Today: # Introduction to Digital Control

Lecture 2
ECE 147B
Digital Control

Reading: FPW 3.1-3.2 (pp.57-65), 4.1 (pp.73-78)
or: FV 2.2, 3.1-3.2, 6.3.1,6.3.2

Topics:
- Wrap up CT Review (see Lecture 1 notes)
- Overview: Sampled data systems
- Difference Equations (vs differential (Lapace)) Domain.
- Sampling: Effect of Zero-Order Hold (ZOH)
- Numerical Integration: Tustin Approx.

Next Class: ~~Difference Equations. Sampling. Tustin (Trapezoid) Approx.~~
~~- FPW: 3.1-3.2 (pp. 57-65), 4.1 (pp. 73-78)~~
~~or: - FV: FV 2.1, 3.1-3.2, 3.4, 6.3.1, 6.3.3~~

See non-annotated slides for next class reading!

Sampled-Data Systems

- We want to use computers to control cool stuff in the real world...

Kuka robot arm

robot arm

dish washers & cloth washers

stand-alone CNC machine millers

"Cousin" of BigDog called LS3

4-legged

6-legged

& more legged robots

Humanoid robots & 2-legged robots

HRP-4C

automated guided vehicles

unmanned aerial vehicles

wheeled robots

caterpillar-tracked robots

Google car

Prof. Byl ☺

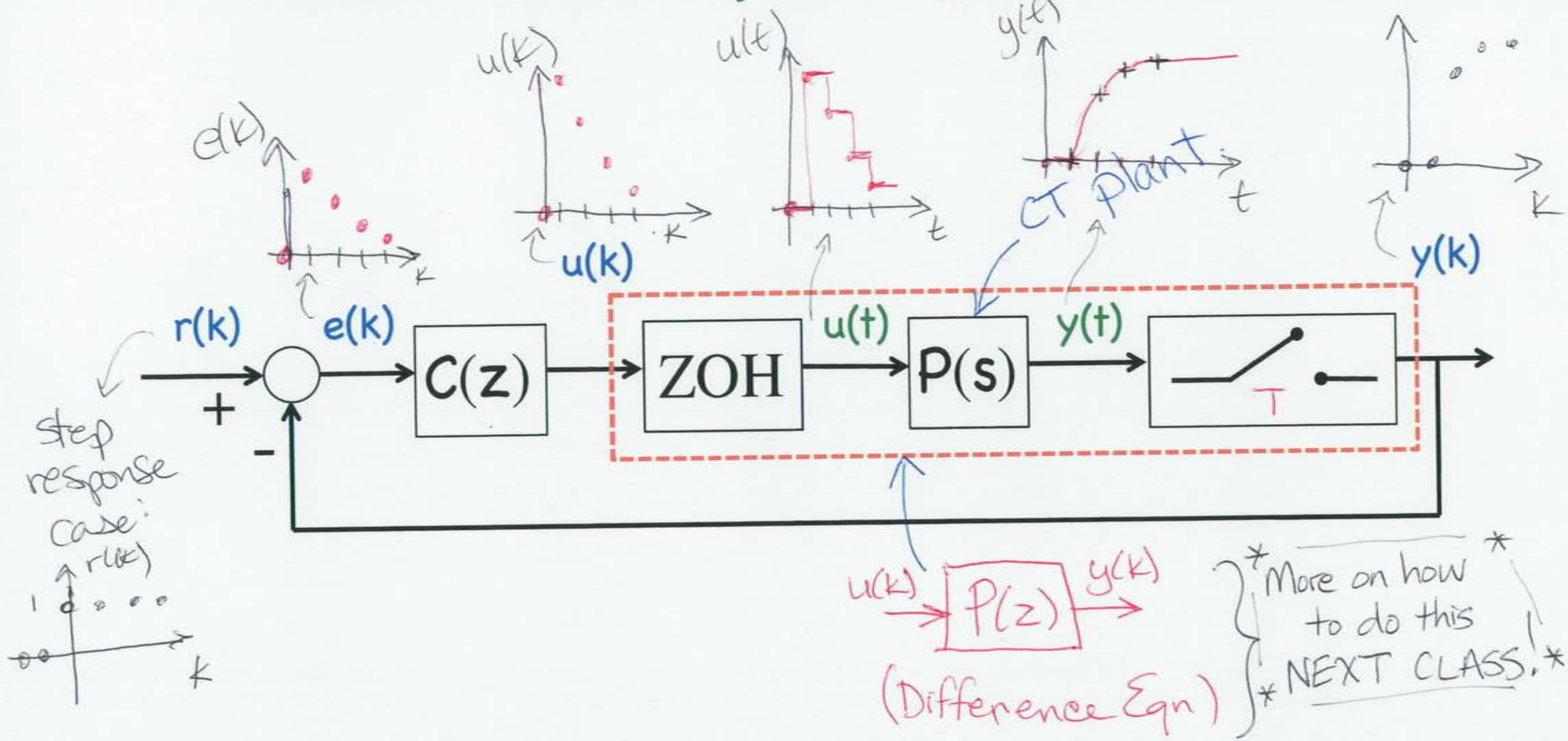
Mars rover (@JPL)

predator drone (in Iraq)

