.e., list the 4 correctly-matched triplets. e.g., if A, 1, and I match, write "A-1-I".)

b) Estimate the continuous-time transfer function for each system.




D-4-I

4: DC gain 10
D: FV 10

$$\frac{2000}{s^2+20s+200}$$

C-3-II

$$\frac{10(s+2)}{(s+20)}$$

3: DC gain 1, high freq 10
C: FV 1
IV 10

A-2-III

2: DC gain 100
A: FV 100

pole: $s \approx -3 \pm 12j$
zero: $s = -3$

$$\frac{5100(s+3)}{(s^2+6s+153)}$$

$\zeta\omega = 3$

$\omega_n = \sqrt{3^2+12^2}$

$\omega_n = \sqrt{153}$

B-1-IV

$$\frac{1}{2}\frac{(s+20)}{(s+1)}$$

1: DC gain 10, high freq < 1
B: FV = 10
IV = ? (can't see it)

BUT → $\tau = 1.0$ sec, & zero @ $s = -20$

So, can calculate exactly here, even w/out Init. Value or high freq. asymptote

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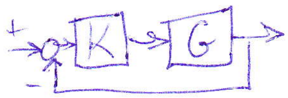
**Problem 2.** Stability for continuous-time (CT) systems.




$$G(s) = \frac{2s-12}{s+3}$$

* closed loop TF: $\Rightarrow \frac{K(2s-12)}{K(2s-12)+(s+3)}$

characteristic eqn: $\Rightarrow (2K+1)s + (3-12K) = 0$

$\therefore \boxed{s = \frac{(12K-3)}{(2K+1)}}$ ← closed loop pole

Assume a unity-feedback configuration with a gain, K, as the (proportional) controller. For what values of K will the resulting closed-loop system be stable? (HINT: assume K might be negative or positive!) Justify your answer via each of the following two methods:

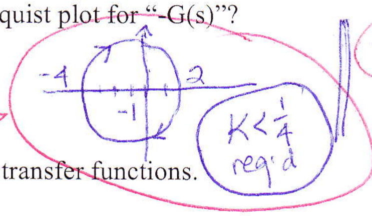
So, K=0 stable. We need: $(12K-3)<0$ and $(2K+1)>0$, so...

a) Algebraically solve for the closed-loop transfer function, keeping K as a variable; then reason (as appropriate) about when the poles would all be stable.

$\therefore K < \frac{3}{12}$ & $K > \frac{-1}{2} \Rightarrow \boxed{-\frac{1}{2} < K < \frac{1}{4}}$ for STABILITY

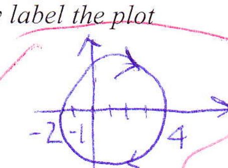
b) Sketch the Nyquist (frequency response) plot for the system; then reason (as appropriate) about when the poles would all be stable. *Hints:* 1. Please feel encouraged to exploit the "nyquist" function in MATLAB to get started, but for your submitted homework, please just *sketch and carefully label the plot by hand*. 2. What is the nyquist plot for "-G(s)"?

