Today: Review of Continuous-Time (CT) Dynamics and Control (ECE 147A material) **Lecture 1**

Reading: FPW (Ch.1 pp.1-8, optional), 2.1-2.5 (pp.11-24, 31-41)
or: FV 2.1, 5.1-5.3 [*Today's readings are review, as needed*.]

Topics:
- Overview: CT vs DT analysis
- Time Response
- Frequency Response
- Block Diagrams
- Stability and Stability Margins
- Control: What (are we trying to do), and How (to do it)

Next Class: Difference Equations. Sampling. Tustin (Trapezoid) Approx.
- FPW: 3.1-3.2 (pp. 57-65), 4.1 (pp. 73-78)

or: - FV: FV 2.1, 3.1-3.2, 3.4, 6.3.1, 6.3.3

147A (CT)

1) Continuous time (CT), x(t).

2) Differential Equations, e.g.,

$$a\ddot{y} + b\dot{y} + cy = d\dot{x} + ex$$

3) Laplace Transforms:

$$Xs \leftrightarrow \frac{d\ x}{dt}$$

$$X\frac{1}{s} \leftrightarrow \int x$$

4) CT Transfer Function (TF):

$$\frac{Y(s)}{X(s)} = \frac{ds + e}{as^2 + bs + c}$$

$$G(s) = \frac{b(s)}{a(s)}$$

← in general →

147B (DT)

1) Discrete time (DT) samples, xk.

2) DIFFERENCE Equations, e.g.,

$$ay_{k+2} + by_{k+1} + cy_k = dx_{k+1} + ex_k$$

3) z-Transforms:

$$Xz^n \leftrightarrow x_{k+n}$$

(e.g., $X\frac{1}{z} = Xz^{-1} \leftrightarrow x_{k-1}$)

3) DT TF: $\frac{Y(z)}{X(z)} = \frac{dz + e}{az^2 + bz + c}$

$$G(z) = \frac{b(z)}{a(z)}$$

147A (CT)	147B (DT)
5) POLES: Roots of DENOMINATOR (i.e., find solutions for s...) $$a(s)=0$$	5) POLES: same, except polynomial is just in "z" instead of "s": $$a(z)=0$$

Poles affect the FREE RESPONSE (t>0).

6) ZEROS: Roots of NUMERATOR $$b(s)=0$$	6) ZEROS: same, except "z", not "s" $$b(z)=0$$

Zeros affect the INITIAL CONDITION (t=0).

Check yourself... $P(s)=\dfrac{s-4}{s^2+4}$ ← CT TF

1. What are the poles and zeros?

2. What order system is this?

Check yourself...

$$P(s) = \frac{s-4}{s^2+4}$$

3. Is P(s) proper?

4. Is P(s) causal?

5. What determines if a CT TF is STABLE? (and Why?)

[*HINT*: e^{st}]

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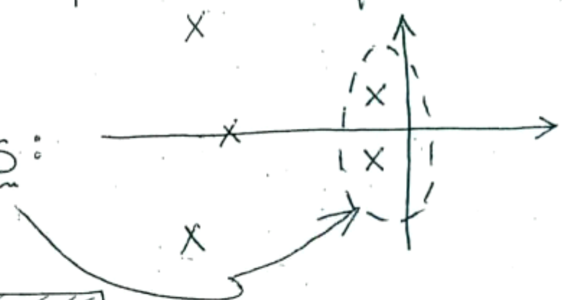
6. Sketch a UNIT STEP response for $P_2(s)$:

[*HINT*: IVT and FVT?]

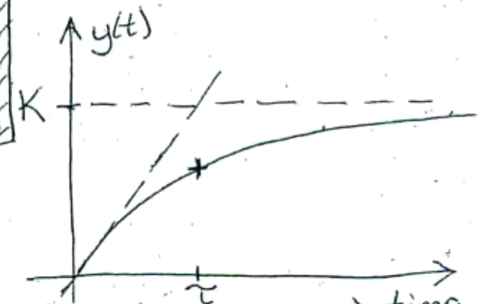
$$P_2(s) = \frac{s-15}{s+5}$$

Time Response (CT, only)

- 1^{st}-order & 2^{nd}-order responses are important!
  - Higher-order systems often have one or TWO DOMINANT POLES:



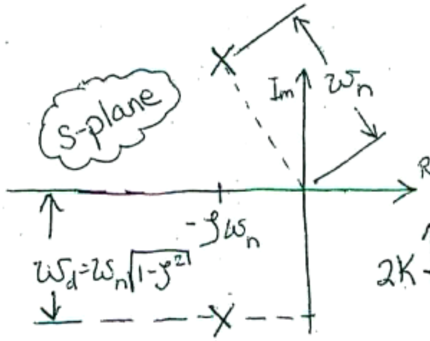
- 1^{st}-order: $$\boxed{H(s) = \frac{K}{\tau s + 1}}$$
  - pole: $s = \frac{-1}{\tau}$
  - unit step response:
    $y(t) = K\left(1 - e^{-t/\tau}\right)$



(unit STEP response)

• 2nd-order:

$$\boxed{H(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$



- 2 poles: $s = -\zeta\omega_n \pm \omega_d j$

$\boxed{\tau = \frac{1}{\zeta\omega_n}}$ ← Real part of poles sets DECAY.

$$M_p = e^{-\left(\frac{t_p}{\tau}\right)} = e^{-\left(\frac{\pi}{\omega_d}\cdot\zeta\omega_n\right)}$$

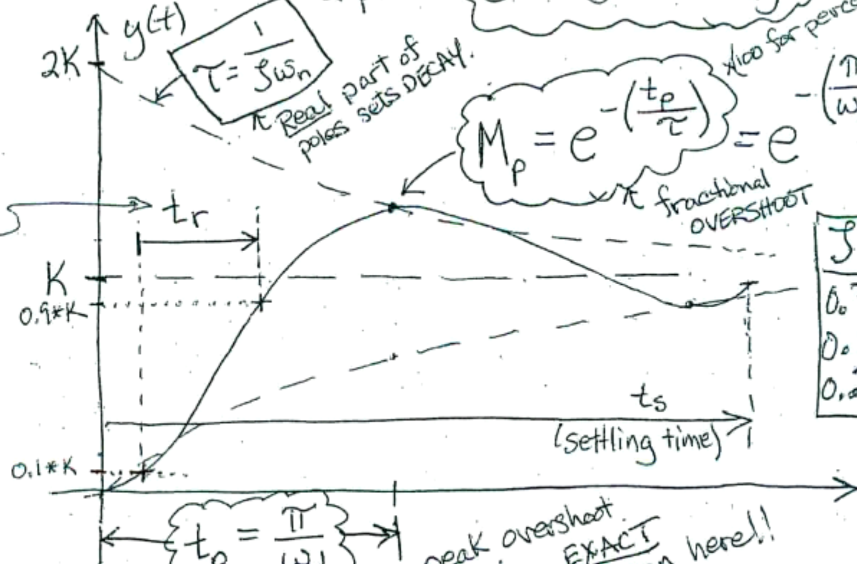
← fractional OVERSHOOT

×100 for percent

ζ	M_p
0.7	.05 = 5%
0.5	.16 = 16%
0.2	.53 = 53%

$\boxed{t_r \approx \frac{1.8}{\omega_n}}$

↑ "rise time" (10% → 90%)



$$t_p = \frac{\pi}{\omega_d}$$

← peak overshoot @ t_p ... EXACT eqn here!!

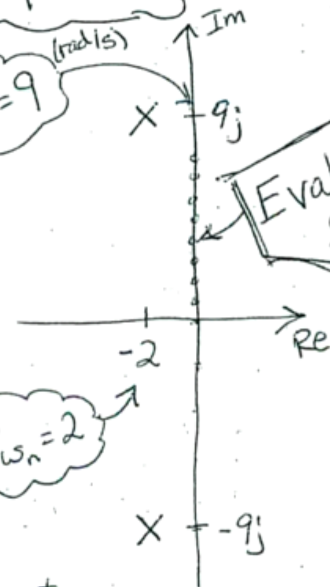
Time Response vs. Frequency Response

For General n^{th}-order system,
"Homogeneous" OR (aka) "Free" Response?

$$y(t)=\sum_{i=1}^{n} K_i e^{s_i t}$$

S-plane

$\omega_d = 9$ (rad/s)



$\zeta\omega_n = 2$

Evaluate $G(s)$ @ "$s=j\omega$", from $\omega = 0$ rad/s (DC) to $\omega = +\infty$ (high freq limit).

$$\omega_n = \sqrt{2^2+9^2}\ \left(\frac{\text{rad}}{\text{sec}}\right)$$

"Break point" @ ω_n.

