Announcements

- Lab Ø Wed 7-9:50 (Not Required, in 3120A HFH)
- Prelab 1 due Monday @ 5pm, outside 3120A (Jan. 14)
- HW1 due next Friday @ 5pm (Jan. 18)

Handouts

1. Syllabus
2. Lecture 1 Notes

Today: Review of Continuous-Time (CT) Dynamics and Control (ECE 147A material) **Lecture 1**

Reading: FPW (Ch.1 pp.1-8, optional), 2.1-2.5 (pp.11-24, 31-41)
or: FV 2.1, 5.1-5.3 [*Today's readings are review, as needed.*]

Either book, not Both!

Topics:
- Overview: CT vs DT analysis
- Time Response
- Frequency Response
- Block Diagrams
- Stability and Stability Margins
- Control: What (are we trying to do), and How (to do it)

Next Class: Difference Equations. Sampling. Tustin (Trapezoid) Approx.
- FPW: 3.1-3.2 (pp. 57-65), 4.1 (pp. 73-78)

or: - FV: FV 2.1, 3.1-3.2, 3.4, 6.3.1, 6.3.3

note similarities

147A (CT)

1) Continuous time (CT), x(t).
2) Differential Equations, e.g.,

$$a\ddot{y} + b\dot{y} + cy = d\dot{x} + ex$$

3) Laplace Transforms:

$$Xs^n \leftrightarrow \frac{d^n x}{dt}$$

$$X\frac{1}{s^m} \leftrightarrow \underbrace{\int x}_{m \text{ integrals}}$$

4) CT Transfer Function (TF):

$$\frac{Y(s)}{X(s)} = \frac{ds+e}{as^2+bs+c}$$

a ratio of polynomials

$$G(s) = \frac{b(s)}{a(s)}$$

$\boxed{\leftarrow \text{in general} \rightarrow}$

147B (DT)

1) Discrete time (DT) samples, xk.
2) DIFFERENCE Equations, e.g.,

$$ay_{k+2} + by_{k+1} + cy_k = dx_{k+1} + ex_k$$

3) z-Transforms:

$$Xz^n \leftrightarrow x_{k+n}$$

(e.g., $X\frac{1}{z^m} = Xz^{-1m} \leftrightarrow x_{k-m}$)

3) DT TF: $\dfrac{Y(z)}{X(z)} = \dfrac{dz+e}{az^2+bz+c}$

$$G(z) = \frac{b(z)}{a(z)}$$

147A (CT)	147B (DT)
5) POLES: Roots of DENOMINATOR (i.e., find solutions for s...) $a(s) = 0$	5) POLES: same, except polynomial is just in "z" instead of "s": $a(z) = 0$

Poles affect the FREE RESPONSE (t>0).

6) ZEROS: Roots of NUMERATOR $b(s) = 0$	6) ZEROS: same, except "z", not "s" $b(z) = 0$

Zeros affect the initial condition ~~**FREE RESPONSE**~~ **(t=0).**, including velocity & higher derivs.

Check yourself... $P(s) = \frac{s-4}{s^2+4}$ ← CT TF

"continuous time transfer fn"

1. What are the poles and zeros? ← Roots of num! $s - 4 = 0$ $\boxed{s = 4}$

Roots of den: $s^2+4=0 \rightarrow \boxed{s = \pm 2j}$

2. What order system is this?

Equals # of poles, 2.

Check yourself... $P(s) = \frac{s-4}{s^2+4}$

3. Is P(s) proper? Yes! STRICTLY proper! (Num. is lower order than denom.)

4. Is P(s) causal? Yes! Current response (output) does NOT depend on future input.

5. What determines if a CT TF is STABLE? (and Why?)

[*HINT*: e^{st}] All poles must have real parts < 0, so "e^{st}" will decay in magnitude as $t \to \infty$.

6. Sketch a UNIT STEP response for $P_2(s)$: $P_2(s) = \frac{s-15}{s+5}$ ← $\tau = \frac{1}{5} = 0.2$

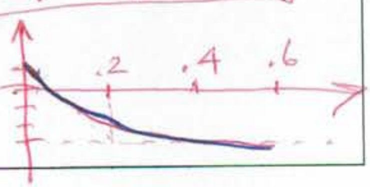
[*HINT*: IVT and FVT?]

Final Value Thm.

$$\boxed{\lim_{s \to 0} \left(s \cdot \frac{1}{s} \cdot \left(\frac{s-15}{s+5}\right)\right) = \frac{-15}{5} = -3}$$

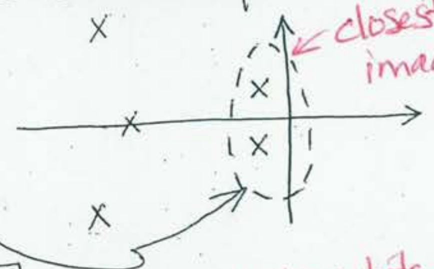
initial value thm

$$\boxed{\lim_{s \to \infty} \left(s \cdot \frac{1}{s} \cdot \left(\frac{s-15}{s+5}\right)\right) = \frac{15}{15} = 1}$$



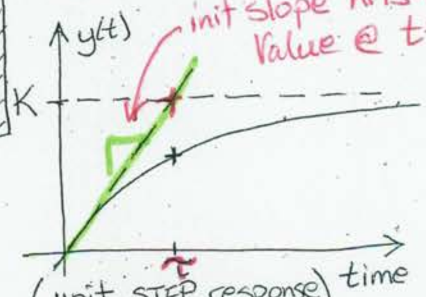
Time Response (CT, only)

- 1^{st}-order & 2^{nd}-order responses are important!
  - Higher-order systems often have one or two DOMINANT POLES:

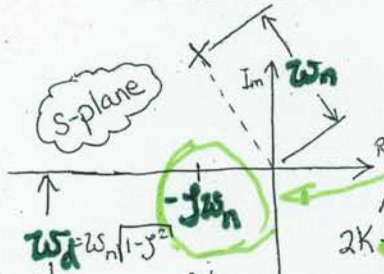


i.e. SLOWEST REAL PART.

- 1^{st}-order: $\boxed{H(s) = \frac{K}{\tau s + 1}}$
  - pole: $s = \frac{-1}{\tau}$
  - unit step response: $y(t) = K(1 - e^{-t/\tau})$



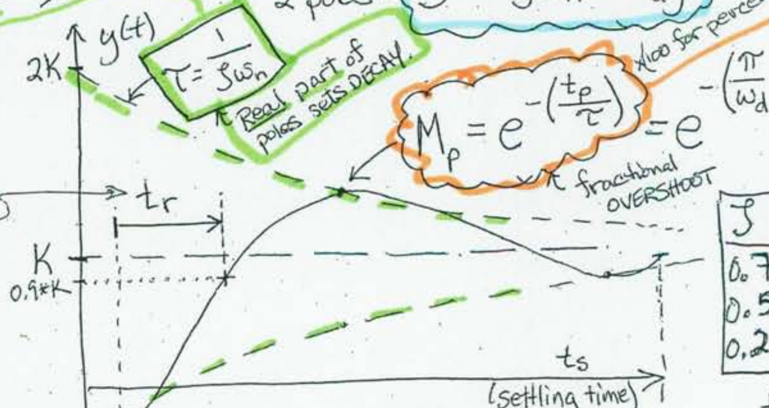
- 2^{nd}-order: $\boxed{H(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$



- 2 poles: $s = -\zeta\omega_n \pm \omega_d j$

$\tau = \frac{1}{\zeta\omega_n}$ ← Real part of poles sets DECAY.

$M_p = e^{-\left(\frac{t_p}{\tau}\right)} = e^{-\left(\frac{\pi}{\omega_d}\cdot\zeta\omega_n\right)}$ Exact! (×100 for percent) ← fractional OVERSHOOT



$t_r \approx \frac{1.8}{\omega_n}$ "rise time" (10% → 90%)

ζ	M_p
0.7	.05 = 5%
0.5	.16 = 16%
0.2	.53 = 53%

$t_p = \frac{\pi}{\omega_d}$ ← peak overshoot @ t_p... EXACT eqn here!!

Exact!

ω_d: Sets oscill. period
$\zeta\omega_n$: Sets decay env.

Time Response vs. Frequency Response

*For General n^{th}-order system, "Homogeneous" OR (aka) "Free" Response?

$$\boxed{y(t)=\sum_{i=1}^{n} K_i e^{s_i t}}$$

S-plane

$\omega_d = 9$ (rad/s)

$\zeta\omega_n = 2$

Evaluate $G(s)$ @ "$s = j\omega$", from $\omega = 0$ rad/s (DC) to $\omega = +\infty$ (high-freq limit).

ω_n important for freq. resp.

$$\omega_n = \sqrt{2^2+9^2}\ \left(\frac{rad}{sec}\right)$$

"Break point" @ ω_n.

In our example,

$$y(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$
$$\dot{y}(t) = s_1 K_1 e^{s_1 t} + s_2 K_2 e^{s_2 t}\ ?$$

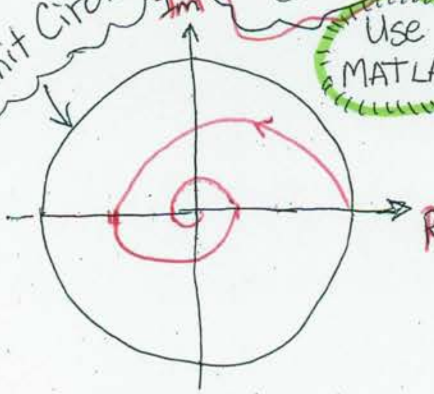
where $\boxed{\begin{array}{l} s_1 = -2+9j \\ s_2 = -2-9j \end{array}}$

MATLAB Reality Check...

Hmmm, what does "$e^{s_1 t}$" look like for $s_1 = -2+9j$?

Use MATLAB!

Unit Circle



w/ s_1 & s_2 combined, we end up w/ $y(t)$ REAL.

*Example Initial Condition (Free) Response for: $\boxed{\begin{array}{l} y(t=0) = 8 \\ \dot{y}(t=0) = 200 \end{array}}$

(@ $t=0$, $e^{st} = e^0 = 1$ ...)

* K_1 & K_2 must satisfy: (Check Yourself!)

$$y(0) = 8 = K_1 e^{s_1 0} + K_2 e^{s_2 0}$$
$$\dot{y}(0) = 200 = s_1 K_1 + s_2 K_2\ ?$$

so...

$$\underbrace{\begin{bmatrix} 1 & 1 \\ s_1 & s_2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} K_1 \\ K_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 8 \\ 200 \end{bmatrix}}_{b}$$

Use MATLAB!

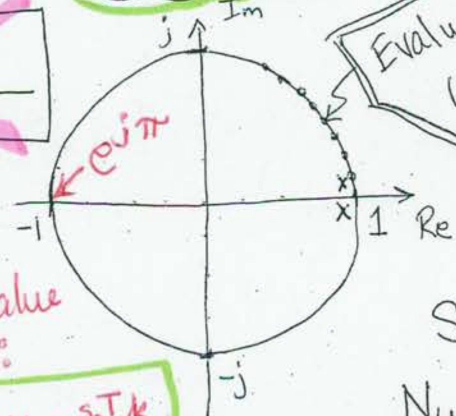
$\boxed{x = A \backslash b} \rightarrow \boxed{K_1 = 4-12j}\ \boxed{K_2 = 4+12j}$

$\boxed{\text{Time Response}}$ vs. $\boxed{\text{Frequency Response}}$

Z-plane

Wrong Question!