G(s): DC = $\frac{-12}{3} = -4$, high freq = 2




$K < \frac{1}{4}$ req'd

-G(s): DC = +4, high freq = -2

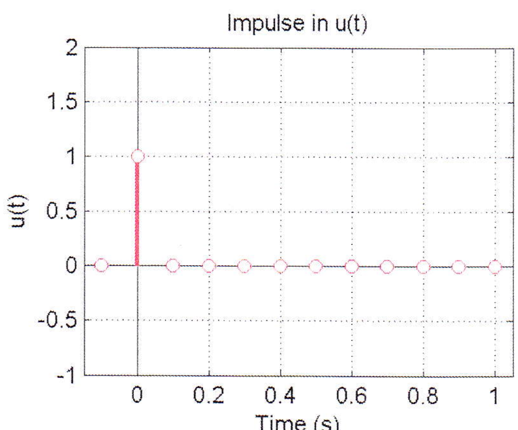


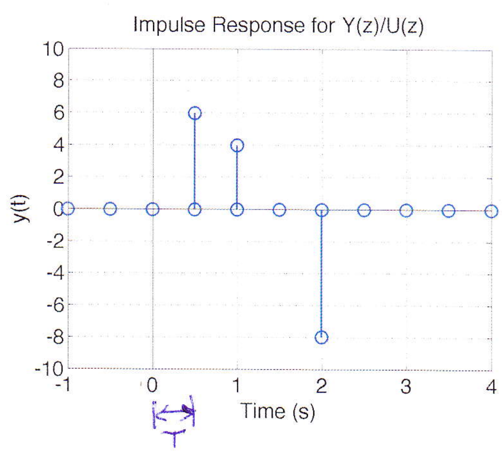

$|K| < \frac{1}{2}$, Since "-G(s)", $\boxed{K > -\frac{1}{2} \text{ needed}}$

$\therefore \boxed{-\frac{1}{2} < K < \frac{1}{4}}$ for STABILITY

**Problem 3.** Discrete-time transfer functions.

Below at left is an impulse in u(t). At right is the response of a discrete-time system, $P(z)=Y(z)/U(z)$, to this impulse, plotted as discrete values over time.




T=0.5 sec

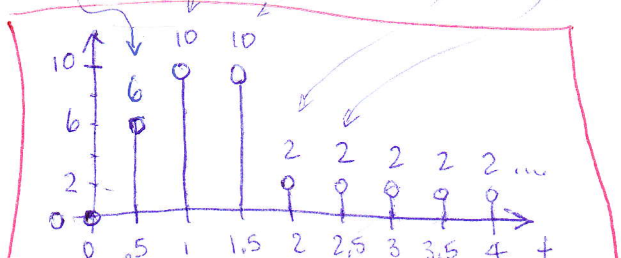
a) What is the sampling time, and what is the discrete-time transfer function, $P(z)=Y(z)/U(z)$. (Assume y(t) continues to be 0 for all $t > 2.0$ sec in the plot at right, above…)

b) By hand, plot the **step response** of P(z).

$$P(z) = 6z^{-1} + 4z^{-2} - 8z^{-4}$$

$$\therefore \boxed{P(z) = \frac{6z^3 + 4z^2 - 8}{z^4}}$$

0, 6, 6+4=10, 10+0=10, 10-8=2, 2+0=2, …

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i) forward: $C(z) = \frac{z-1+T}{z-1+16T}$ $\boxed{\text{i)} = \frac{z-.97}{z-.52}}$

ii) $C(z) = \frac{z-1+zT}{z-1+16zT}$ $\boxed{\text{ii)} = \frac{1.03z-1}{1.48z-1}}$

iii) $C(z) = \frac{2(z-1)+T(z+1)}{2(z-1)+16T(z+1)}$ $\boxed{\text{iii)} = \frac{2.03z-1.97}{2.48z-1.52}}$

**Problem 4.** Numerical integration approximations.

Consider the following continuous-time lead controller:

b) Phase: $\phi_{num}$ minus $\phi_{den}$

$\phi_{num} - \phi_{den} = \operatorname{atan}(\frac{4}{1}) - \operatorname{atan}(\frac{4}{16}) = 1.08 \text{ rad} = 61.93°$

$$C(s) = \frac{(s+1)}{(s+16)}$$

a) By hand, calculate the change (i.e., magnitude of increase or decrease, as appropriate) in DC gain this controller would provide to the loop transmission, *C(s)P(s)*. for DC, $s=0$ → $\boxed{\text{Multiply DC gain by } \frac{1}{16}.}$

b) By hand, calculate the maximum phase change i.e., magnitude of increase or decrease, as appropriate that occurs due to this controller, and at what frequency does this occur? (We will call this frequency $\omega_a$.) $\boxed{\omega_a = \sqrt{1 \times 16} = 4 \text{ rad/s}}$

(See above...) $\boxed{\text{Phase @ } 4 \text{ rad/s is } +61.93°}$

c) Recall from lecture that i) Forward Euler, ii) Backward Euler, and iii) Tustin (trapezoid rule) approximations results in the following mappings between the differential operator, s, and the difference equation operator, z, respectively:

i) $s \leftrightarrow \frac{z-1}{T}$  ii) $s \leftrightarrow \frac{z-1}{zT}$  iii) $s \leftrightarrow \frac{2(z-1)}{T(z+1)}$

For the remainder of the problem, use a sample period, T, of 0.03 seconds. $\boxed{z_a = e^{j\omega_a T} = .9923 + .1197j}$

See above...

b) Calculate the forward difference approximation, $C_{forward}(z)$, for C(s). What is its phase at $z_a = e^{j\omega_a t}$?

$\boxed{\text{phase: } 65.0044°}$

c) Calculate the backward difference approx., $C_{backward}(z)$, for C(s). What is its phase at $z_a = e^{j\omega_a t}$?