In our example,

$y(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$
$\dot{y}(t) =$ ____________________ ?

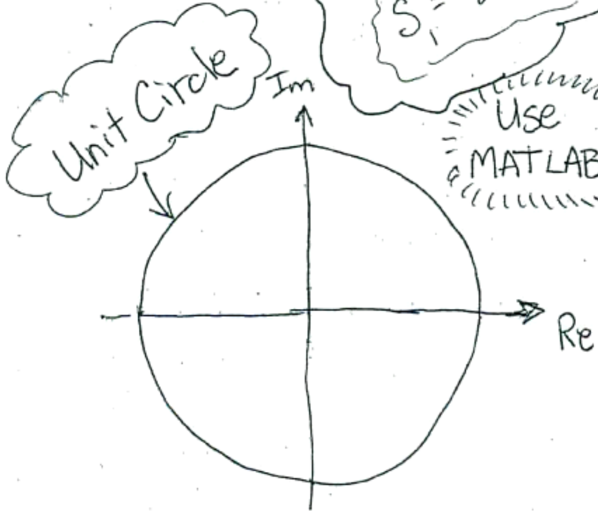
where $\boxed{\begin{array}{l} s_1 = -2+9j \\ s_2 = -2-9j \end{array}}$

MATLAB Reality Check...

?

Hmmm, what does "$e^{s_1 t}$" look like for $s_1 = -2+9j$?

Use MATLAB!



* Example Initial Condition (Free) Response for: $\boxed{\begin{matrix} y(t=0)=8 \\ \dot{y}(t=0)=200 \end{matrix}}$

(@ $t=0$, $e^{st}=e^{0}=1\ldots$)

* K_1 & K_2 must satisfy: (Check Yourself!)

$y(0)=8=K_1e^{s_1 0}+K_2e^{s_2 0}$

$\dot{y}(0)=200=$ ______________ ?

so...

$$\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \end{bmatrix}=\begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$$

?? ?

$A \quad x = b$

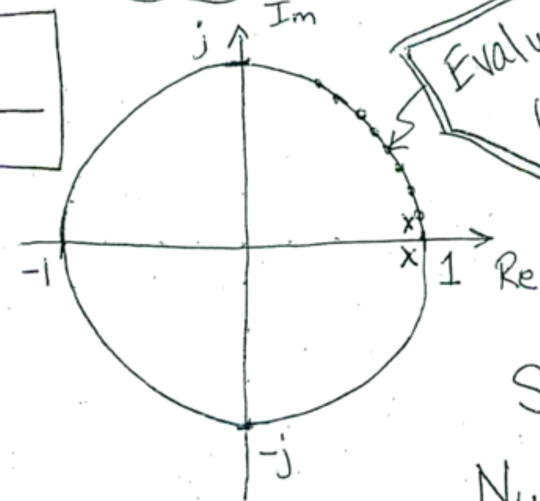
Use MATLAB!

$\boxed{X = A \backslash b} \rightarrow \boxed{K_1=4-12j}\ \boxed{K_2=4+12j}$

Time Response vs. Frequency Response

Z-plane

$y(t) = \underline{\ ?\ }$



Evaluate $G(z)$ @ "$z = e^{j\omega T}$"
(from $\omega = 0$ to $\omega = +\infty$)

These "$e^{j\omega T}$" points all lie on the UNIT CIRCLE

Sample time $= T$

Nyquist freq. $= \dfrac{\pi}{T} =$ "ω_{Ny}"

* @ ω_{Ny}, $z = e^{j\frac{\pi}{T}T} = e^{j\pi} =$ ______ ?

* Imagine free response of

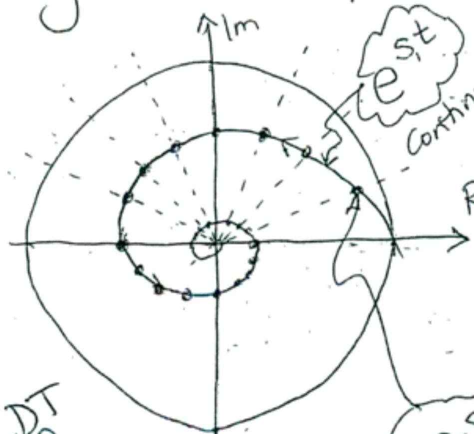
$$\boxed{G(s) = \frac{85}{s^2+4s+85}}$$

(same poles as s-plane time resp. example.)

$s_1 = -2+9j$

$s_2 = -2-9j$

$e^{s_1 t}$ continuous

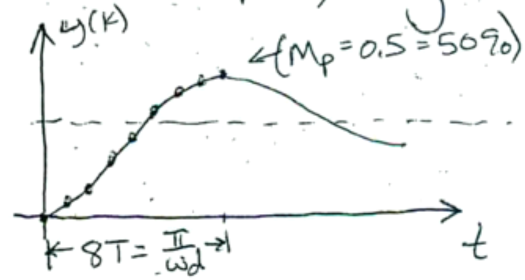


Poles of DT system w/ free response shown are:

$z_1 = e^{s_1 T}, \; z_2 = e^{s_2 T}$

$e^{s_1 T k}$ ← for $k = 0, 1, 2, \ldots$

(Discrete samples, only!)



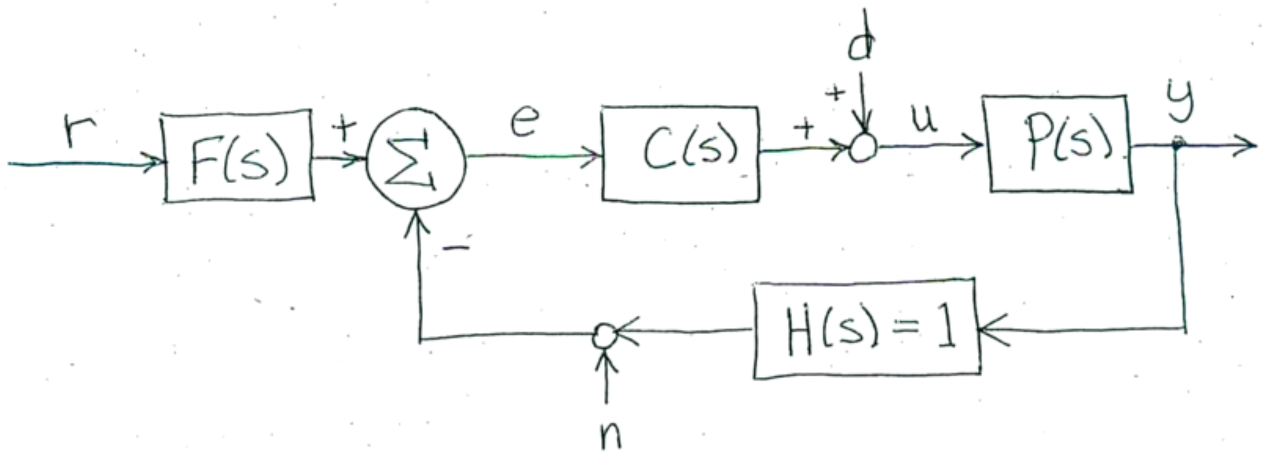
$e^{s\Delta} \approx 1 + s\Delta$, for $|s\Delta| << 1.0$

$e^{s_1 * .001} \approx$ ______________ ? MATLAB check!

BLOCK DIAGRAMS

No sensor GAIN nor DYNAMICS

"Unity Feedback Configuration", i.e., $\boxed{H=1}$ in feedback path.



BLOCK DIAGRAM ALGEBRA

$$\frac{Y(s)}{R(s)} =$$

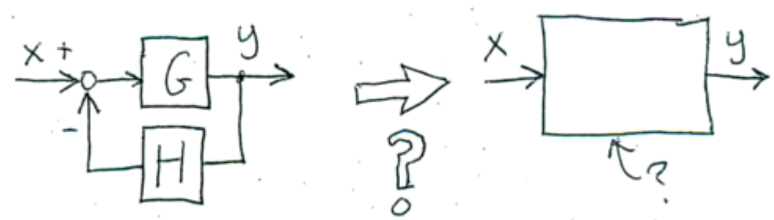
Set all other inputs to ZERO & solve algebraically for $\frac{Y(s)}{R(s)}$...

