Today: Reduced-Order Estimators

Lecture 15
ECE 147B
Digital Control

Reading: FPW 8.2.5, 8.3
or: FV 9.5.2, 9.6 (RS-13)

Topics:
- Separation Principle
- Guidelines for pole placement (control and est.)
- Reduced-Order Estimator
- Frequency Response Methods

Next Class: Frequency Response Methods.
- FPW 7.4 <or> FV (6.4), 6.5, (6.6) (RS-1)

Using the state estimate for feedback...

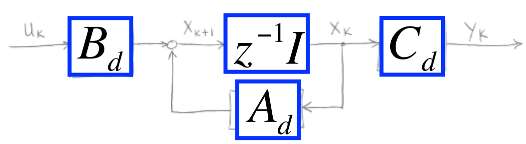
Recall STATE FEEDBACK
Control uses the true state:

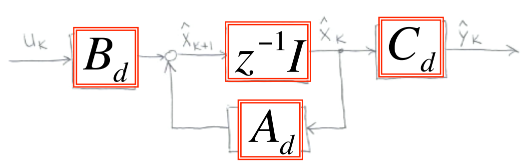
$$\boxed{u(k) = -Kx(k)}$$

What if we use the state ESTIMATE
for feedback control, instead?

$$\boxed{u(k) = -K\hat{x}(k)}$$

Block Diagram:





Closed-loop Dynamics, for state (x) and error (e_x)

MATRIX form for DYNAMICS: eigenvalues?

$$\boxed{\begin{bmatrix} x_{k+1} \\ e_{k+1} \end{bmatrix} = \begin{bmatrix} (A-BK) & BK \\ 0 & (A-LC) \end{bmatrix} \begin{bmatrix} x_k \\ e_k \end{bmatrix}}$$

NOTE - Matrix above has "Upper Block Diagonal" form. Examples:

$$\boxed{\det\left(\begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} a & c \\ 0 & b \end{bmatrix} \right) = 0}$$

$$\boxed{\det\left(\begin{bmatrix} z & 0 & 0 & 0 \\ 0 & z & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & z \end{bmatrix} - \begin{bmatrix} a_1 & a_2 & c_1 & c_2 \\ a_3 & a_4 & c_3 & c_4 \\ 0 & 0 & b_1 & b_2 \\ 0 & 0 & b_3 & b_4 \end{bmatrix} \right) = 0}$$

Eigenvalue examples, via MATLAB

Below is code to "see" some of the ideas for yourself – at least for our toy examples:

$$\boxed{\det\left(\begin{bmatrix} z & 0 & 0 & 0 \\ 0 & z & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & z \end{bmatrix} - \begin{bmatrix} a_1 & a_2 & c_1 & c_2 \\ a_3 & a_4 & c_3 & c_4 \\ 0 & 0 & b_1 & b_2 \\ 0 & 0 & b_3 & b_4 \end{bmatrix} \right) = \det(zI - M) = 0}$$

```
syms a1 a2 a3 a4 b1 b2 b3 b4 c1 c2 c3 c4 z
A = [a1 a2; a3 a4];
B = [b1 b2; b3 b4];
C = [c1 c2; c3 c4];
M = [A C; zeros(2,2) B]
M =
[ a1, a2, c1, c2]
[ a3, a4, c3, c4]
[  0,  0, b1, b2]
[  0,  0, b3, b4]
zI = z*eye(4)
zI =
[ z, 0, 0, 0]
[ 0, z, 0, 0]
[ 0, 0, z, 0]
[ 0, 0, 0, z]

det(zI-M)
ans =
z^4 - a4*z^3 - b1*z^3 - b4*z^3 - a1*z^3 + a1*a4*z^2 - a2*a3*z^2 + a1*b1*z^2 + a1*b4*z^2
+ a4*b1*z^2 + a4*b4*z^2 + b1*b4*z^2 - b2*b3*z^2 + a1*a4*b1*b4 - a1*a4*b2*b3 -
a2*a3*b1*b4 + a2*a3*b2*b3 - a1*a4*b1*z + a2*a3*b1*z - a1*a4*b4*z + a2*a3*b4*z -
a1*b1*b4*z + a1*b2*b3*z - a4*b1*b4*z + a4*b2*b3*z

simplify(det(zI-M))
ans =
(a2*a3 - a1*a4 + a1*z + a4*z - z^2)*(b2*b3 - b1*b4 + b1*z + b4*z - z^2)
```

Separation Principle

For feedback with an ESTIMATOR,

1) CONTROL for nth-order system has n poles, determined by eig(A-BK).