$\boxed{\text{phase: } 58.9362°}$

d) Calculate the bilinear difference approximation, $C_{bilinear}(z)$, for C(s). What is its phase at $z_a = e^{j\omega_a t}$?

$\boxed{\text{phase: } 61.9275°}$

See attached plot...

e) Use MATLAB to generate a Bode plot for the original C(s) along with all 3 approximations on a single graph, using a frequency range from 0.1 (rad/s) up to the Nyquist frequency. Discuss the most important effect(s) of each transform. Which one works "best", and why? (Be sure to solve for and label the Nyquist frequency on your plot!) Well, bilinear (Tustin) matches C(s) best!

**Problem 5.** DT control design via emulation. $\boxed{C(s) \approx 5000 \frac{(s+6.2)}{(s+160)}}$ ... See Matlab plot ~~code~~ attached.

a) Design a continuous-time lead compensator, *C(s)*, for the plant $P(s) = 5/s^2$ so that the overshoot, $M_p$, is about 15% (between 10% and 20% is OK…) and the peak time, $t_p$, is *approximately* 0.1 seconds (e.g., in the range 0.08 to 0.12 seconds). Include a MATLAB step response plot to verify your answer.

b) What are the poles and zeros of the closed-loop system? What effect do the zeros have on overshoot, and why? $\boxed{\text{poles: } s = -120, -31, -8.3}$ ← all REAL, $\boxed{\text{zeros: } s = -6.2}$ ← Zero causes overshoot. Poles are strictly real!

Repeat c) and d) for sampling rates of **T=.01**, **T=.05**, and **T=.06** seconds.

c) By hand, convert P(s) to P(z), assuming a zero-order hold, and convert C(s) to C(z), using the Tustin approximation. (Show work by hand, but please DO use MATLAB to *check* your work.)

d) Calculate the closed-loop transfer function for the discrete-time (DT) system, and generate a step response for 5 seconds, using MATLAB. (e.g., "step(P,5)" What are the peak time and percent overshoot

See Matlab plot ~~code~~

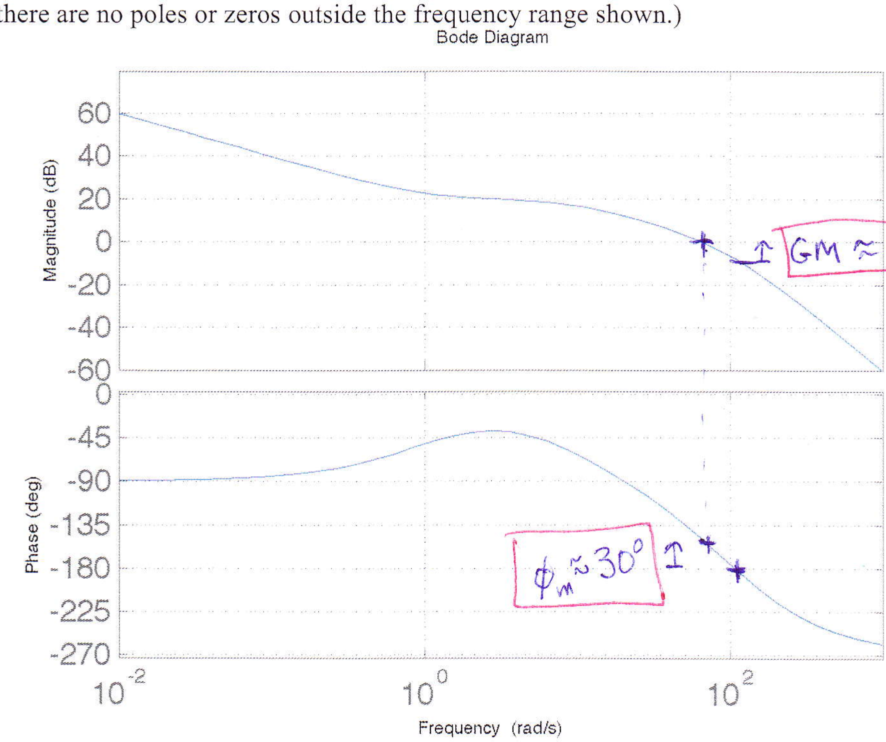
for the DT system? **Put all 4 step responses (the CT step and all three DT sampling rate cases) on a single plot, clearly labeled.**

e) What is the phase margin and overshoot for each of the 4 cases. (Hint: You can use "margin(P*C)" in MATLAB for each loop transmission...) Which closed-loop systems are STABLE?