Sensitivity:

Complementary Sensitivity:

Characteristic Equation:

More Generally:



"Black's Formula"

$$\frac{Y(s)}{X(s)} = \underline{\qquad\qquad}$$

Frequency Response

Bode & Nyquist plots

Radians per Second! (not Hz)

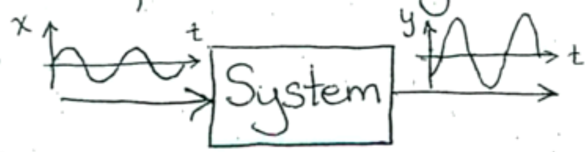
* CT procedure for freq. resp. of CT TF $G(s)$:

1. Sequentially "plug in" $\boxed{s=j\omega}$, using ω s.t. $0 \le \omega \le \infty$, into $G(s)$.
2. Calculate magnitude: $\boxed{|G(j\omega)| = \sqrt{(Re(G(j\omega)))^2 + (Im(G(j\omega)))^2}}$
3. Calculate phase: $\boxed{\angle G(j\omega) = \tan^{-1}\left(\frac{Im(G(j\omega))}{Re(G(j\omega))}\right)}$

* Bode plot: Plot MAGNITUDE vs ω on log-log scale.
  AND Plot PHASE vs ω on semilogx scale.
* Nyquist plot: Plot only MAGN. & PHASE, on polar plot, as $\omega \uparrow$.

- What is a freq. resp. showing?

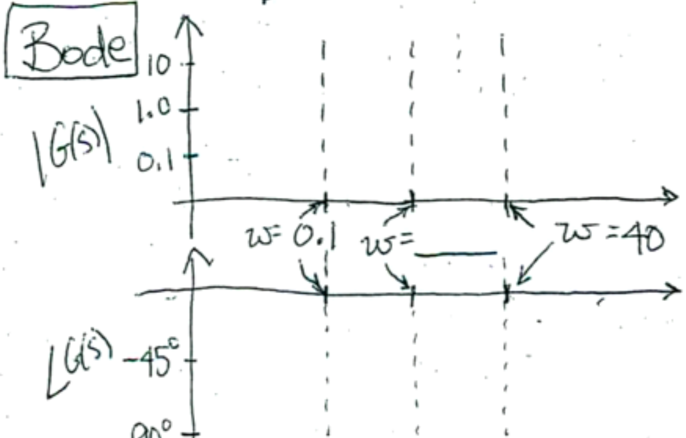
pure sinusoid in @ ω



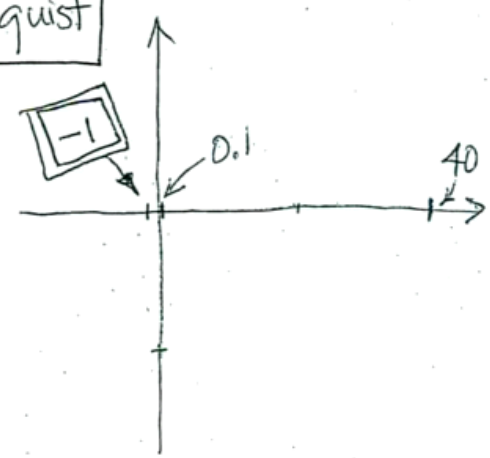
find change in amplitude & phase of sinusoid AT SAME ω in output. (i.e., the Fourier component @ freq ω...)

Example: $\boxed{G(s) = \dfrac{2s+80}{20s+2}}$

Bode



Nyquist



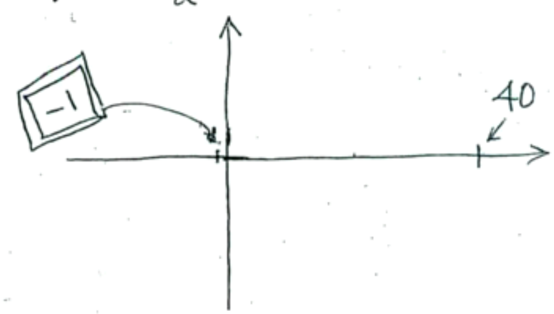
Check Yourself...