ASIMO (Honda)

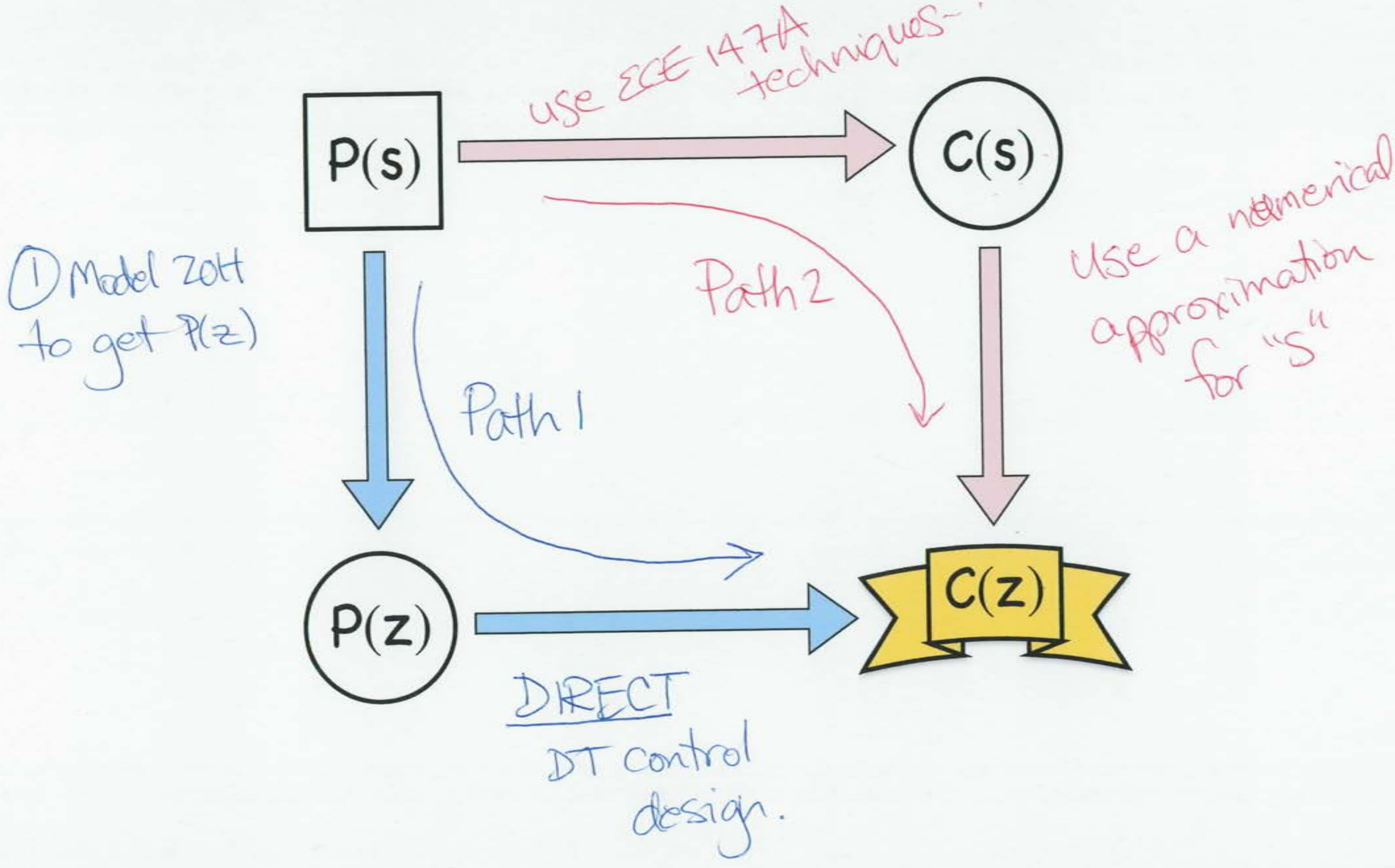
Sampled-Data Systems

- Such sampled-data systems are a combo of:
  - Discrete-time (DT) signals
  - Continuous-time dynamic system