* T=.06 might or might not be, depending on YOUR C(s)... others are stable

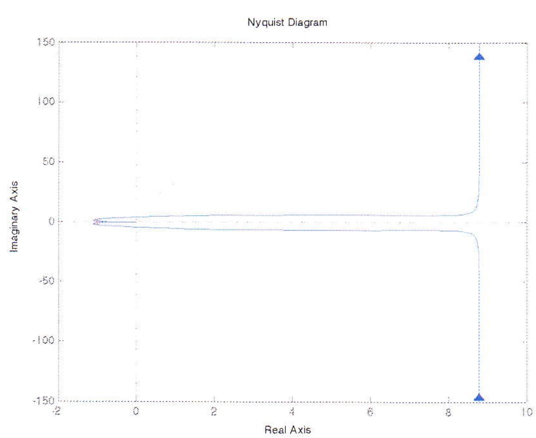
**Problem 6.** Gain and phase margins.

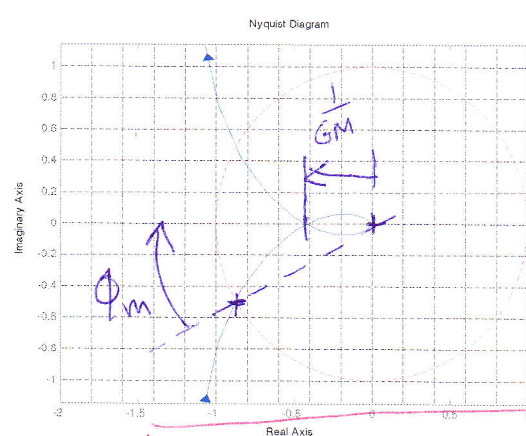
a) From the Bode plot below, estimate gain and phase margin. Also indicate the crossover frequency. (Assume there are no poles or zeros outside the frequency range shown.)




(hard to tell... See Nyquist!)

b) From the Nyquist plot below, again estimate gain and phase margin. (At right is "zoom" view...)




