

Today: Frequency Response Methods

Lecture 16

ECE 147B

Digital Control

Reading: RS-13 (!!!), FPW 7.4 <or> FV (6.4), 6.5, (6.6) (RS-1)

Topics:

- Reduced-Order Estimator (Review)
- Frequency Response Methods
- Reference Tracking

Next Class:

- Reference Tracking (continued)

*** Review ***

Reduced-order Estimator

What if you only want to "estimate" a subset of the entire state vector? e.g., You only want to estimate "unmeasured" states, which are not inside y . For example, in LAB:

Position: encoder - measured, x

Velocity: Not measured! $\leftarrow$ unmeasured, $\dot{x}$

Let's partition the states, $x(k) = \begin{bmatrix} x_a(k) \\ x_b(k) \end{bmatrix}$ a : directly measured
 b : not measured $\leftarrow$ only estimate $\hat{x}_b$ w/ red-ord. est.

We will get this directly from measurement

$$\begin{bmatrix} x_a(k+1) \\ x_b(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} A_{aa} & A_{ab} \\ A_{ba} & A_{bb} \end{bmatrix}}_A \begin{bmatrix} x_a(k) \\ x_b(k) \end{bmatrix} + \underbrace{\begin{bmatrix} B_a \\ B_b \end{bmatrix}}_B u(k)$$

" x_{kn} "

#1 $x_b(k+1) = A_{bb} x_b(k) + (A_{ba} x_a(k) + B_b u(k))$

unknown unknown known

#2 $\rightarrow$ write an eqn for $\hat{x}_b(k+1) \dots \hat{x}_b(k)$ then #3 is calculate by #1 minus #2

Error dynamics

*** Review ***

Reduced-order Estimator

This looks a lot like

$$x(k+1) = Ax(k) + Bu(k)$$

$$y(k) = Cx(k) + Du(k)$$

...if we make the following SUBSTITUTIONS:

Was → Now

- 1) $x(k) \rightarrow x_b(k)$ ← states TO BE ESTIMATED
- 2) $A \rightarrow A_{bb}$ ← a sub-part of the full "A" matrix.
- 3) $Bu(k) \rightarrow (A_{ba}x_a(k) + B_b u(k))$ ← "Known" inputs now include BOTH "Bu" and also terms depending on $x_a(k)$ [known/measured].
- 4) $y(k) \rightarrow (x_a(k+1) - A_{aa}x_a(k) - B_a u(k))$ ← What is left on LHS of * once "A_{ab}x_b(k)" (like "Cx") is moved to RHS
- 5) $C \rightarrow A_{ab}$ ← "A_{ab}x_b(k)" is now analogous to "Cx"

GOAL is to match THIS form:
(But estimate ONLY $x_b(k)$...)

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L(y(k) - C\hat{x}(k))$$

Use piece ① through ⑤ above!

from last page

$$x_a(k+1) = A_{aa}x_a(k) + A_{ab}x_b(k) + B_a u(k)$$

Think of this as "Cx" in "y=Cx".

$$x_b(k+1) = A_{bb}x_b(k) + (A_{ba}x_a(k) + B_b u(k))$$

$$(x_a(k+1) - A_{aa}x_a(k) - B_a u(k)) = A_{ab}x_b(k)$$

*** Review ***

Reduced-order Estimator

Traditional (not "reduced") form, which we wish to match.

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L(y(k) - C\hat{x}(k))$$

#2 $\hat{x}_b(k+1) = A_{bb}\hat{x}_b(k) + (A_{ba}x_a(k) + B_b u(k)) + L(x_a(k+1) - A_{aa}x_a(k) - B_a u(k) - A_{ab}\hat{x}_b(k))$

We have a new estimator equation, as desired, but there is a PROBLEM! We do not have access to " $x_a(k+1)$ " yet! (Causality...)
(To be discussed NEXT PAGE...)

Recall:

#1 $x_b(k+1) = A_{bb}x_b(k) + (A_{ba}x_a(k) + B_b u(k))$

So, use "place" with
"A" = A_{bb}
"C" = A_{ab}

Error Dynamics? $e_b \equiv x_b - \hat{x}_b$, So, take #1 minus #2

$$x_b(k+1) - \hat{x}_b(k+1) = A_{bb}(x_b(k) - \hat{x}_b(k)) - L_r(A_{ab}x_b(k) - A_{ab}\hat{x}_b(k))$$

$$\therefore e_b(k+1) = [A_{bb} - L_r A_{ab}]e_b(k)$$

poles set by " $A_{bb} - L_r A_{ab}$ "

Uh oh!

from eqn *, last page...

***** Review *****

Reduced-order Estimator

⊗ Issue: How can we include " $x_a(k+1)$ " in our actual implementation of the reduced-order estimator?

Ans: Let $x_c(k) \equiv \hat{x}_b(k) - L_r x_a(k)$

$\therefore x_c(k+1) = \hat{x}_b(k+1) - L_r x_a(k+1)$

And note, $x_a(k+1) = A_{aa}x_a(k) + A_{ab}x_b(k) + B_a u(k)$

Get $\hat{x}_b(k+1)$ from #2

(cancels out!)

