Today: Estimator design via pole placement

Lecture 13
ECE 147B
Digital Control

Reading: FPW 8.2.1-8.2.4
or: FV 9.5.1

Topics:
- Pole Placement (to control system dynamics)
- Controllability
- State estimation: open-loop approach
- State estimation: feedback approach
- Estimator design via pole placement
- Observability (compared to Controllability)

Next Class: Reduced-order estimators. Separation Principle.
- FPW 8.2.5, 8.3 <or> FV 9.5.2, 9.6 (RS-13)

"Control Canonical Form" – QUICK REVIEW

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & -a_3 & -a_4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

1. Off-diagonal "ONES", and otherwise all ZEROS below the first row.

2. B matrix ONLY non-zero in top row.

3. All states are interrelated directly via either "s" (CT) or "z" (DT) operator:
   - Highest deriv state (CT) [largest "k" index (DT)] is "x1".
   - Lowest deriv state (CT) [lowest "k" index state (DT)] is "xn".

4. First row (therefore) gives NEGATIVE coeffs of char. eqn. (open-loop sys).

"Control Canonical Form" - REVIEW

Example: For state feedback control "u=-Kx", solve for K such that closed-loop poles are at s=-3, s=-3.

$$K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -5 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad A - BK = \begin{bmatrix} 3-K_1 & -5-K_2 \\ 1 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ -5 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad A - BK = \begin{bmatrix} 0 & 1 \\ -5-K_1 & 3-K_2 \end{bmatrix}$$

"Control Canonical Form" – DT example

Example:

$$K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}$$

$$A_d = \begin{bmatrix} 0.6 & -1.2 \\ 1 & 0 \end{bmatrix} \quad B_d = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad A - BK = \begin{bmatrix} & \\ & \end{bmatrix}$$

For state feedback control "u=-Kx", solve for K such that closed-loop poles are at z=0, z=0. This is "deadbeat" control!

Pole Placement – for system dynamics

1. For systems written in CONTROL CANONICAL form:
   - EASY to solve for "K" for (arbitrary) closed-loop poles! You are DIRECTLY setting coefficients of the characteristic equation!
2. For "MESSIER" cases (i.e., A matrix NOT with an off-diagonal of "ones"):
   - Solving for "K" BY HAND would be HARD and TEDIOUS.

General solution? → "pole placement".
First, let's look at MATLAB commands:

```
A = rand(4,4); % some random, 4th-order sys…
B = rand(4,1); % …including actuator inputs
pnow = eig(A)  % open-loop poles
pdes = [-1 -2 -3 -4]; % desired poles (4 poles; 4th-order system…)
K = place(A,B,pdes) % "pole placement" command in matlab


K2 = acker(A,B,pdes) % alternate "pole placement" command


pact = eig(A-B*K)  % ACTUAL closed-loop poles!
```

Controllability

Given matrices A and B (for CT), or A_d and B_d (for DT), the CONTROLLABILITY matrix is:

$$C \equiv [B, \quad AB, \quad A^2B, \quad \ldots, \quad A^{n-1}B]$$

The system is "CONTROLLABLE" if and only if
- this matrix has FULL RANK
- (meaning it is INVERTIBLE).

Note the CHARACTERISTIC EQUATION for your desired CLOSED-LOOP (still nth-order) dynamics can be written as:

$$z^n + \alpha_1 z^{n-1} + \alpha_2 z^{n-2} + \ldots + \alpha_n = 0$$

GAIN MATRIX "K" needed will be:

$$K = [0, \ldots 0, 0, 1] \;\; C^{-1} \alpha_c(A)$$

Where:

$$\alpha_c(A) \equiv A^n + \alpha_1 A^{n-1} + \alpha_2 A^{n-2} + \ldots + \alpha_n I$$

Controllability

What is the definition of "CONTROLLABILITY"?

For a LINEAR system, there are two, equivalent definitions:

1.

2.

Controllability – Pole Placement equations

On web: (1) google "byl". (2) Click "ECE Robotics at UCSB" link. (3) Click "ECE 147B" link. (4) goto "Lectures" tab. (5) Under "Lecture 13", download "example_controllability.m"

```
n=4;
A = rand(n,n)   % n states
B = rand(n,1)   % single input here...
Co = ctrb(A,B)   % CONTROLLABILITY matrix
```

$$\boxed{C \equiv [B, \quad AB, \quad A^2B, \quad ..., \quad A^{n-1}B]}$$

```
RankCo = rank(Co)   % iff full rank -> Controllable (invertible)!
sdes = [1 2 3 4 5] % desired characteristic eqn
```

$$\boxed{z^n + \alpha_1 z^{n-1} + \alpha_2 z^{n-2} + ... + \alpha_n}$$

```
pdes = roots(sdes) % desired poles
alpha_c = 0*A; % initialize
for ni = 1:length(sdes)
    alpha_c = alpha_c + sdes(ni)*A^(5-ni);
end
```

