Today: The Z Transform

Why not Start up MATLAB?

Lecture ~~3~~ 4

ECE 147B

Digital Control

Reading: FPW 6.3.1, 4.6.1, 4.2.1

or: FV 2.3, 2.5, 2.8, 6.3.3

Topics:

- Definition of Z Transform (Review from Lecture 3)
- Some classic (and important) examples
- Block Diagram Algebra is the same (G(z) vs G(s))
- Properties of the Z Transform
- Tustin with Pre-Warp

HW 1 due TOMORROW! Prelab 2 due Tues.

(Keep a copy for yourself. Solutions out in lab...)

Next Class: Time response: s-plane vs z-plane.

- FPW: 4.4, 7.3.1

or: - FV: 2.6

teaching the 'Z-TRANSFORM'...

Focus on EXAMPLES.
(not tables, etc...)

→ MATLAB your way thru class...

CT	DT
Gs = tf(a, b)	Gz = tf(a, b, T)
[1,2,3], [4,5,6]	T = .0 or T = 1

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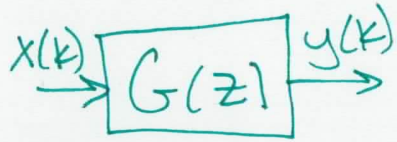
Definition of Z Transform

Given a sequence of samples, x_k:

$$\frac{1}{z^k}$$

$$X(z) \equiv \mathcal{Z}\{x(k)\} \equiv \sum_{-\infty}^{\infty} x_k z^{-k}$$



Example:

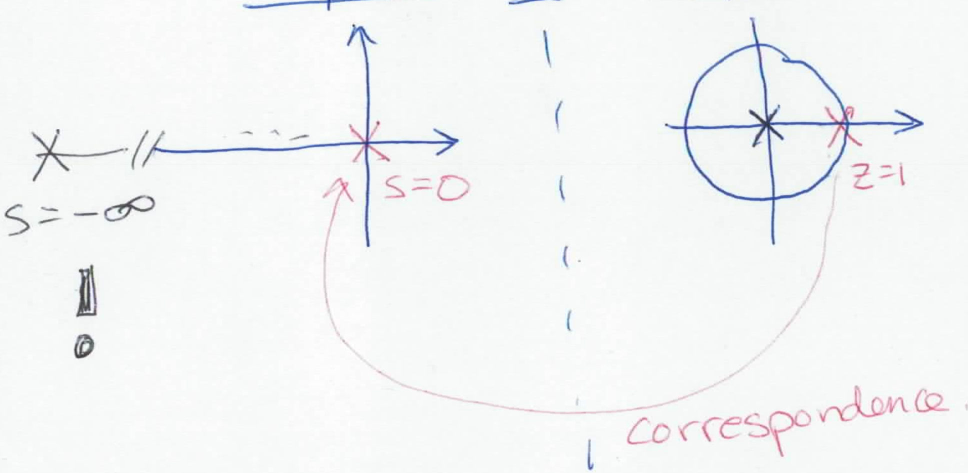
$$G_1(z) = \frac{Y(z)}{X(z)} = \frac{4z^3 + 2z^2 + 5z + 3}{\boxed{z^3}}$$

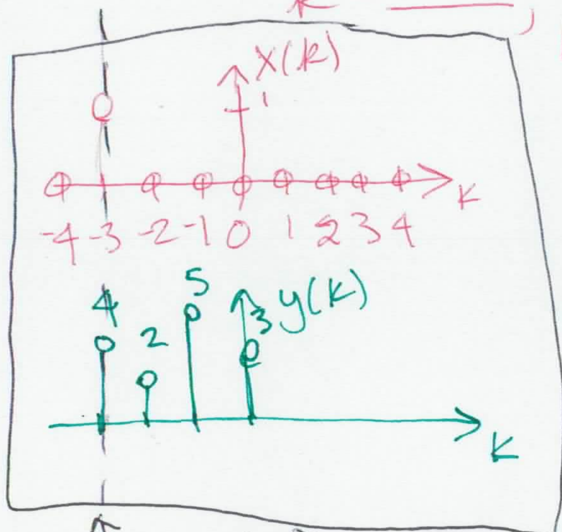
$y(k)$: $\sum \overbrace{y_{k=-3}}^{4} z^3 + \overbrace{y_{-2}}^{2} z^2 + \overbrace{y_{-1}}^{5} z^1 + \overbrace{y_0}^{3} z^0$

$x(k)$: x is non-zero only @ $k = -3$, $\boxed{x(k=-3)=1}$

$x(k \neq -3) = 0$

s-plane vs z-plane





can shift k-axis for both.