Digital Control Design Approaches

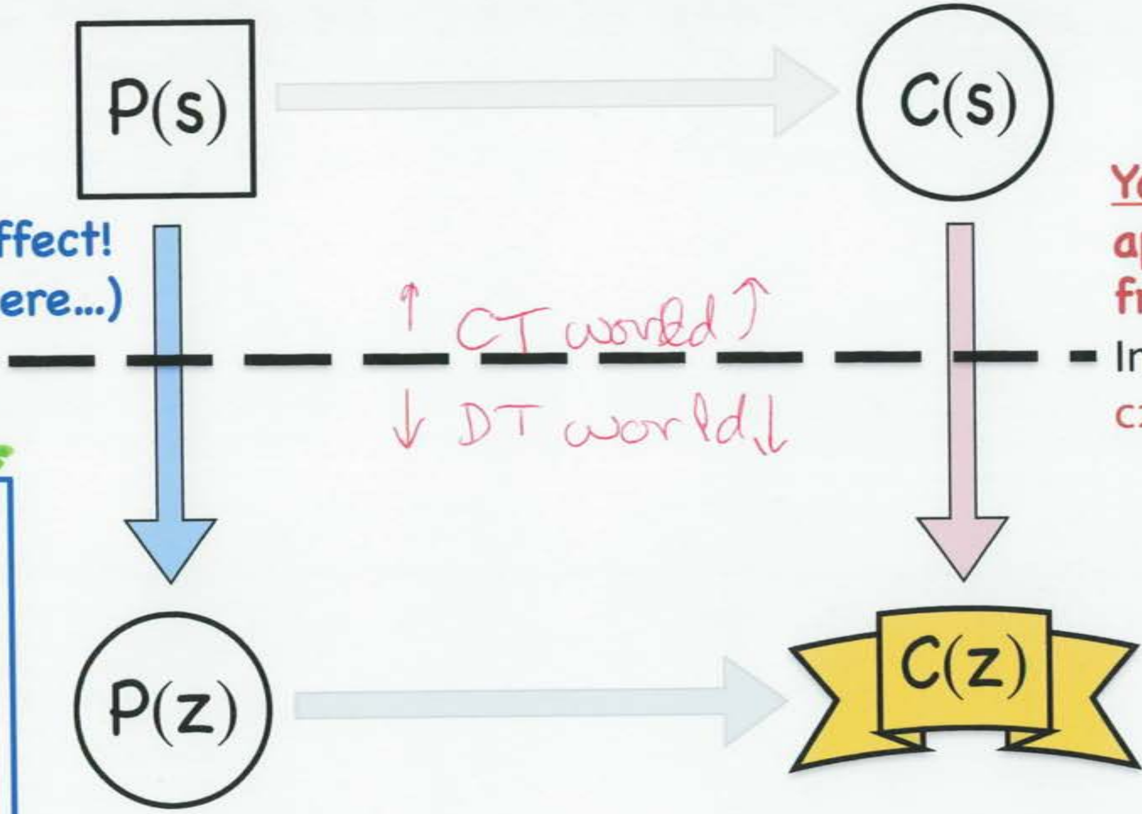
- We will look at two paths for design…



MATLAB c2d...

- There are two DIFFERENT ways we will convert a TF from CT to DT...

Model ZOH effect!
(No "choice" here...)
In MATLAB:
c2d(Ps,T,'zoh')



You choose an approximation, from a "menu".
In MATLAB:
c2d(Cs,T,'tustin')

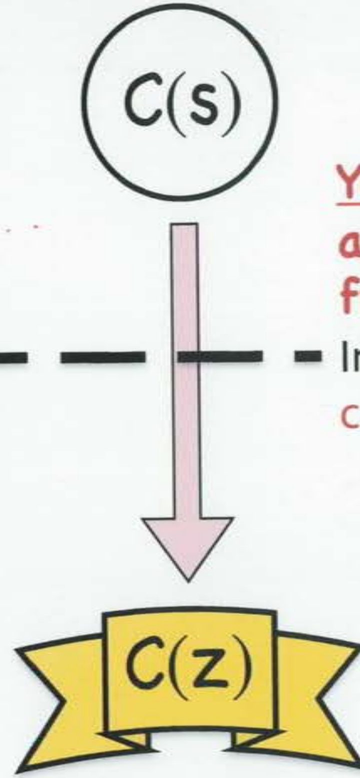
LAW
There is just one "right way" to do this! You find an exact answer to model the actual dynamics.

MENU
- Forward Euler
- Backw. Euler
- Tustin
- P-Z matching
- Your choice!

First, consider the "MENU"...

- To convert C(s) to C(z), you use a numerical approximation to estimate any derivative (s) or integral (1/s) operations.

Tustin is probably your safest bet...



You choose an approximation, from a "menu".

In MATLAB:
c2d(Cs,T,'tustin')

