Homework 8 – Pick 4 of the 5 problems, only!

Due: **FRIDAY**, March 15, 2013, **at 5 pm**, in the homework boxes outside Harold Frank Hall 3120A.

Problem 1. For the state space system below, design a controller and estimator to design specifications, using the steps listed below.

$$A_d = \begin{bmatrix} 0.5 & -1.2 \\ 1 & 0 \end{bmatrix} \qquad B_d = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad C_d = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D_d = \begin{bmatrix} 0 \end{bmatrix} \qquad \text{T=0.1 seconds}$$

a) What are the OPEN-LOOP system poles, and is the system STABLE?

b) Calculate z-plane poles corresponding to a closed-loop step response with peak time 1 second and about 5% overshoot.

c) Solve BY HAND for the gain matrix, K, that will result in these closed-loop poles, from part b. (Hint: This is relatively easy, since the system is in Control Canonical form.)

d) Now, design for ESTIMATOR poles that are "twice as fast"; i.e., same percentage overshoot, but with a peak time of only 0.5 seconds.

e) Solve BY HAND for the gain matrix, Lp, that gives these estimator poles. (This is more challenging than part c, so write out the equations for "det(sI-(A-Lp*C))" BY HAND.)

Problem 2. For the discrete-time, SISO transfer function $G(z) = \dfrac{0.1}{(z-0.8)^2}$,

a) Use MATLAB to create a Nyquist plot and print a copy, to label by hand, as required below.

b) By zooming in on this plot, calculate the range of values for K for which the closed-loop system (given a unity feedback configuration with proportional control gain K).
Label by hand the required points (where you zoomed in), needed to answer this.

c) For what range of values of K does the system have TWO unstable poles? And for what range of values of K (if any) does the system have ONE unstable pole?
Label by hand the required points (where you zoomed in), needed to answer this.

d) Sketch a root locus by hand, considering both positive and negative values for K are possible, and label the regions of K values found in b) and c) on your plot.

Problem 3. For open-loop system $A_d = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ check controllability for:

a) $B_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ vs for b) $B_d = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

c) Sketch a block diagram for each case, clearly labeling states "x1" and "x2", and use this to explain your results in a) and b), intuitively, based on how information flows (i.e. how input "u" propagates).

Problem 4. Given the SISO system $P(z) = \dfrac{z+0.5}{z^3+1.2z^2+0.5z-1}$, with T=0.01 seconds,

a) Calculate the Control Canonical form state space matrices, A_d, B_d, C_d, and D_d.
b) By hand, calculate the gain matrix, K, for deadbeat control. (Easy!)
c) By hand, calculate the gain matrix, K, to "cancel" the zero in P(z) and to place the other two poles at z=0.8.
d) Simulate an initial condition response (using MATLAB command "initial") for the closed-loop system of c) for X0 = [1;1;1]. Closed loop, Ad is now "Ad-Bd*K", and Bd is just empty (Bd=[]).
e) On the same plot as d), compare an initial condition response for a SECOND-ORDER CONTINUOUS-TIME system with 2 s-plane poles corresponding to "z=0.8, T=0.01 seconds", using an initial condition with x (position) set to match the response in d) and dx/dt=0 (zero for the initial velocity state). The CT response should look similar to d)…
f) On a second plot, simulate 3 initial condition responses, one each for the DT state-space system for X0=[1;0;0], X0=[0;1;0], and X0=[1;0;0]. (Easy, if you have already done d)!)
g) By definition, X0 is giving the 3 values of state vector "X" at k=0 (t=0*T). By HAND, calculate X(k=1), using the closed-loop state space (difference) equations.
h) By HAND, calculate output Y for each case in f) at k=0 (t=0) and at k=1 (t=T). Show these Y values match your MATLAB plots. (recall Y=CX.)

Problem 5. Given the CT plant $P(s) = \dfrac{10}{s^2+2s+30}$,

a) Convert the system to a DT state space system, assuming a ZOH and T=0.1 seconds.
b) Determine Nu and Nx for the system, and sketch a block diagram for reference trajectory following control.
c) Determine K to place poles for a critically damped response corresponding to wn=30 rad/sec.
d) Using MATLAB, simulate the system in Problem 5 to demonstrate that there is no steady state error (despite the fact that this is a Type 0 system, with no integral control), due to the feedforward term. (You might use simulink, or you might evaluate the difference equations using a for loop, for example.)