Deadbeat control: poles @ $z=0$

(like $s=-\infty$

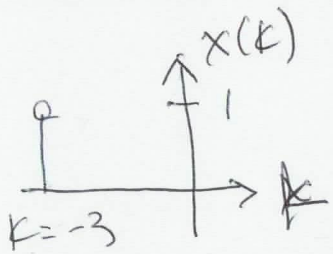
Impulse Response of G(z): Inputs Z-xform is 1

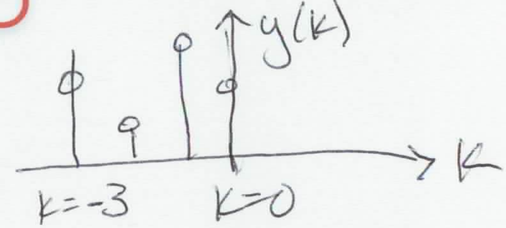
Same Example:

$$G_1(z) = \frac{Y(z)}{X(z)} = \frac{4z^3 + 2z^2 + 5z + 3}{z^3}$$

$$X(z) \equiv \mathcal{Z}\{x(k)\} \equiv \sum_{-\infty}^{\infty} x_k z^{-k}$$

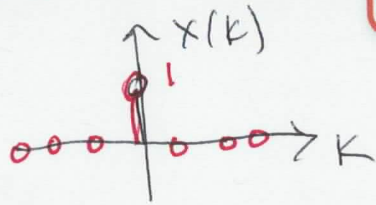
Imagine some u(k) for which $\boxed{\mathcal{Z}(\cancel{x(k)}\ u(k))=z^3}$ ...What is y(k)?

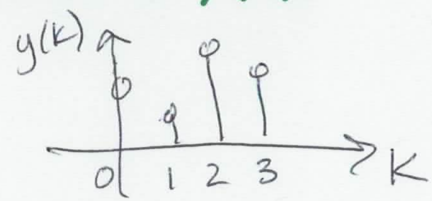




Imagine some u(k) for which $\boxed{\mathcal{Z}(\cancel{x(k)}\ u(k))=1}$ ...What is y(k)?

This is an IMPULSE in x

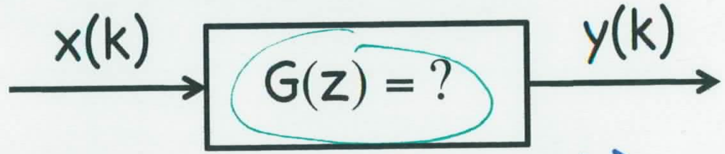




put an IMPULSE in → $\boxed{G(z)=?}$ → y

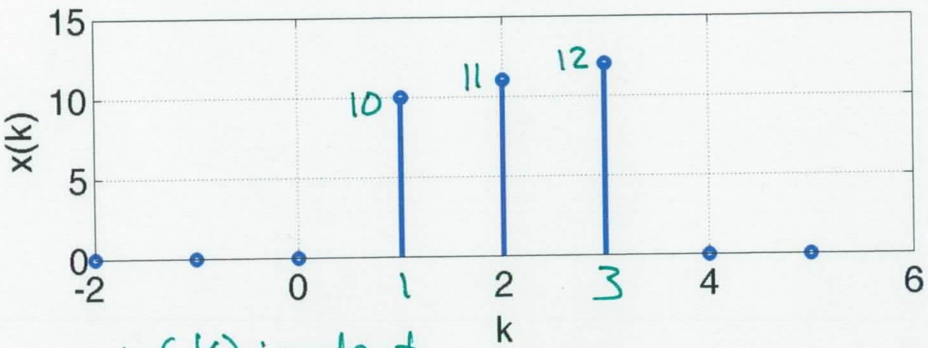
Examples: Find Z xforms for x(k) and y(k)

1. Apply the definition to signal x(k).
2. Apply the definition to signal y(k).
3. Divide 2 by 1 to get the TRANSFER FUNCTION.