1. Is $C(s) = \dfrac{2s+80}{20s+2}$ a
   - ☐ Lead compensator?
   - ☐ Lag compensator?

2. Say $\boxed{G_a(s) = \dfrac{as+80}{20s+2}}$ and $\boxed{G_a(\infty j) = -2}$.

   What value is "a"? ans: a = ________

3. Sketch a <u>NYQUIST</u> plot for $G_a(s)$:



4. Would the closed-loop system: 

be <u>STABLE</u>?

a) $N \equiv$ # CW encirclements of $\boxed{\boxed{-1}}$ = ________

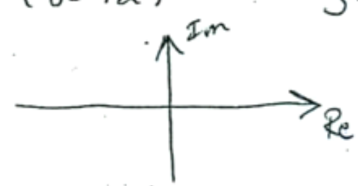
$P \equiv$ # unstable poles in $G(s)$ (open-loop) = ________

$Z \equiv N + P$ = # unstable closed-loop poles = ________

$\therefore$ ☐ stable

☐ unstable

b) $P_{CL} = \dfrac{G}{1+G} = \dfrac{as+80}{(a+20)s+(80+2)}$ → Closed-loop pole @ $s = \dfrac{-82}{a+20}$ = ________

5. Root Locus for $G_a(s)$? 

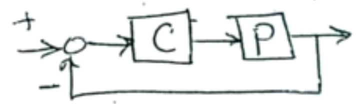
$\boxed{\text{Loop Transmission}}$ → The negative L.T. is "CP", below.

(loop transmission)

(aka "Loop Gain")

((But really, it is dynamic – not a "static gain"...))

check yourself...