$\boxed{\cancel{y(t) = \ ?}}$



Evaluate $G(z)$ @ "$z = e^{j\omega T}$" (from $\omega=0$ to $\omega = +\infty$)

These "$e^{j\omega T}$" points all lie on the <u>UNIT CIRCLE</u>

Only meaningful to think about value at integer samples:

TBD...

$$\boxed{y(k) = K_1 e^{s_1 T k} + K_2 e^{s_2 T k}}$$

Sample time = T

Nyquist freq. $= \frac{\pi}{T}$ "$= \omega_{Ny}$"

* @ ω_{Ny}, $z = e^{j\frac{\pi}{T}T} = e^{j\pi} = \underline{-1}$?

(Go below first...)

* Imagine free response of $\boxed{G(s) = \dfrac{85}{s^2+4s+85}}$

(Same poles as s-plane time resp. example.) $s_1 = -2+9j$, $s_2 = -2-9j$

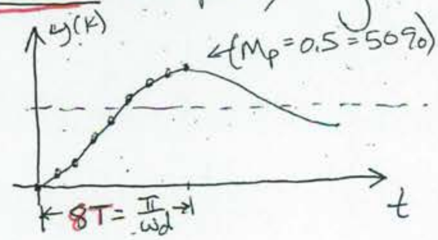
Same angle @ each step!

$\boxed{M_p = 0.5}$

Poles of DT system w/ free response shown are: $z_1 = e^{s_1 T}$, $z_2 = e^{s_2 T}$

$e^{s_1 T k}$ ← for $k = 0, 1, 2, \ldots$

(<u>Discrete</u> samples, only!)



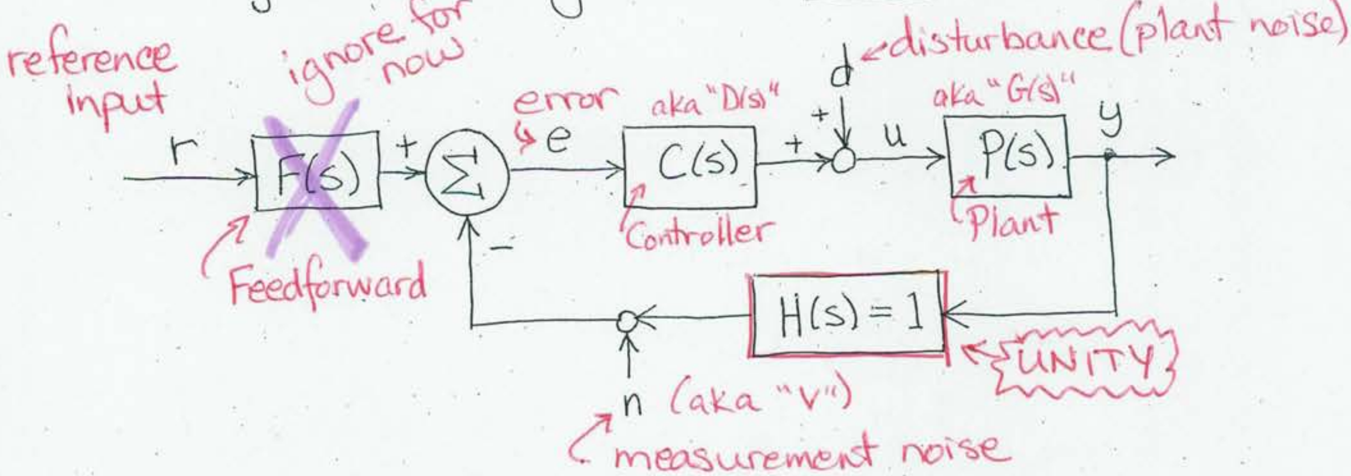
$$\boxed{\begin{array}{l} e^{s\Delta} \approx 1 + s\Delta, \text{ for } |s\Delta| << 1.0 \\ e^{s_1 * .001} \approx \underline{.998 + .009j} \text{ ? MATLAB check!} \end{array}}$$

$(1 + \frac{-2}{1000}) + (\frac{9}{1000})j$

No sensor GAIN nor DYNAMICS

BLOCK DIAGRAMS

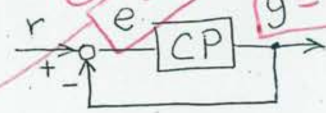
"Unity Feedback Configuration", i.e., $\boxed{H=1}$ in feedback path.