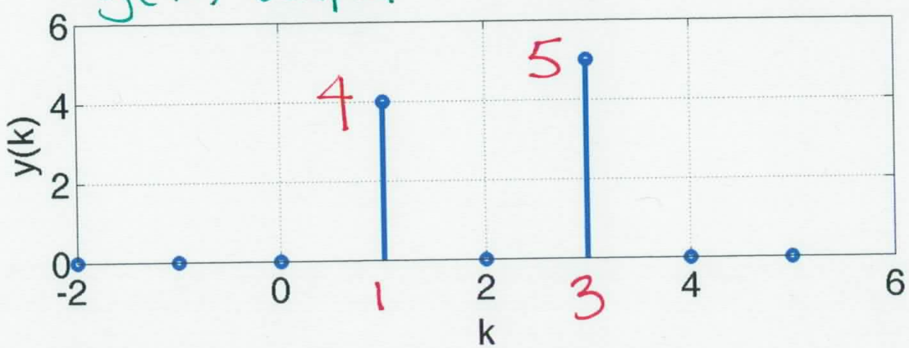


$$G(z) \equiv \frac{Y(z)}{X(z)}$$

x(k): input



y(k): output



$$\sum_{-\infty}^{\infty}(x_k \cdot z^{-k})$$

$$Z(x(k)) = 10z^{-1} + 11z^{-2} + 12z^{-3}$$

$$= \frac{10}{z} + \frac{11}{z^2} + \frac{12}{z^3} = \frac{10z^2+11z+12}{z^3}$$

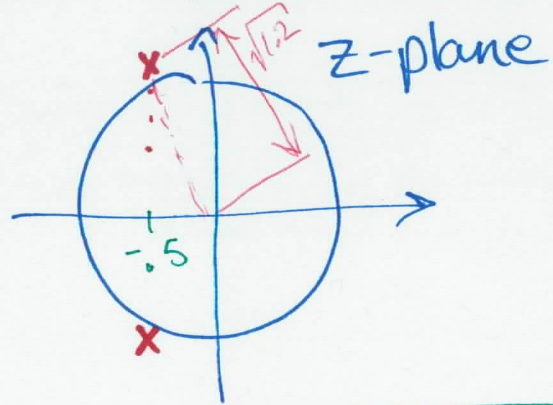
$$Z(y(k)) = 4z^{-1} + 5z^{-3}$$

$$= \frac{4}{z} + \frac{5}{z^3} = \frac{4z^2+5}{z^3}$$

$$\boxed{\therefore G(z) = \frac{(4z^2+5)(\cancel{z^3})}{(\cancel{z^3})(10z^2+11z+12)}}$$

Examples: Find Z xforms for x(k) and y(k)

Now, does this **impulse response** below for this G(z) make sense?