2) ESTIMATION of n states results in another n poles, determined by eig(A-LC)

3) TOTAL system now has 2*n poles:

How to place poles, to set "K" and Lp"?

Controller poles:

Estimator poles:

Effects of Noise?

$$\boxed{\begin{bmatrix} x_{k+1} \\ e_{k+1} \end{bmatrix} = \begin{bmatrix} (A-BK) & BK \\ 0 & (A-LC) \end{bmatrix} \begin{bmatrix} x_k \\ e_k \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \end{bmatrix} n_k}$$

Example: control and estimation, w/ MATLAB

$$A = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$p_{ct} = [-10 + 10j, \quad -10 - 10j]$$

CT (continuous time) – n states (x), m inputs (u), l outputs (y)

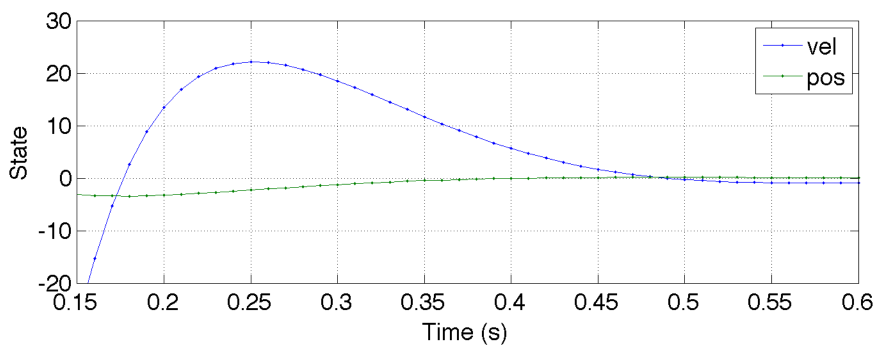
DT (discrete time), [T = 0.01 sec] – system resulting from ZOH

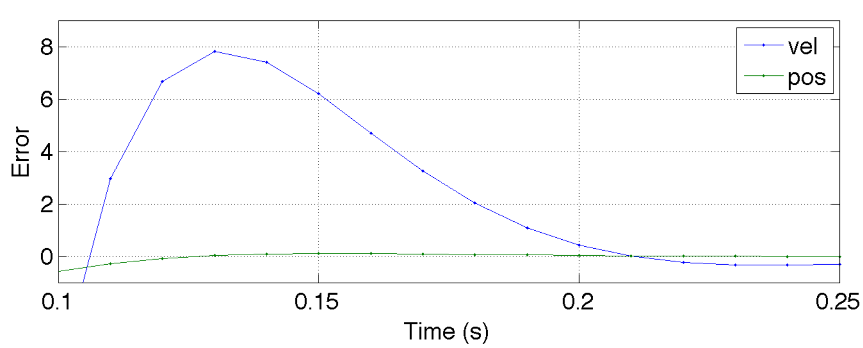
```
M=expm([A, B; zeros(m,n+m)]*T)
Ad = M(1:n, 1:n);
Bd = M(1:n, n+1:end);

pdt = exp(pct*T);



Kd = place(Ad, Bd, pdt)
```





Reduced-order Estimator

What if you only want to "estimate" a subset of the entire state vector? e.g., You only want to estimate "unmeasured" states, which are not inside y. For example, in LAB:

Position:

Velocity:

Let's partition the states, $x(k) = \begin{bmatrix} x_a(k) \\ x_b(k) \end{bmatrix}$

Reduced-order Estimator

This looks a lot like

$$\boxed{\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}}$$

...if we make the following SUBSTITUTIONS:

<u>Was</u> → <u>Now</u>

1) x(k) →

2) A →

3) Bu(k) →

4) y(k) →

5) C →

GOAL is to match THIS form: $\boxed{\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L(y(k) - C\hat{x}(k))}$

(But estimate ONLY $x_b(k)$...)

Reduced-order Estimator

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L(y(k) - C\hat{x}(k))$$

$$\hat{x}_b(k+1) =$$

Reduced-order Estimator

Frequency Response Methods

Frequency Response Methods