BLOCK DIAGRAM ALGEBRA

$$\frac{Y(s)}{R(s)} = \boxed{\frac{CP}{1+CP}}\,④$$

② $\boxed{e = r - eCP}$ $\boxed{y = e \cdot CP}$ ①



↑ Set all other inputs to ZERO & solve algebraically for $\frac{Y(s)}{R(s)}$...

$\therefore e + eCP = r$

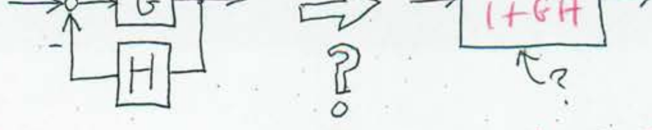
$e(1+CP) = r$

Sensitivity: ③ $\boxed{\frac{E}{R} = \frac{1}{1+CP}}$ → ∴ write ① as: $y = \left(\frac{1}{1+CP}\right) CP \cdot R$

Complementary Sensitivity: $1 - \frac{E}{R} = \frac{1+CP}{1+CP} - \frac{1}{1+CP} = \boxed{\frac{CP}{1+CP} = \frac{Y}{R}}$ ⑤

Characteristic Equation: $\boxed{1 + CP = 0}$ ← same for all TF's! Since same denom. same poles

More Generally:



"Black's Formula"

$$\frac{Y(s)}{X(s)} = \frac{G}{1+GH}$$

* Whole system has same poles, regardless of particular input or output of a

Frequency Response — Bode & Nyquist plots

Radians Per Second! (not Hz)

*** CT procedure for freq. resp. of CT TF $G(s)$:**

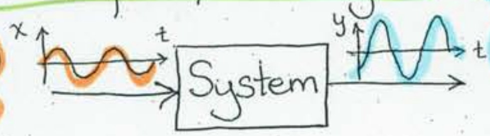
1. Sequentially "plug in" $\boxed{s = j\omega}$, using ω s.t. $0 \le \omega \le \infty$, into $G(s)$.
2. Calculate magnitude: $\boxed{|G(j\omega)| = \sqrt{(Re(G(j\omega)))^2 + (Im(G(j\omega)))^2}}$
3. Calculate phase: $\boxed{\angle G(j\omega) = \tan^{-1}\left(\frac{Im(G(j\omega))}{Re(G(j\omega))}\right)}$

* Bode plot: Plot MAGNITUDE vs ω on log-log scale.
AND Plot PHASE vs ω on semilogx scale.

* Nyquist plot: Plot only MAGN. & PHASE, on polar plot, as $\omega \uparrow$.

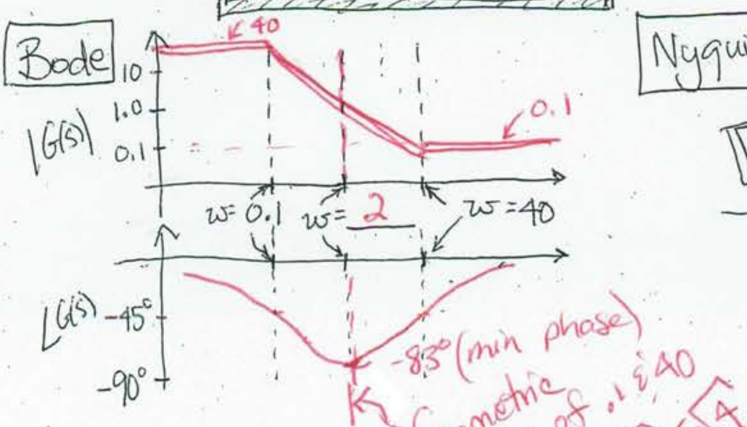
· What is a freq. resp. showing?