$$\boxed{\alpha_c(A) \equiv A^n + \alpha_1 A^{n-1} + \alpha_2 A^{n-2} + ... + \alpha_n I}$$

```
K = [zeros(1,n-1), 1] * inv(Co) * alpha_c

pact = eig(A-B*K)
```

$$\boxed{K = [0, ...0,0,1] \; C^{-1} \alpha_c(A)}$$

Controllability - More MATLAB examples...

```
A = [-2 -5; 1 0];    % control canonical form
B = [1; 0];
Co = ctrb(A,B)
% Co: CONTROLLABILITY matrix
```

$$C \equiv [A, \quad AB] = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

```
RankCo = rank(Co)  % iff full rank -> Controllable (invertible)!
sdes = [1 20 100]
% sdes: desired char eqn coef's
pdes = roots(sdes)
% pdes: desired poles
alpha_c = 0*A; % initialize
for ni = 1:length(sdes)
    alpha_c = alpha_c + sdes(ni)*A^(5-ni);
end
```

$$s^2 + 20s + 100 = 0$$

$$\alpha_c(A) \equiv A^2 + 20A + 100A^0 = \begin{bmatrix} 59 & -90 \\ 18 & 95 \end{bmatrix}$$

```
K = [zeros(1,n-1), 1] * inv(Co) * alpha_c

pact = eig(A-B*K)
% pact: CL poles
```

$$K = [0,1]C^{-1}\alpha_c(A) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 59 & -90 \\ 18 & 95 \end{bmatrix}$$

State Estimation - Problem Statement

Sometimes, feedback control requires we "estimate" some of the states because, for example:

1. We might not sense all states in the output vector y, e.g.:

$$X = \begin{bmatrix} \dot{x} \\ x \end{bmatrix} \qquad y = [0 \quad 1]X + [0]u$$

2. Our measurements might be noisy, e.g.:

$$Y = CX + DU + V$$

PROBLEM STATEMENT: If we start with some "initial value" as an ESTIMATE of the state vector (for k=0), how can ESTIMATE the states for k>=0 (over time)?

State Estimation – "Open loop" approach

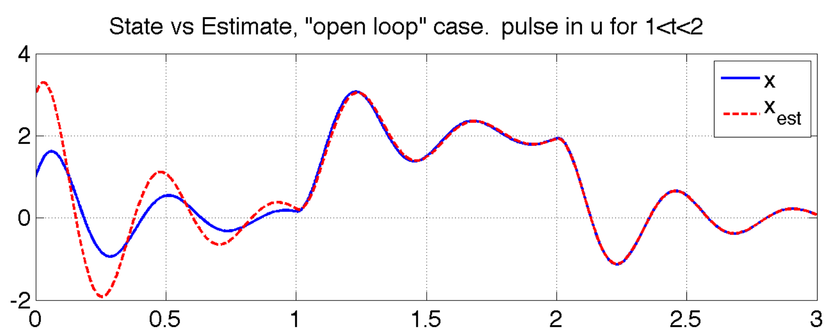
Start with some "initial value" as an ESTIMATE of the states…
…and assume this vector of value evolves via "Ax+Bu" dynamics.

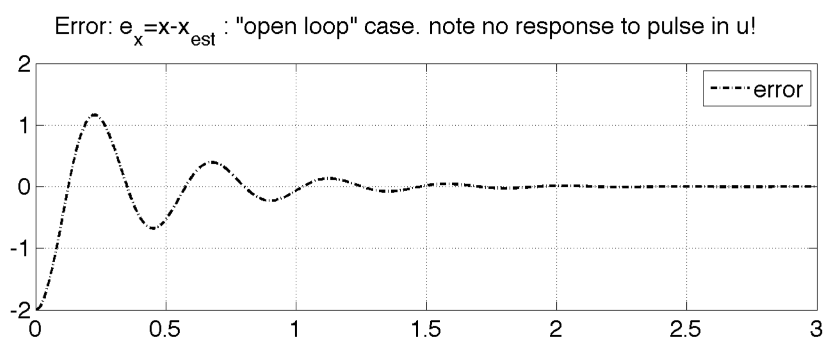
State: $x(k+1)$

Estimate of state: $\bar{x}(k+1)$

Error in the esimate: $e_x(k+1) = x(k+1) - \bar{x}(k+1)$ or: $\tilde{x}(k+1)$

Error dynamics?





State Estimation – "feedback" approach

Start with some "initial value" as an ESTIMATE of the states…
…use BOTH a MODEL ("Ax+Bu") and a MEASUREMENT ("y=Cx+Du").

Estimate of state WITH MEASUREMENT FEEDBACK : $\hat{x}(k+1)$

But how do we "feed back" MEASUREMENT info?

State Estimation – "feedback" approach

(Continued…)

Error dynamics?

Looks similar to pole placement?

Let's compare:

- pole placement for state feedback

- pole placement for estimator design

MATLAB commands?

Let's compare:

- pole placement for state feedback

- pole placement for estimator design

Observability (vs Controllability)

→ When can we set estimator poles arbitrarily?
"Controllability" question is ANALOGOUS to "Observability"

Observability (vs Controllability)

"Controllability" vs "Observability"