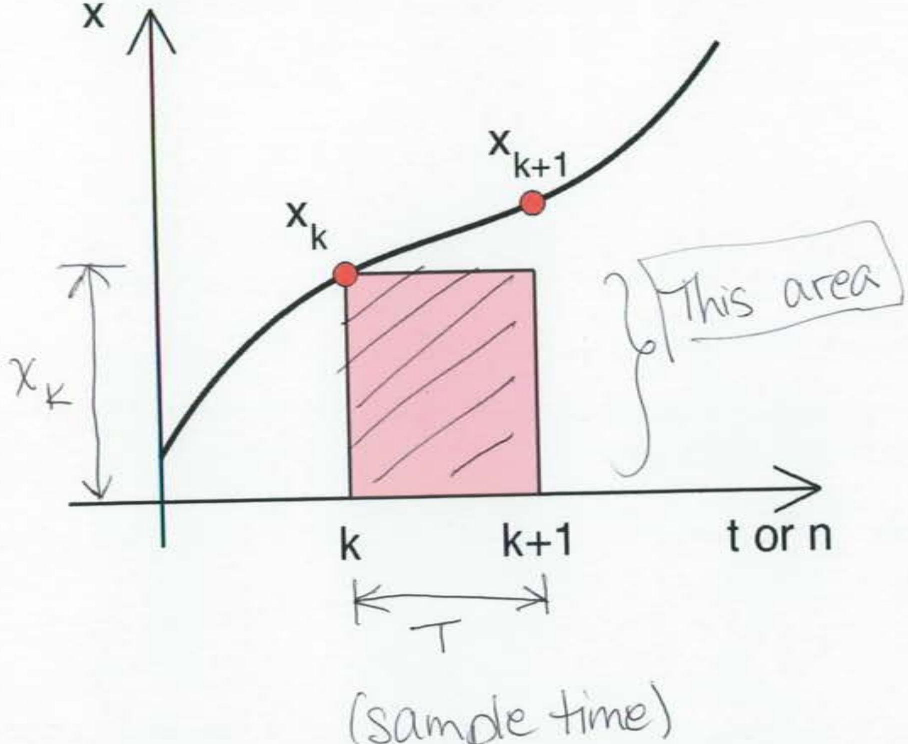
Probably do NOT use 'zoh' here!

MENU

- Forward Euler
- Backw. Euler
- Tustin
- P-Z matching
- Your choice!

Forward Euler Approximation

- To approximate integration of x(t), draw a box going "forward" from x(k), as shown:



$$\boxed{\begin{array}{c} y = \int x\,dt \\ Y(s) = \frac{1}{s} X(s) \\ \boxed{X = sY} \end{array}}$$

$$y_{k+1} = y_k + T x_k$$

$$y_{k+1} - y_k = T x_k$$

$$\downarrow z$$

$$(z^1 - z^0) Y = T z^0 X$$

$$\boxed{(z-1) Y = T X}$$

$$\boxed{\frac{Y}{X} = \frac{1}{s}} \longleftrightarrow \boxed{\frac{Y}{X} = \frac{T}{z-1}}$$

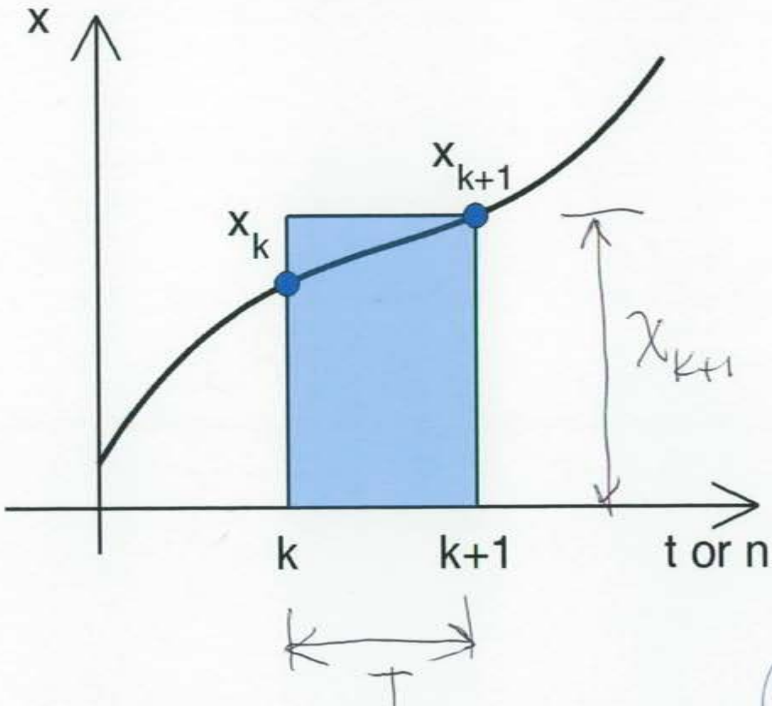
$$\boxed{\frac{X}{Y} = s} \longleftrightarrow \boxed{\frac{X}{Y} = \frac{z-1}{T}}$$

$$\boxed{s \approx \frac{z-1}{T}}$$

Backward Euler ~~Forward Euler~~ Approximation

Ooops! ☺

- To approximate integration of x(t), draw a box going "backward" from x(k), as shown:



$$y_{k+1} = y_k + T x_{k+1}$$

$$(z-1)Y = Tz \cdot X$$

$$\frac{Y}{X} = \frac{Tz}{(z-1)} \Leftrightarrow \frac{1}{s}$$

$$\boxed{\frac{(z-1)}{Tz} \Leftrightarrow s}$$

Preview:

	FW	BW	Aver.
$\frac{1}{s}$	$\frac{T}{z-1}$	$\frac{Tz}{z-1}$	$\frac{T(z+1)}{2(z-1)}$
			?

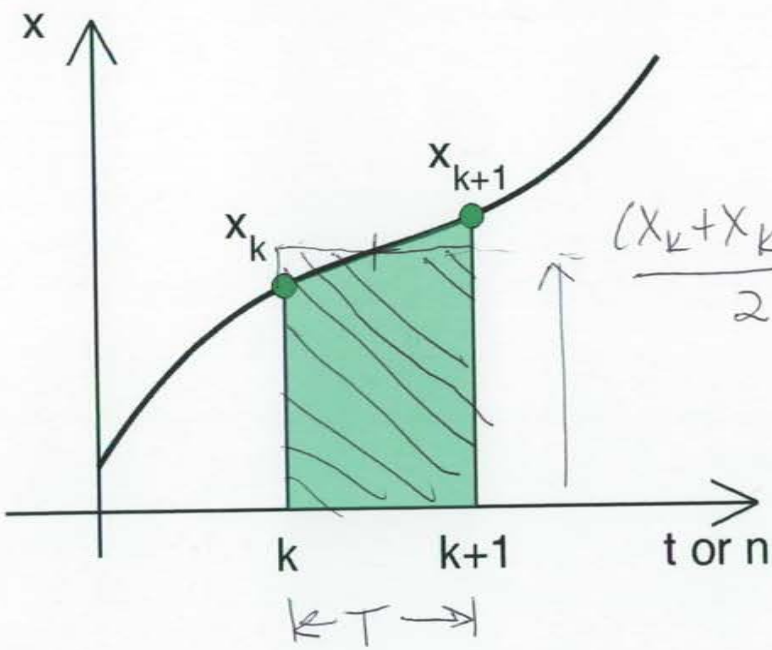
(Maybe)

e.g.

$$C(s) = K\frac{(s+a)}{(s+b)} \Leftrightarrow K\frac{\left(\frac{z-1}{Tz}+a\right)}{\left(\frac{z-1}{Tz}+b\right)} = C(z)$$

Tustin (aka bilinear [aka "Trapezoid Rule"])

- To approximate integration of x(t), draw a box using a trapezoid, as shown:



$$\frac{(x_k + x_{k+1})}{2}$$

$$y_{k+1} = y_k + \frac{T}{2}(x_k + x_{k+1})$$

$$(z-1)Y = \frac{T}{2}(z+1)X$$

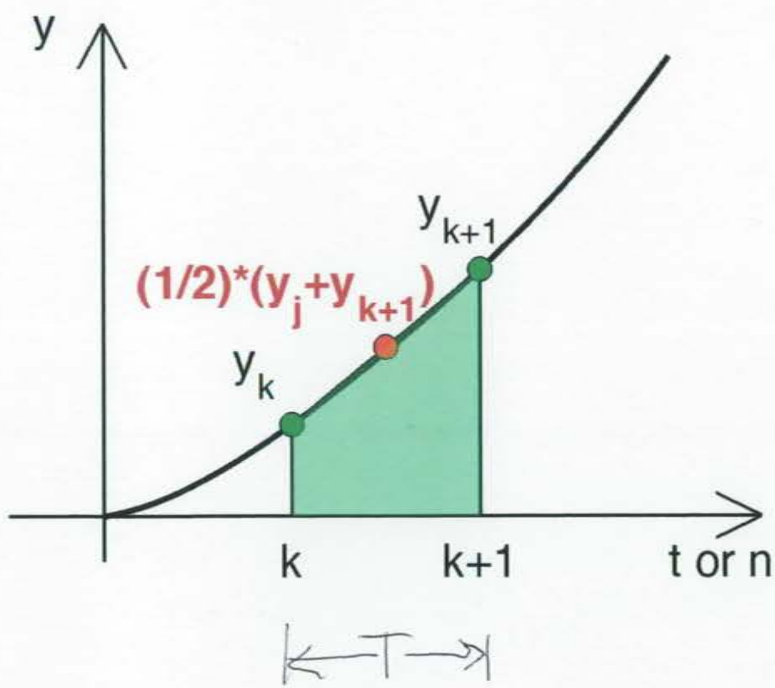
$$\boxed{\frac{Y}{X} = \frac{T}{2}\frac{(z+1)}{(z-1)} \Longleftrightarrow \frac{1}{s}}$$

$$s \leftrightarrow \frac{2}{T}\frac{(z-1)}{(z+1)}$$

(There is a "Tustin with prewarp" that preserves Mag. & Phase of C(s) [so C(z) matches] at a particular freq.)

Another viewpoint: Approx. Differentiation

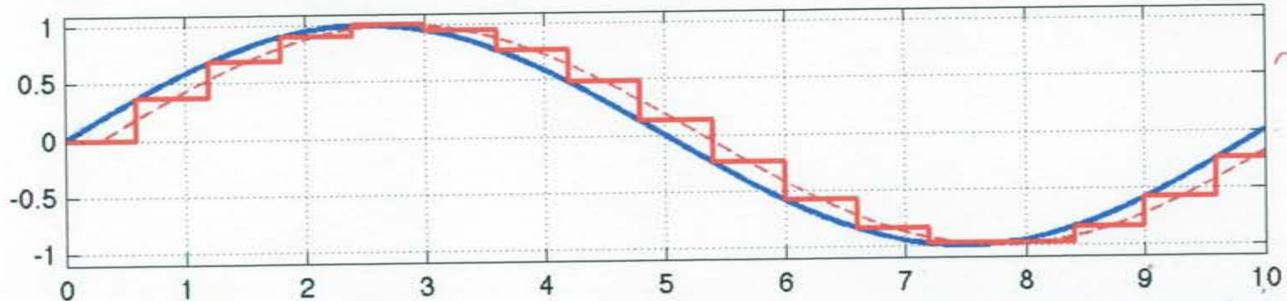
- To approximate differentiation of y(t), slope applies to midpoint between y(k) and y(k+1):



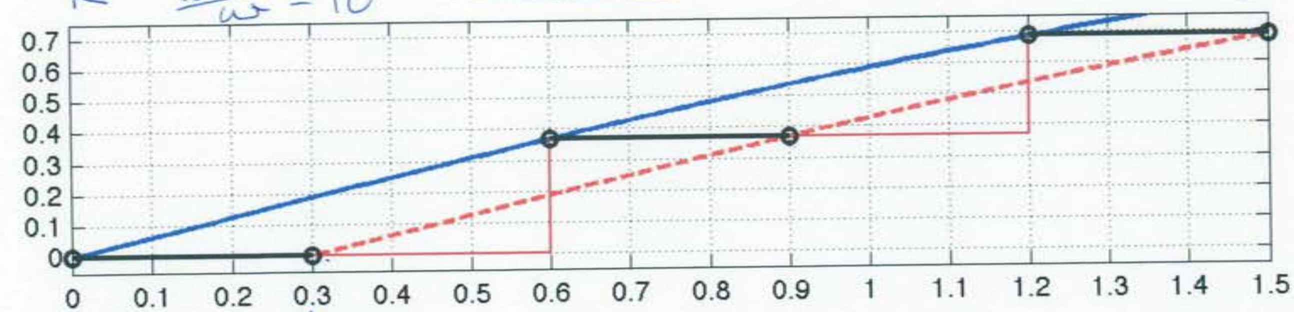
Do @ home if interested.

Sampling: Zero-Order Hold (ZOH)

1. Fourier: CT signal can be broken into sinusoids.
2. After ZOH, the CT waveform of a sinusoid has the original (primary) freq plus higher freqs, too.
3. The primary freq sinusoid is now shifted in time...

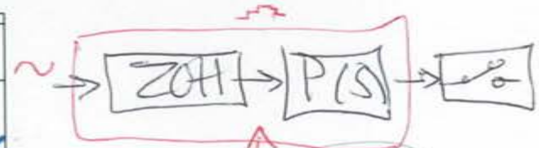


$\leftarrow \frac{2\pi}{\omega} = 10 \rightarrow$



delay ≈ 0.3

T



"same" amp but diff phase @ primary freq.
(i.e., P(s) gets a phase-shifted sin as input → so output also has same shift!)

We have a half sample delay.

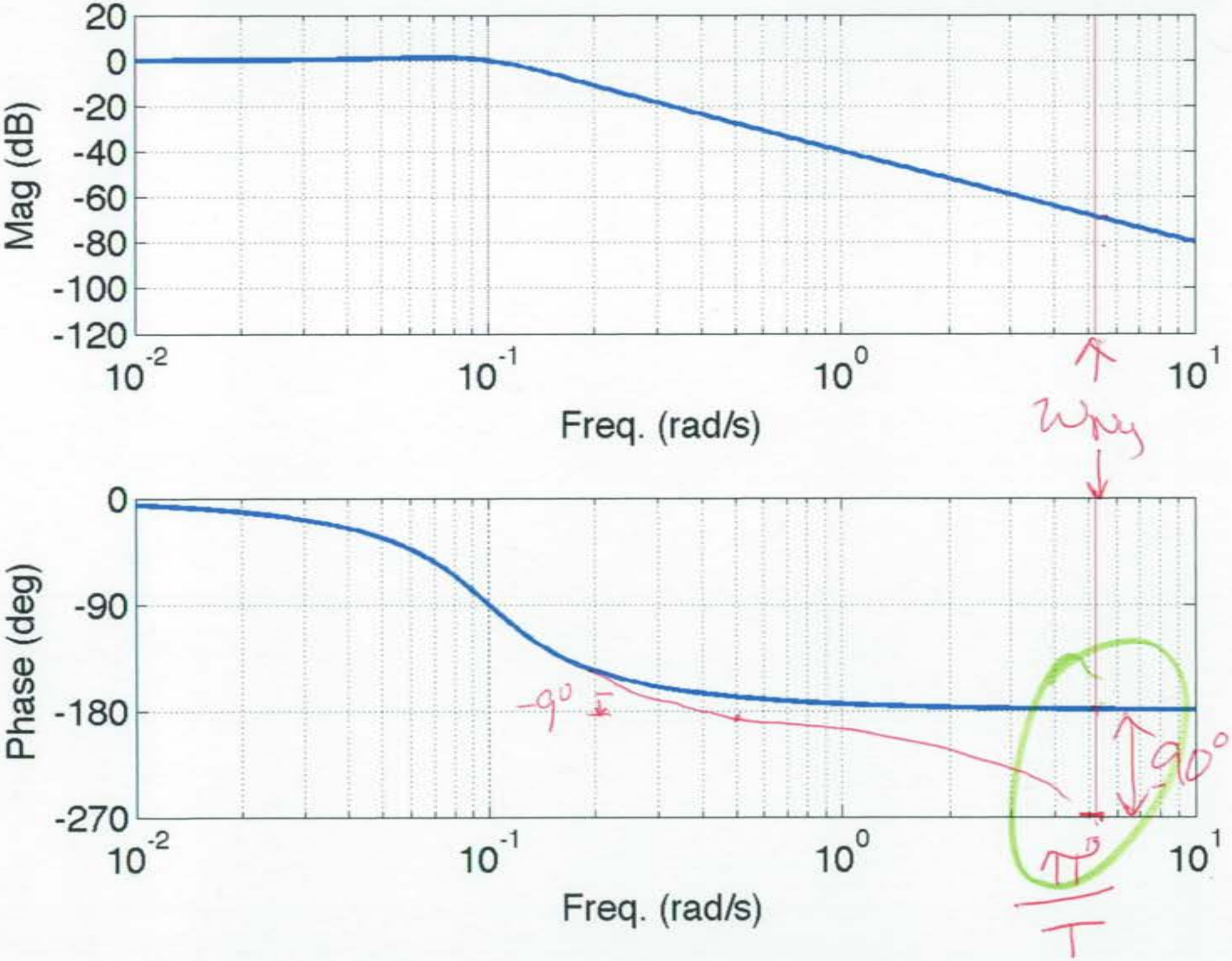
It takes 10 sec to go "360°"....