$$G(z) = \frac{(4z^2+5)}{\underbrace{(10z^2+11z+12)}}$$

poles @ $\boxed{z = -\frac{1}{2} \pm .94j}$

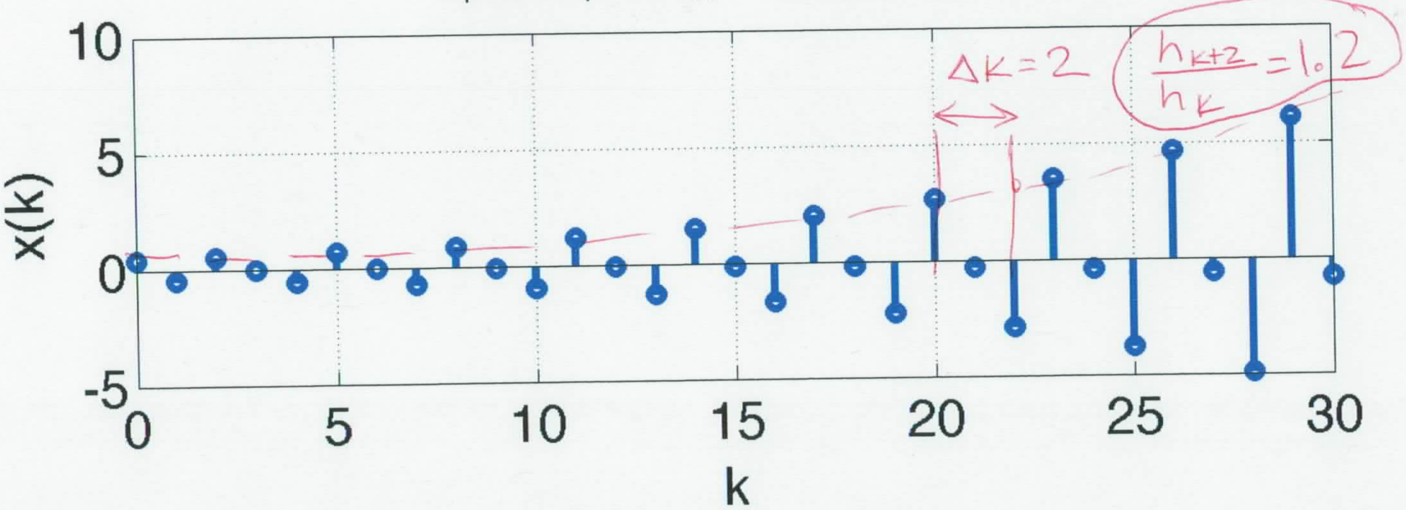
(roots([10, 11, 12]))

char. eqn $\boxed{z^2 + 1.1z + 1.2 = 0}$

$|z| = \sqrt{1.2} > 1.0$

$\therefore$ UNSTABLE!

Impulse Response: x(k) = 1 for k=0, otw x(k) = 0



Examples: Find Z xforms for x(k) and y(k)

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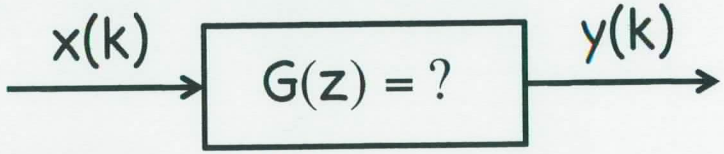
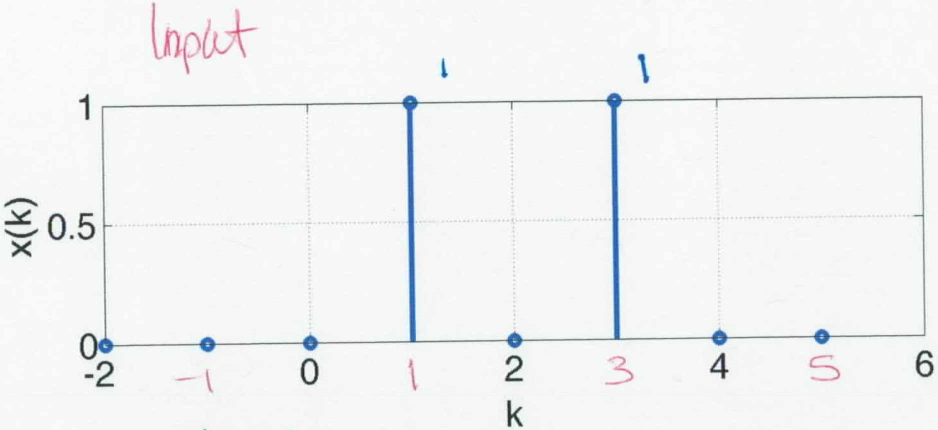
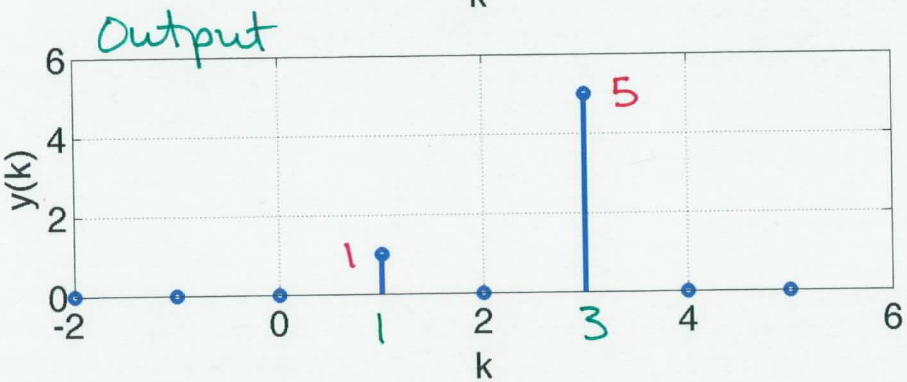




$$\boxed{Z(x(k)) = \frac{1}{z} + \frac{1}{z^3} = \frac{z^2+1}{z^3}}$$



$$Z(y(k)) = \frac{1}{z} + \frac{5}{z^3} = \frac{z^2+5}{z^3}$$

$$G(z) = \frac{z^2+5}{z^2+1}$$

$$= \frac{z^2}{z^2+1} + \frac{5}{z^2+1}$$

Examples: Find Z xforms for x(k) and y(k)

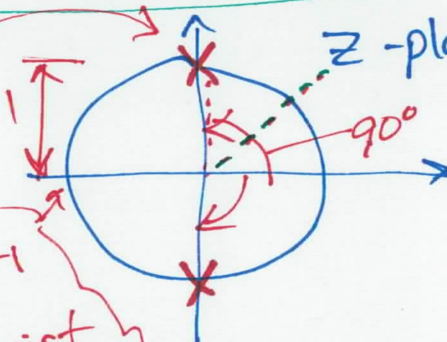
Now, does this **impulse response** below for this G(z) make sense?

$$G(z) = \frac{z^2+5}{z^2+1}$$

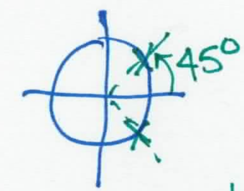
$$(z^2+1=0)$$

(char. eqn ...)

$\boxed{\text{poles: } z = \pm 1 \cdot j}$ ← Marginally Stable

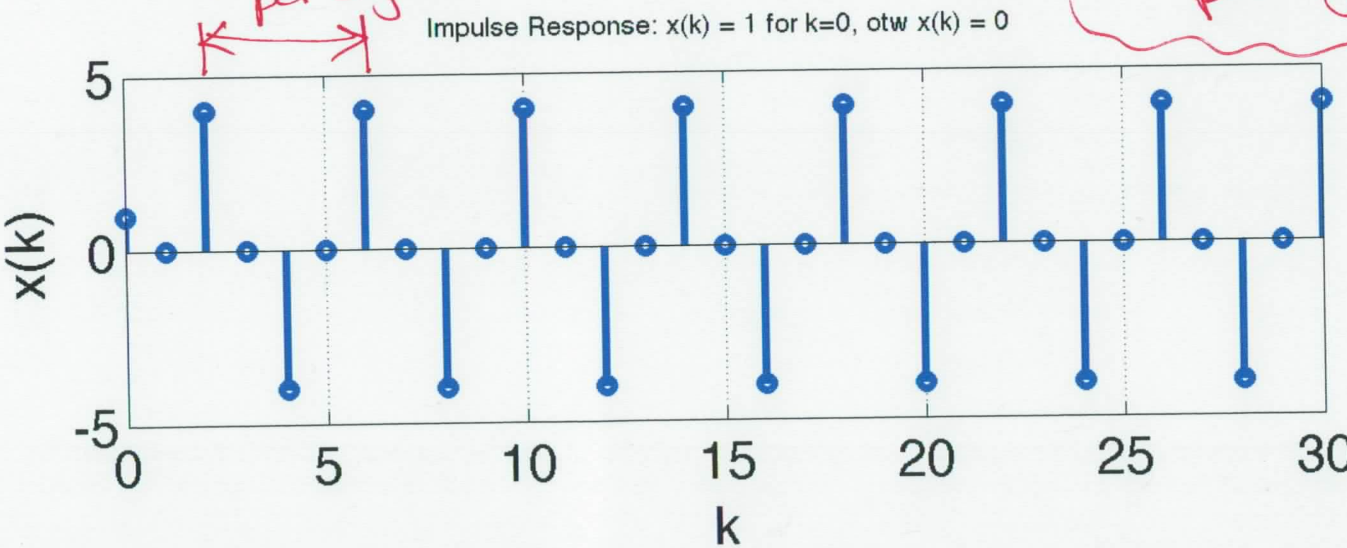


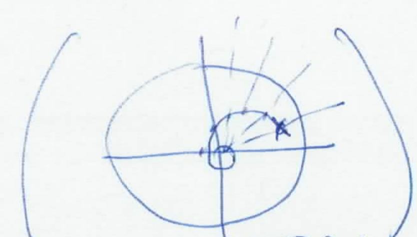
z=-1 is Nyquist (2 samples per cycle)



8 samples per cycles

4 time steps per cycles





TBC'd ...

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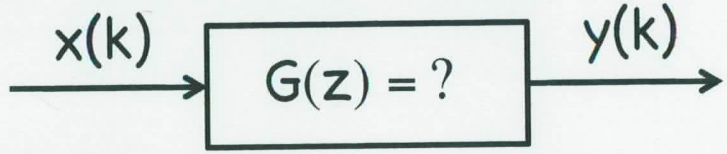
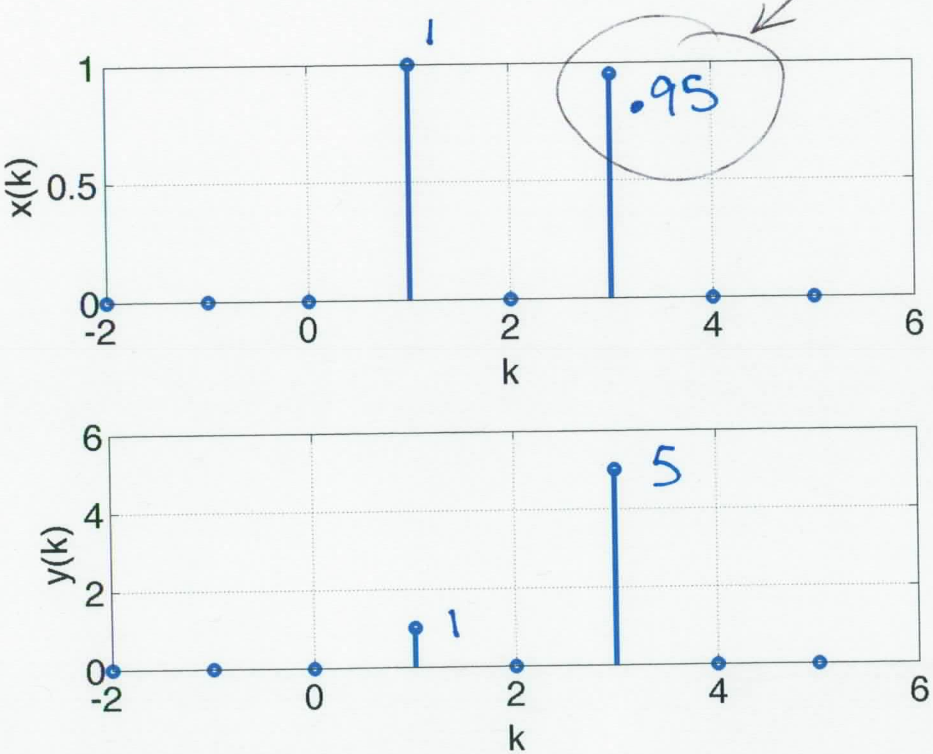


only changed, compared w/ 2 slides back...

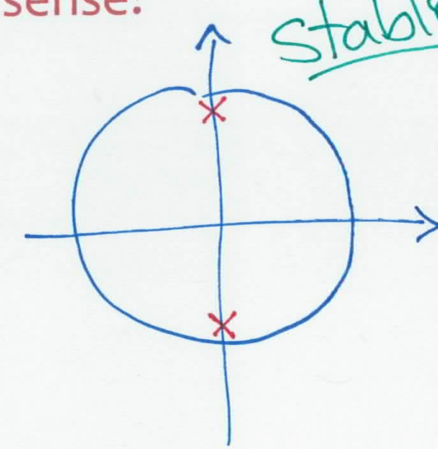


$$G(z) = \frac{z^2+5}{z^2+0.95}$$

Examples: Find Z xforms for x(k) and y(k)

Now, does this **impulse response** below for this G(z) make sense?