Regrouping:

$$x_c(k+1) = (A_{bb} - L_r A_{ab})\hat{x}_b(k) + (A_{ba} - L_r A_{aa})x_a(k) + (B_b - L_r B_a)u(k)$$

And then,

$$\hat{x}_b(k) = x_c(k) + L_r x_a(k)$$

$u(k) = -K \begin{bmatrix} x_a(k) \\ x_b(k) \end{bmatrix}$

See Roy Smith's
"Lecture 13"
notes! (RS-13)

Frequency Response Methods

We focus more on what is "New" about control design for discrete-time systems (as compared with design for continuous-time systems...

HOWEVER, "loop shaping" is still a valid approach! (i.e., gain and phase margin...)

Below are Quick Tips, summarizing the key ideas to apply toward DT systems.

1. Recall how to evaluate frequency response for some transfer function, " $G(z)$ ":

$$z = e^{j\omega T}$$

2. Primary effect of a ZOH on frequency response is the HALF-SAMPLE delay: -90 degree change at Nyquist frequency, so...

Frequency Response Methods

3. Only SOME rules for CT Bode (and Nyquist) plots STILL apply (for DT plots):

CT	DT
a. Can plot gain and phase BY HAND easily.	a. No. (well, not <u>"directly"</u> ...)
b. Can take an EXPERIMENT Bode plot.	B. Yes!
c. Nyquist stability criterion applies...	c. Yes – but be careful [See next page...]
d. Gain and phase margins can be used to interpret closed-loop response.	d. Yes!
e. Error constants (K_p or K_d) can be read DIRECTLY from Bode plot.	d. Yes!
f. Gain and phase curve computed easily for extra compensator poles or zeros.	e. No...
g. Effect of pole or zero in compensator easy to predict via Bode sketch.	f. No...

Frequency Response Methods

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d. Gain and phase margins can be used to interpret closed-loop response.	d. Yes!
e. Error constants (K_p or K_d) can be read DIRECTLY from Bode plot.	e. Yes!
f. Gain and phase curve computed easily for extra compensator poles or zeros.	f. No...
g. Effect of pole or zero in compensator easy to predict via Bode sketch.	g. No...

Frequency Response Methods

4. More details on NYQUIST CRITERION for DT systems:

$$Z = P + N_{CW}$$

5. Rules to sketch a Bode plot for a discrete-time (DT) system, $G(z)$, BY HAND.

a) Use “ $z=1$ ” to evaluate DC gain and phase.

b) Use “ $z=-1$ ” to evaluate gain and phase at NYQUIST frequency.

c) Use “ $z=e^{sT}$ ” mapping: $z = e^{sT}$ $s = \frac{1}{T} \log(z)$

d) For poles and zeros “near” $z=1$ (within .15 “or so”):
 $z \approx 1 + sT$ $s \approx \frac{1}{T}(z - 1)$

e) Don't confuse “half-sample” delay stuff into all this! That is for the SPECIFIC CASE of taking a CT plant, $G(s)$, and preceding it with a ZOH and then sampling the output...

Freq. Resp. - Examples

Use MATLAB to check these!!

$$G_1(z) = \frac{K}{(z - 0.9)(z - 0.9)}$$

Stable for what “K” values?

Sketch Nyquist for “K=1” case.

Sketch Bode for “K=1” case.



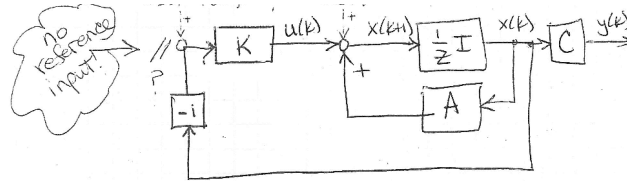
Freq. Resp. - Examples

Use MATLAB to check these!!

Sketch Bode plot BY HAND: $G_2(z) = \frac{10(z - 0.98)}{(z - 0.9)}$

Reference Tracking

So far, we have only considered state space control for a “regulator”:

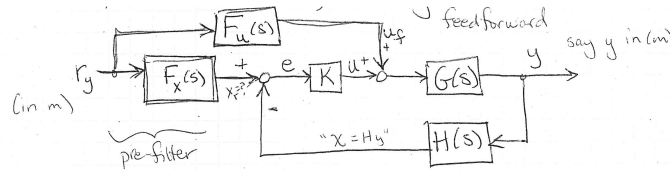


Questions: How can we

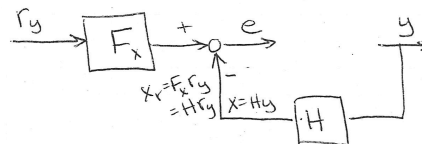
- 1) Compare $x(k)$ (or an ESTIMATE of $x(k)$) with a desired REFERENCE, $x_r(k)$?
- 2) Generate a feedforward term, $u_f(k)$, to eliminate steady state error?

Reference Tracking

First, consider the familiar "SISO" system case:

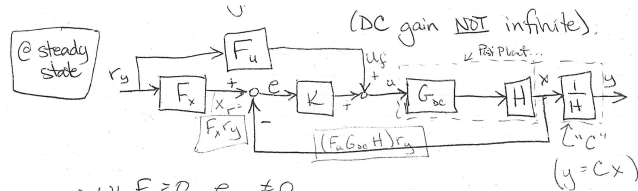


F_x should scale output "y" to match (in units) the feedback term:



Reference Tracking

...And F_u should result in NO FEEDBACK ERROR at steady state:



- w/ $F_u > 0$, $e_{ss} \neq 0$.
- If we want $e_{ss} = 0$, $F_x r_y - (F_u G_{dc} H) r_y = 0$

MIMO equivalent:

Now, consider a MIMO state space system form:

