Today: State (Space) Feedback

Lecture 11
ECE 147B
Digital Control

Reading: FPW 7.2, 8.1
or: FV 6.5, (9.1), 9.2

Topics:
- Quick, last-minute REVIEW! ("Pop Quiz")
- "Direct Design" revisited: unstable poles in G(z)?
- State space block diagrams
- Modeling the ZOH via ss equations
- State feedback: u=-Kx
- Control Canonical Form

Next Class: MIDTERM EXAM.
- You may bring one, single-single, self-prepared 8 ½ x 11 sheet of notes. No calculators, cell phones, etc.

Pop Quiz!

1. If the final value to a unit step input to G(z) is 5, then the final value to an IMPULSE input is:
   a. 5
   b. 0
   c. ∞
   d. There is not enough information to determine this.

2. If the final value to an impulse input to G(z) is 5, then the final value to a unit step input is:
   a. 5
   b. 0
   c. ∞
   d. There is not enough information to determine this.

Pop Quiz!

3. The Final Value Theorem says:
   a. Just blindly plug in "z=0", always.
   b. Just blindly plug in "z=1", always.
   c. Given transfer function G(z)=Y(z)/X(z) has some input X(z), the final value of Y(z) will be: $\lim_{z \to 1}\left(\frac{z-1}{z} X(z) \cdot G(z)\right)$
   d. Use definition in "c.", but only apply IF POLES ARE STABLE!

Pop Quiz!

4. For "Direct Design" of controller C(z) of an open-loop plant, G(z), to yield some desired closed-loop plant, P(z), which statements are TRUE:
   a. P(z) cannot respond faster than G(z), since C(z) is causal.
   b. Zeros of G(z) outside the unit circle cannot be "canceled" using poles of C(z), since C(z) can't have unstable poles.
   c. Unstable poles of G(z) should just become zeros in C(z), using "pole-zero cancellation".

Direct Design: One last thing!

State space: Block diagram of equations

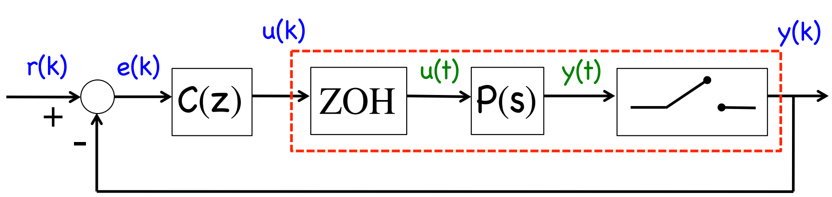
1. Dynamics. (What internal system does, "open loop".)

2. Outputs. (Things we can MEASURE.)

State space: Block diagram of equations

1. Control: A way to MODIFY the dynamics. i.e., set u as a function of X, via a "control law", e.g., a "PD controller" (for example)...

"Case 2": Modeling the ZOH plus sampling



"Case 2": Modeling the ZOH plus sampling

The Matrix Exponential:

$$e^{M} = \sum_{k=0}^{\infty} \frac{1}{k!} M^{k}$$

Example 1: M=1
(M just a 1x1 matrix, i.e., a scalar…)

Properties:

$$e^{M \times 0} = I$$

$$e^{M(s+t)} = e^{Ms} e^{Mt}$$

$$e^{-Mt} e^{Mt} = I$$

$$\frac{de^{Mt}}{dt} = M e^{Mt}$$

Example 2: $\ddot{x} + 5\dot{x} + 10x = u \quad (T = 1\,\mathrm{sec})$

Approximations, for "small T"…

Note: Analogous approximation trick applies for "e^{AT}" when "AT" has "small" entries as for "e^{sT}" when "sT" has small magnitude:

Example: $\ddot{x} + 2\dot{x} + 5x = u \quad (T = 0.01\,\mathrm{sec})$

Forward Euler approximation:

State space: Poles and Zeros

1. Dynamics. (What internal system does, "open loop".) -> **POLES**

   Solutions for $x \neq 0$, such that $u=0$?

   $$sx = Ax + Bu \qquad\qquad zx = A_d x + B_d u$$

2. Outputs. (Things we can MEASURE.) -> **ZEROS**

   Solutions for $u \neq 0$, such that $y=0$?

   $$Y = Cx + Du \qquad\qquad Y = C_d x + D_d u$$

State space: CLOSED-LOOP Poles (and Zeros)

1. Control: A way to MODIFY the dynamics. i.e., set u as a function of X, via a "control law", e.g., a "PD controller" (for example)...

   State feedback: $u=-KX$

$$sX = Ax + Bu = Ax - BKx \qquad\qquad zX = A_d x + B_d u = A_d x - B_d Kx$$

State feedback example

$$m\ddot{x} + b\dot{x} + kx = f$$

$$X \equiv \begin{bmatrix} \dot{x} \\ x \end{bmatrix}$$

$$K = [?]$$

- Say you are given the system dynamics and state variable definition at left. (m=1, b=2, k=5)
- What size must matrix "K" be?
- What values in K would result in closed loop poles with ω_n=10 (rad/s) and ζ=1?

"Control Canonical Form"

$$A = \begin{bmatrix} -a_1 & -a_2 & & -a_{n-1} & -a_n \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & & 0 & 0 \\ & \vdots & \ddots & 0 & 0 \\ 0 & 0 & & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$K = \begin{bmatrix} K_1 & K_2 & \cdots & K_{n-1} & K_n \end{bmatrix}$$

$$A - BK = \begin{bmatrix} -a_1 - K_1 & -a_2 - K_2 & & -a_{n-1} - K_{n-1} & -a_n - K_n \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & & 0 & 0 \\ & \vdots & \ddots & 0 & 0 \\ 0 & 0 & & 1 & 0 \end{bmatrix}$$

“Control Canonical Form”

Example:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

“Control Canonical Form”

Example:

$$A = \begin{bmatrix} 3 & -5 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

For state feedback control “u=-Kx”, solve for K such that closed-loop poles are at s=-3, s=-3.

"Control Canonical Form"

Example:

$$A = \begin{bmatrix} 0 & 1 \\ -5 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

For state feedback control "u=-Kx", solve for K such that closed-loop poles are at s=-3, s=-3. HINT: Same dynamics but different definition of "X" here!

"Control Canonical Form"

Example:

$$A_d = \begin{bmatrix} 0.6 & -1.2 \\ 1 & 0 \end{bmatrix} \qquad B_d = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

For state feedback control "u=-Kx", solve for K such that closed-loop poles are at z=0, z=0. This is "deadbeat" control!