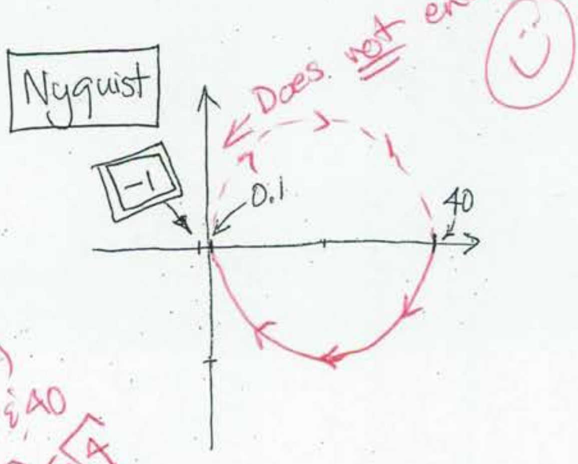
pure sinusoid in @ ω



find change in amplitude & phase of sinusoid AT SAME ω in output. (i.e., the Fourier component @ freq ω...)

Example: $\boxed{G(s) = \frac{2s + 80}{20s + 2}}$





$s = 0 \rightarrow \frac{80}{2} = 40 = 32\text{dB}$

$s = \infty \rightarrow \frac{2}{20} = 0.1 = -20\text{dB}$

Check Yourself...

1. Is $C(s) = \frac{2s+80}{20s+2}$ a $\begin{cases} \boxed{N} \text{ Lead compensator?} \\ \boxed{Y} \text{ Lag compensator?} \end{cases}$ Increases DC gain...

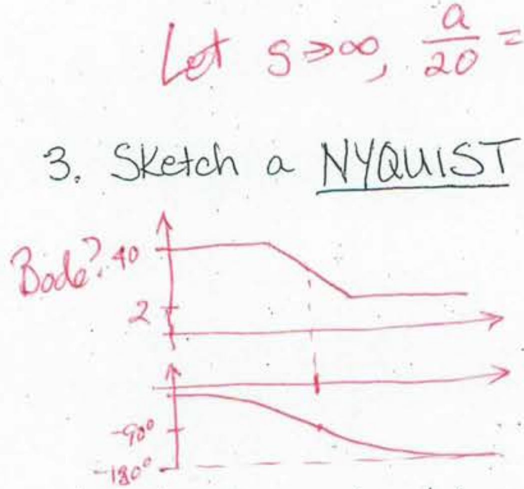
2. Say $\boxed{G_a(s) = \frac{as+80}{20s+2}}$ and $\boxed{G_a(\infty j) = -2}$.

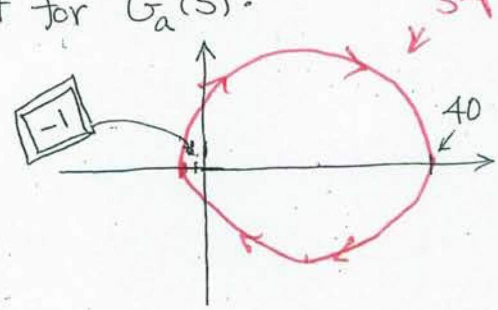
What value is "a"? ans: $a = \underline{-40}$

Let $s \Rightarrow \infty$, $\frac{a}{20} = -2$, $a = -2 * 20$

3. Sketch a NYQUIST plot for $G_a(s)$:

supposed to be a circle





4. Would the closed-loop system:

be STABLE?

a) $N \equiv$ #CW encirclements of $\boxed{-1}$ = $\underline{1}$

$P \equiv$ #unstable poles in $G(s)$ (open-loop) = $\underline{0}$

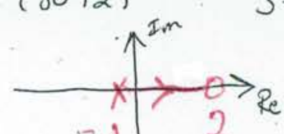
$Z \equiv N + P$ = #unstable closed-loop poles = $\underline{1}$ ← one unstable RHP pole