$\therefore$ $GM \approx \frac{1}{.4} = 2.5 = 8\,dB$

$\phi_m = \operatorname{atan}\left(\frac{.5}{.88}\right) = 30^\circ$

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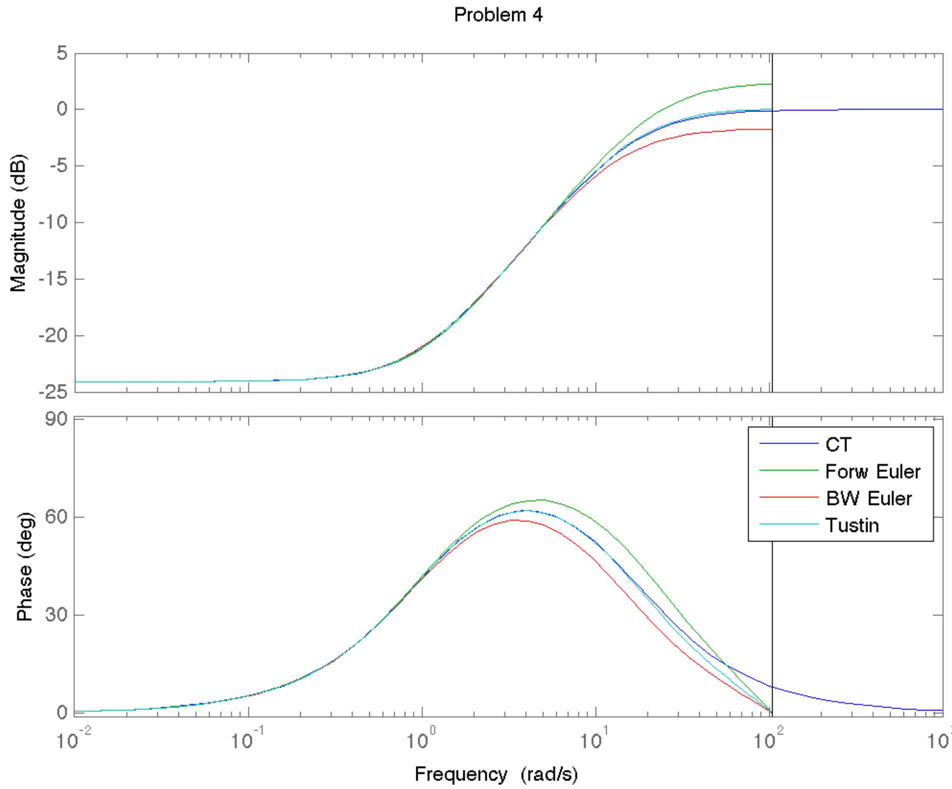
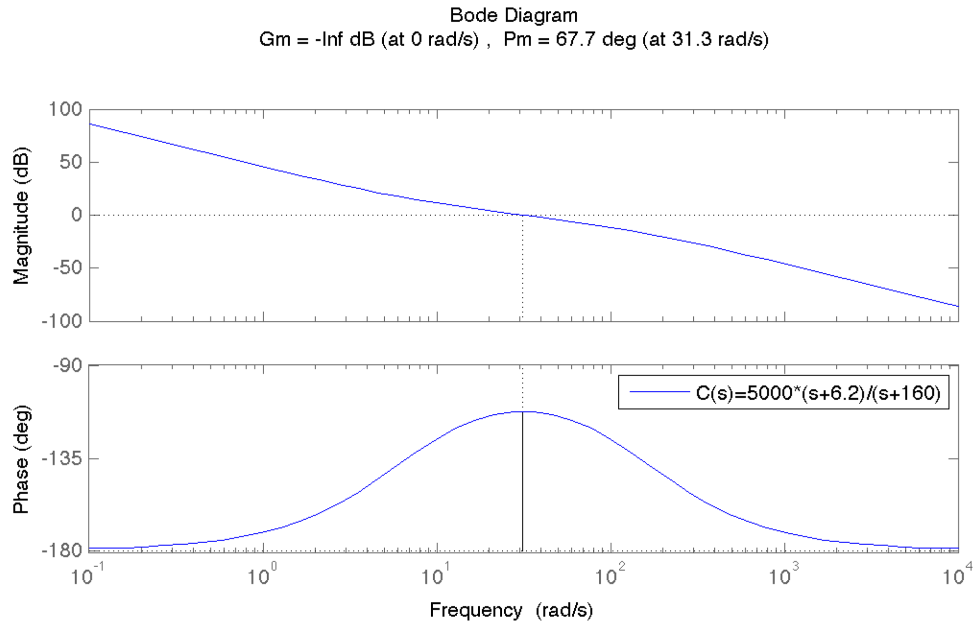
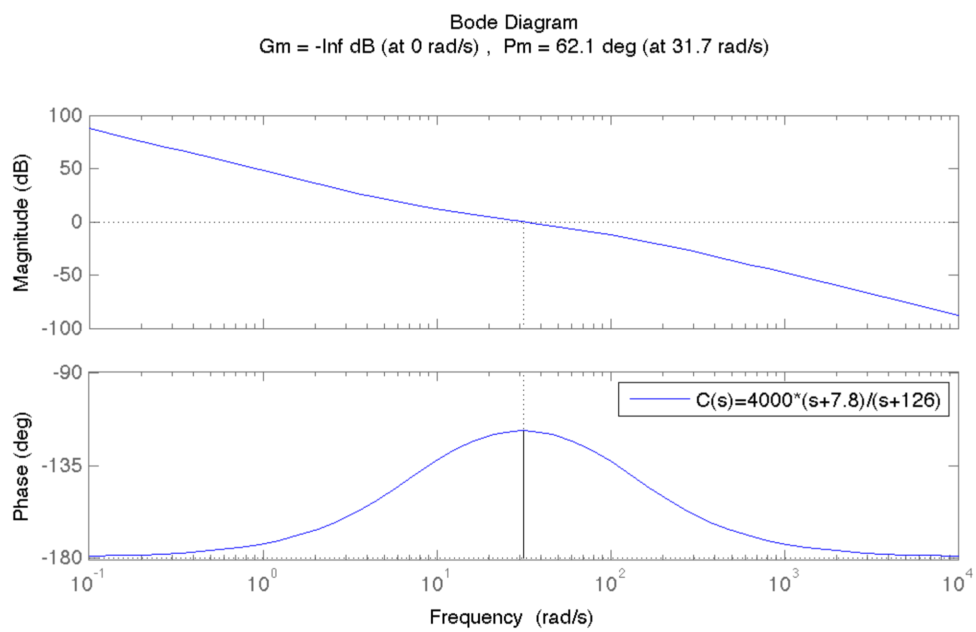
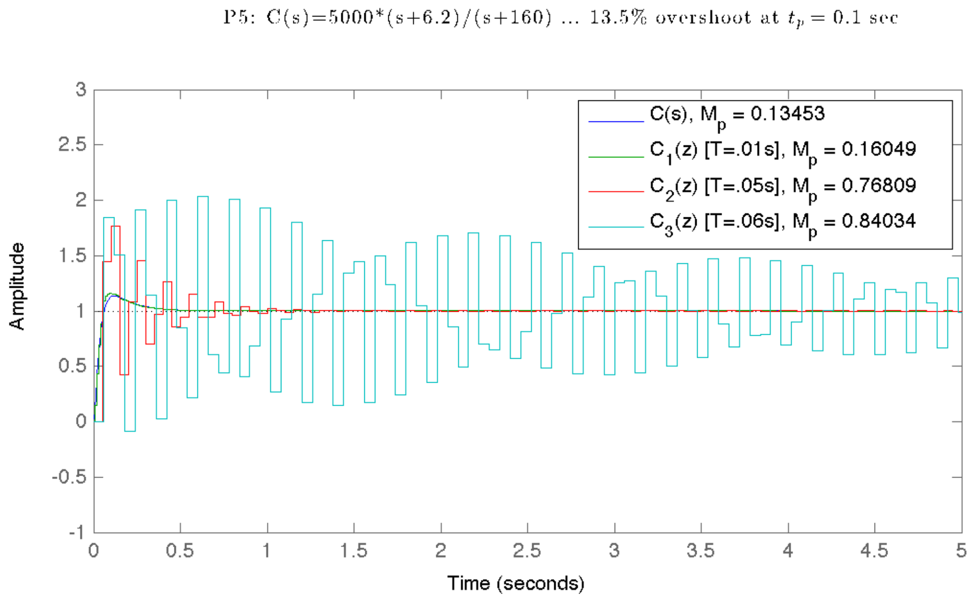