So 0.3 sec delay is $\frac{.3}{10} * 360° \approx$ XXXX $= \Delta$ phas

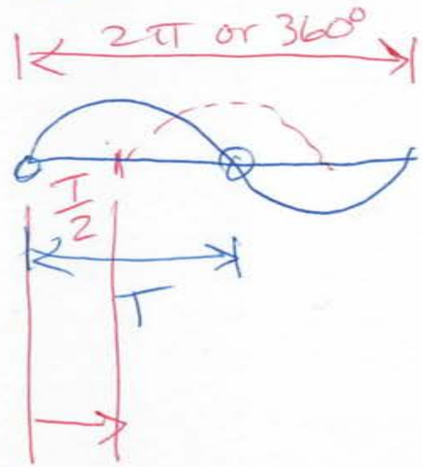
10.8 degrees!!!
XXXX
XXXX

ZOH Consequences in FREQUENCY DOMAIN

- The PHASE LOSS due to half-sample delay may DESTABILIZE the closed-loop system!



@ Nyquist freq.

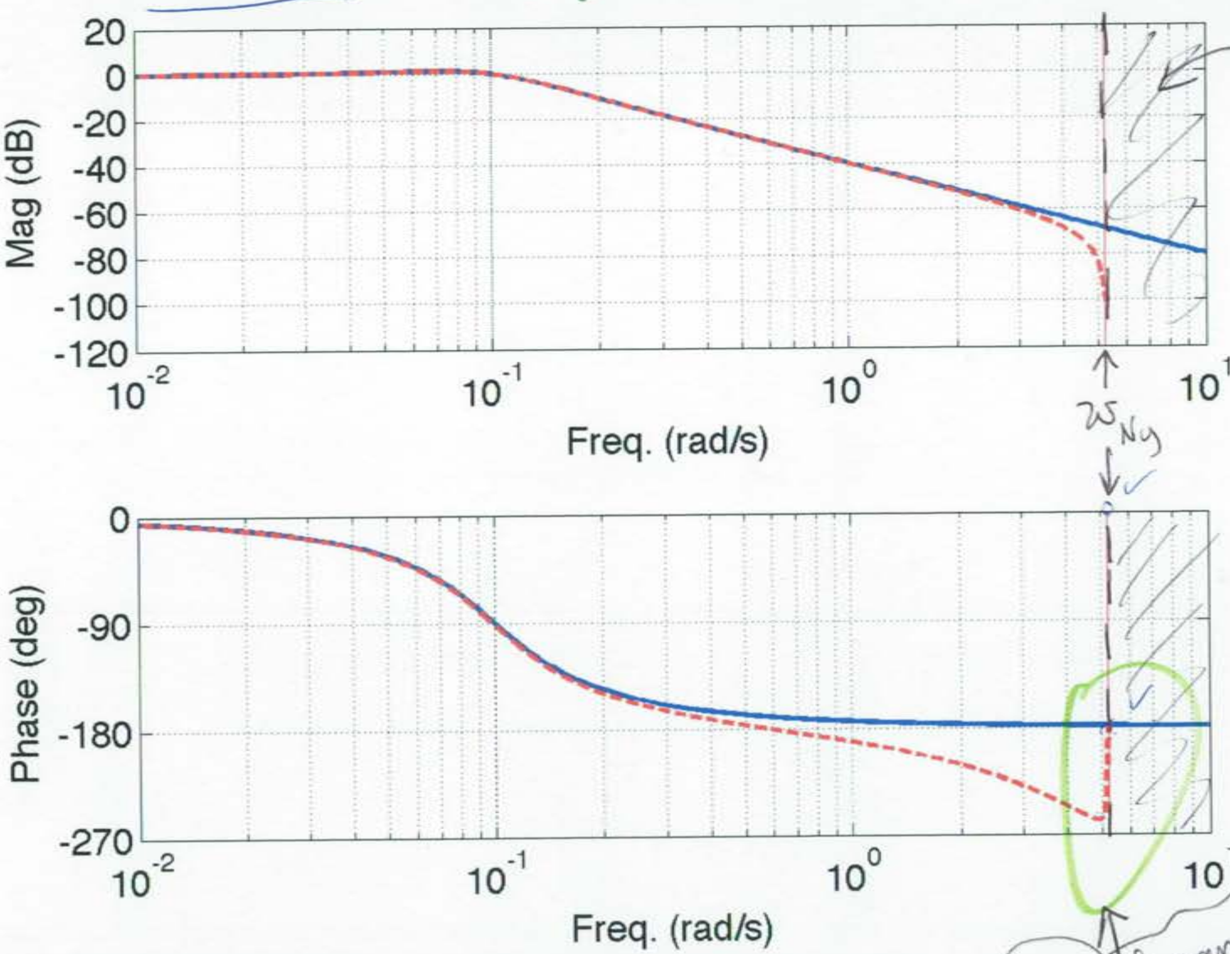


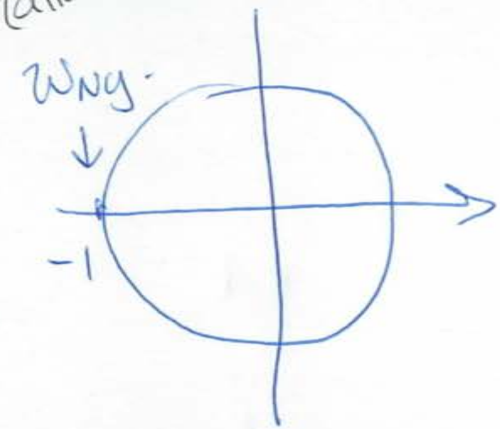
-90° @ ω_{Nyq}.

T=.6 sec
last p.

ZOH Consequences in FREQUENCY DOMAIN

- But wait, G(z) at z=-1 (Nyquist) must be REAL-VALUED! i.e., phase of 0 or +/-180 deg.





$\boxed{Z = -1}$ @ ω_{Ny}