$\therefore$ ☐ stable

☑ unstable ←

b) $P_{CL} = \frac{G}{1+G} = \frac{as+80}{(a+20)s + (80+2)}$ → Closed-loop pole @ $s = \frac{-82}{a+20} = \frac{-82}{-40} \approx +2.05$

5. Root Locus for $G_a(s)$?



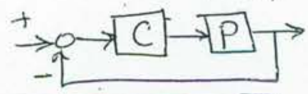
pole: $s = \frac{-2}{20} = -.1$

zero: $s = \frac{80}{40} = 2$

Loop Transmission → The negative L.T. is "CP", below. (loop transmission)

(aka "Loop Gain")

((But really, it is <u>dynamic</u> - <u>not</u> a "static gain"...))



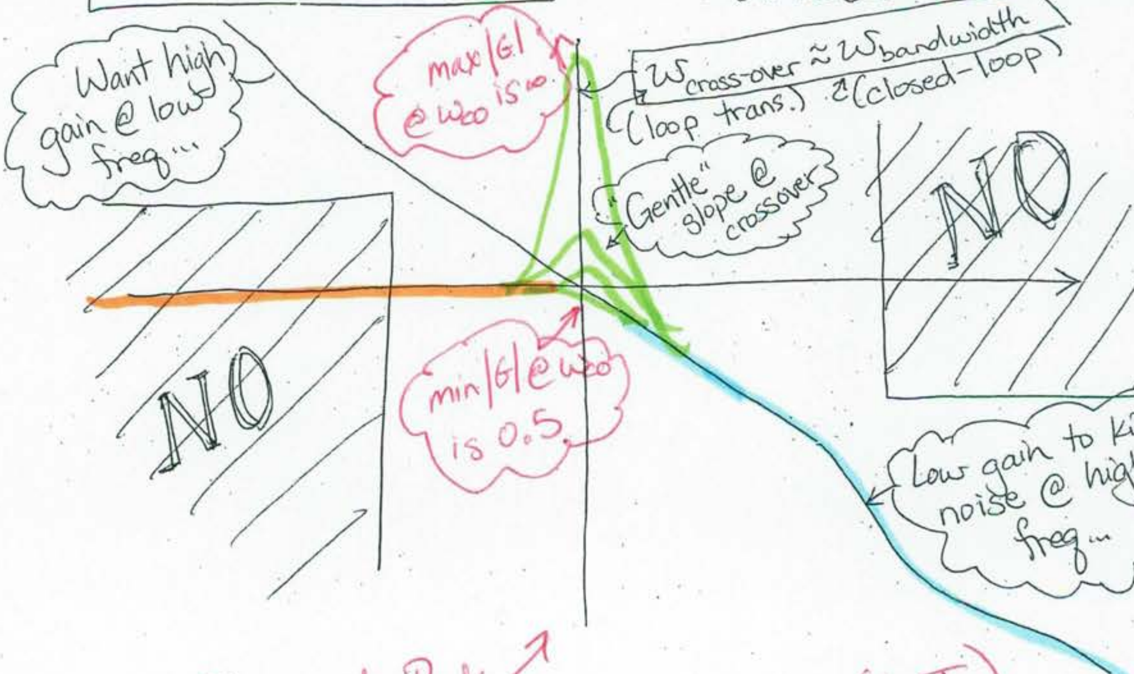
$$\boxed{G_{CL} = \frac{CP}{1+CP}}$$

check yourself...

• What is a good approx'n for G_{CL} when:

1. $\lvert CP\rvert >> 1$	$\frac{G}{1+G} \approx \boxed{\frac{1}{\frac{1}{G}+1} \approx 1}$	← Slightly less than 1, unless $G(0)=\infty$!
2. $\lvert CP\rvert \approx 1$	$G=1$, $G=j$, $G=-1$, $G=-j$ $\frac{G}{1+G} \approx$ min → $\frac{1}{2}$, $.5+.5j$, $\frac{-1}{0}=\infty!$ ← max, $.5-.5j$	
3. $\lvert CP\rvert << 1$	$\frac{G}{1+G} \approx \boxed{\frac{G}{1+\Delta} \approx G}$	$\boxed{\frac{\varepsilon}{1+\varepsilon} \approx 0?}$ ← No, let's be more precise...)

Bode Obstacle Course - AKA "LOOP SHAPING"!



Resonant Peak ↗
Depends on <u>phase</u> of CP (-L.T.)
@ crossover.

Controller Design

(aka "compensator")

$$\boxed{C(s) = K\frac{s+a}{s+b}}$$

Example lag or lead...

1. Lag compensation

Which are true? b, c, f

2. Lead comp.

Which are true? a, d, g

a. Adds phase @ ω_{co}, so we are not near "$G=-1$" case.
crossover

b. Increases gain at low (e.g. DC) freq.

c. $a > b$ (in $C(s) = K\frac{s+a}{s+b}$)

d. $b > a$

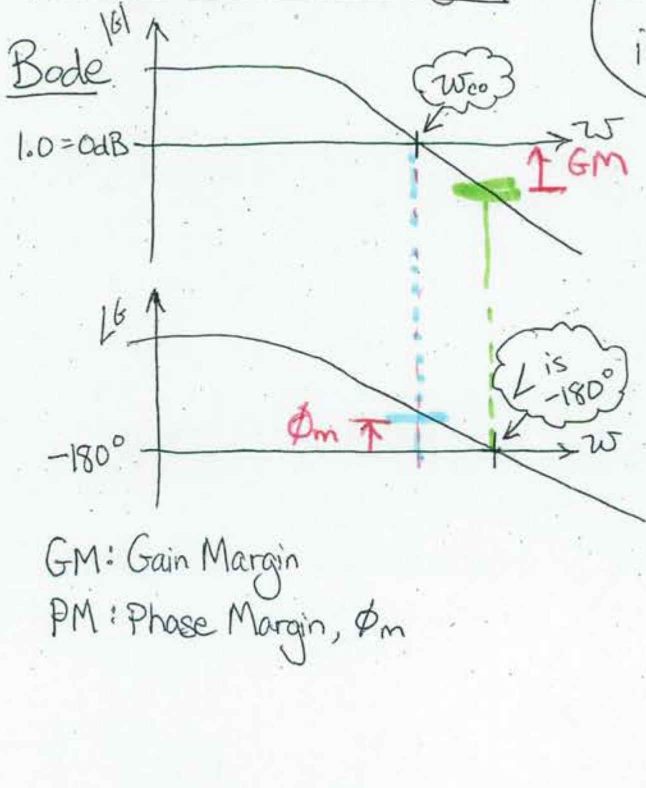
f. Similar to "PI" (proportional + integral)

g. Similar to "PD" (proportional + derivative)

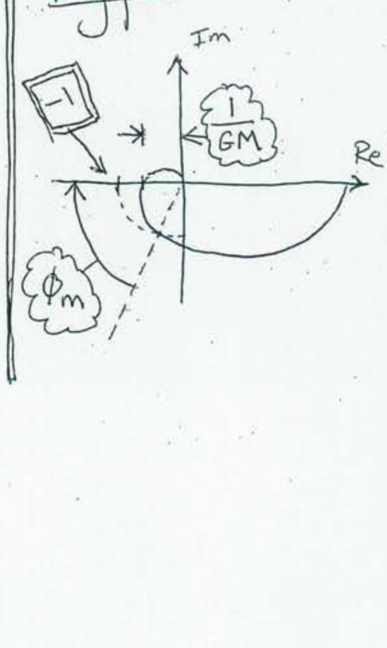
Gain & Phase Margin

A positive margin is GOOD (safe)!

Bode



Nyquist:



GM: Gain Margin

PM: Phase Margin, ϕ_m