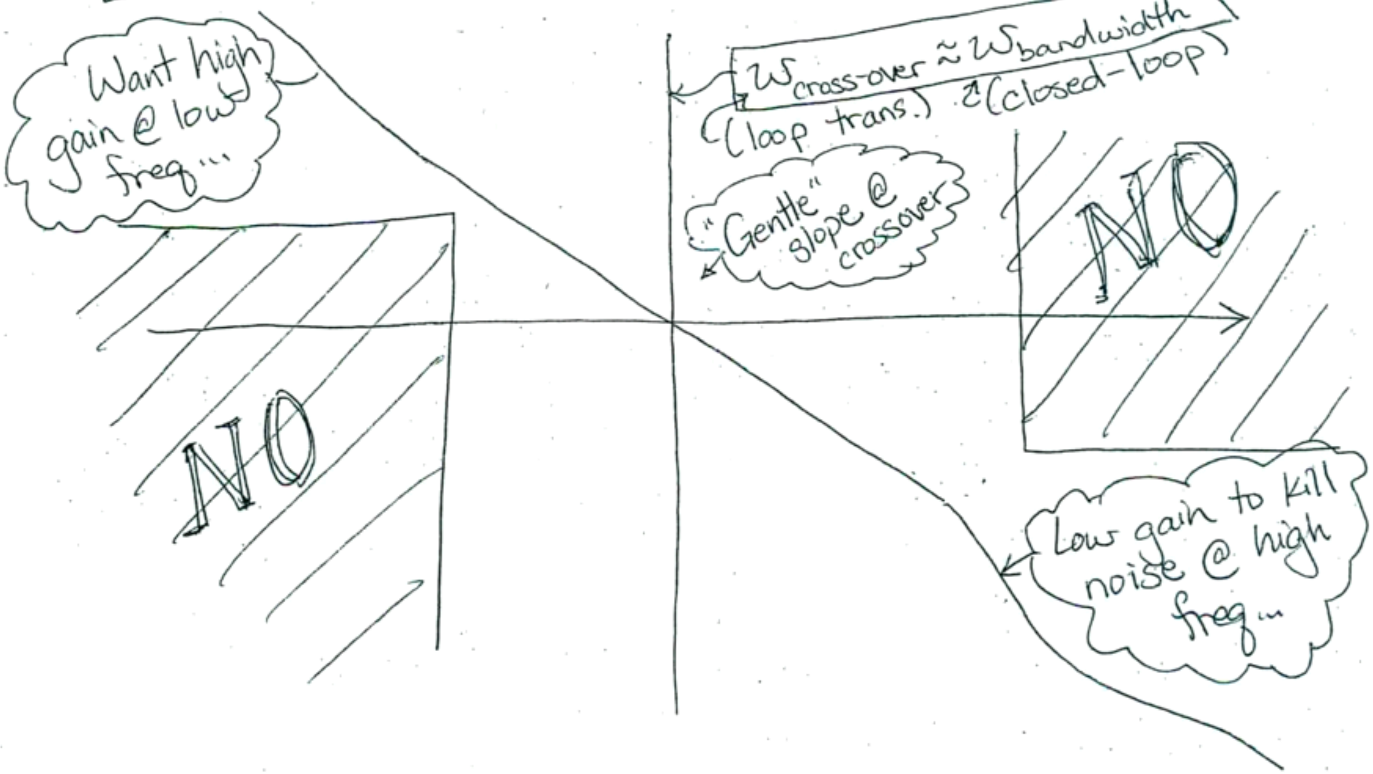


$$\boxed{G_{CL} = \frac{CP}{1+CP}}$$

- What is a good approx'n for G_{CL} when:

1. $\lvert CP \rvert >> 1$	$\frac{G}{1+G} \approx$
2. $\lvert CP \rvert \approx 1$	$\frac{G}{1+G} \approx$
3. $\lvert CP \rvert << 1$	$\frac{G}{1+G} \approx$

Bode Obstacle Course – AKA "LOOP SHAPING"!



$\boxed{\text{Controller Design}}$

(aka "compensator")

$$\boxed{C(s) = K\frac{s+a}{s+b}}$$

Example lag or lead...

1. Lag compensation

Which are true? ______

______

2. Lead comp.

Which are true? ______

______

a. Adds phase @ w_{co}, so we are not near "$G=-1$" case.

crossover

b. Increases gain at low (e.g. DC) freq.

c. $a > b$ } (in $C(s) = K\frac{s+a}{s+b}$)

d. $b > a$ }

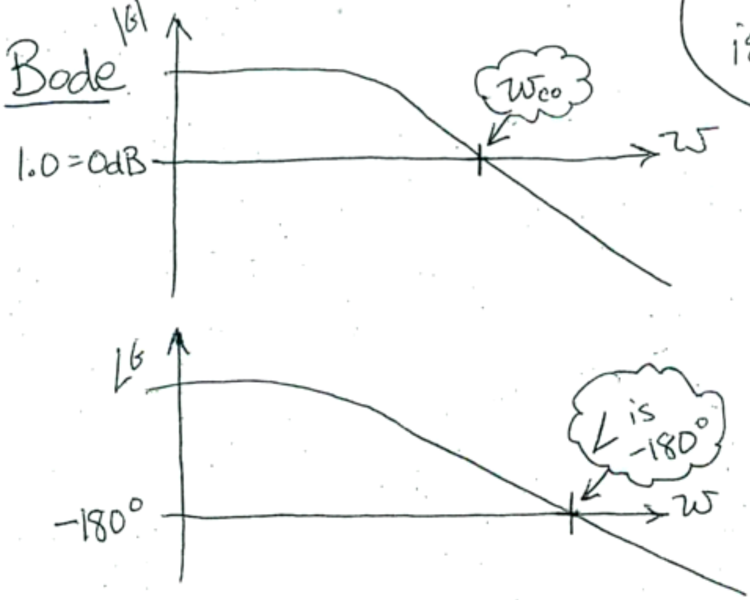
f. Similar to "PI" (proportional + integral)

g. Similar to "PD" (proportional + derivative)

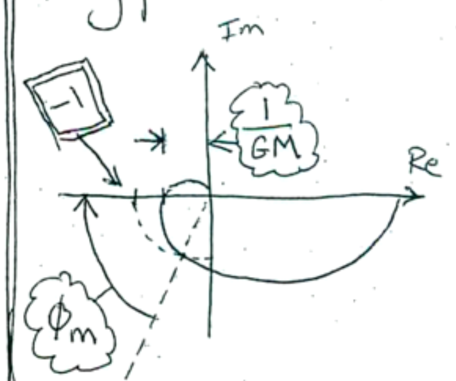
Gain & Phase Margin

A **positive** margin is **GOOD** (safe)!

Bode



Nyquist:



GM: Gain Margin

PM: Phase Margin, ϕ_m