Nyquist frequency is $\boxed{\omega_{Ny} = \frac{\pi}{T}}$

"half-sample" delay NOT EXACT! Breaksdown near ω_{Ny}

To Be Continued... Next Class!

ZOH Equivalent, P(z)

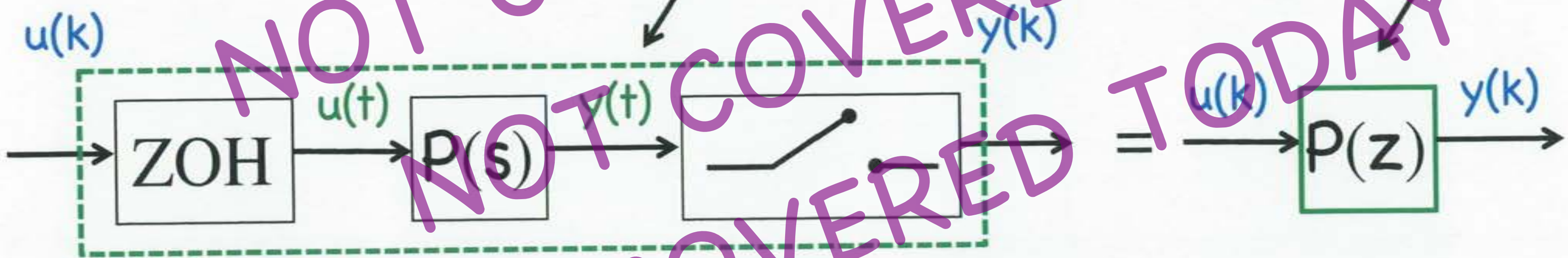
COMING SOON...

$$P_{ZOH}(z) = (1 - z^{-1}) \cdot Z\left\{\frac{P(s)}{s}\right\}$$

Zero-Order Hold Equivalent:

- u(k) gives a "pulse", to be integrated over "T".

- ZOH into P(s), then sampled (rate T) EQUAL P(z).



NOT COVERED TODAY

NOT COVERED TODAY

NOT COVERED TODAY

NOT COVERED TODAY

NOT COVERED TODAY

NOT COVERED TODAY