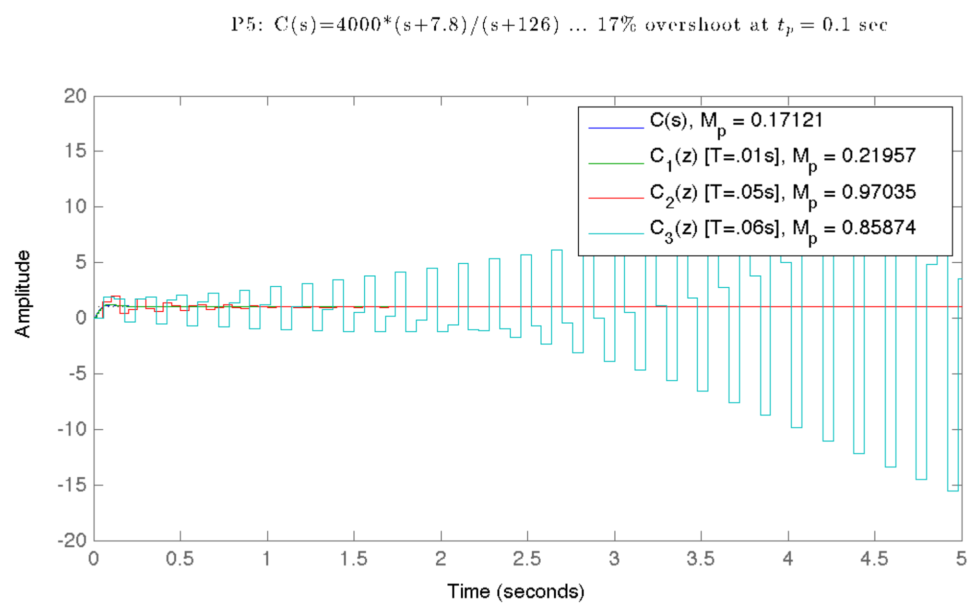

See preceding pages for rest of the solution to Problem 4.




Above: The Bode plots are for JUST C(s), only. Your C(s) may vary! Here are two examples. In both cases, the zero and pole are spaced as "wa/a" and "wa*a" (geometrically equal spacing) wrt crossover, wa.




Problem 5: Depending on your C(s), C(z) with T=0.06 may or may not be stable! Note that having more phase margin (upper plot previous page) allows one to be stable using a larger time step (upper plot THIS page).