stable



char. eqn.: $z^2 + 0.95 = 0$

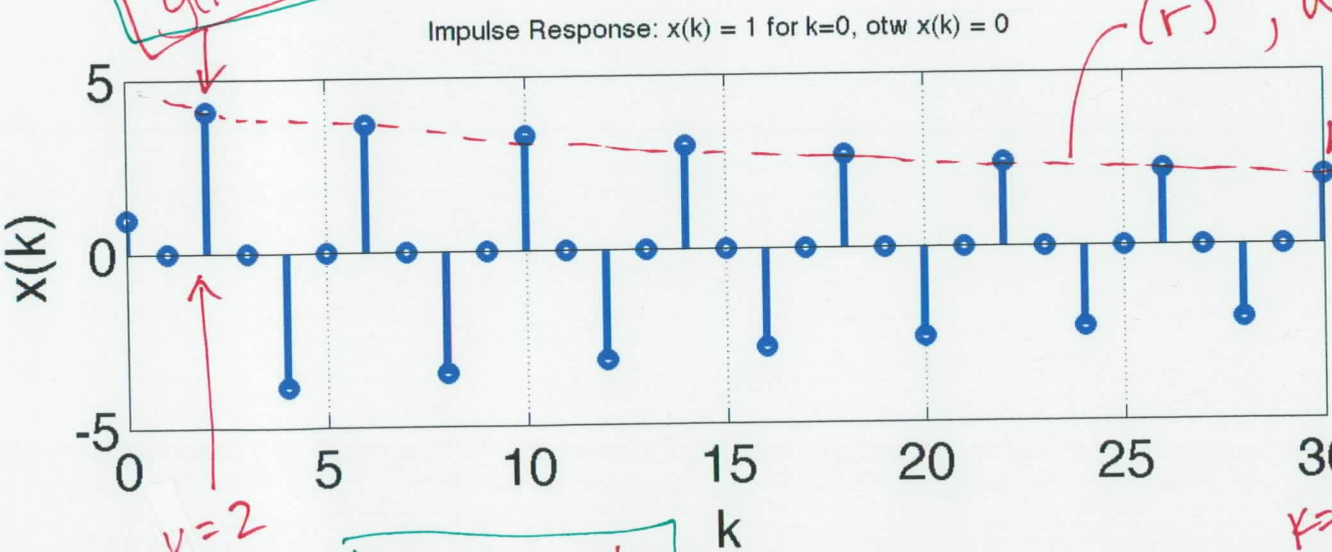
poles: $z = \pm\sqrt{.95}\,j \approx \pm .975j$

$y(k=2) \approx 4.05$

$(r)^k$, where $r = \sqrt{.95}$

$y(k=30) \approx 1.9751$

$\boxed{(.95)^{14} \approx .4877}$



$k=2$

$k=30$

$\xleftrightarrow{\text{28 samples}}$

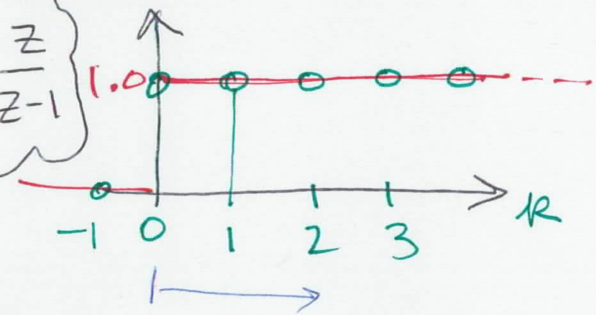
∴ after 28 samples envelope @ $k = N+28$ is .4877 times envelope @ $k=N$

Example: impulse response is a UNIT STEP

$$X(z) \equiv \mathcal{Z}\{x(k)\} \equiv \sum_{-\infty}^{\infty} x_k z^{-k}$$

Output y(k) are SAMPLES of the IMPULSE response for: $G(s) = \frac{1}{s}$

impulse resp. for G(s)



$$G(z) = \sum_{k=-\infty}^{\infty} (y_k z^{-k})$$

$$= \sum_{k=0}^{\infty} (y_k z^{-k})$$

$$= \sum_{k=0}^{\infty} (1 \cdot z^{-k})$$

$$= z^0 + z^{-1} + z^{-2} + z^{-3} + \dots$$

$$= 1 + \frac{1}{z} + \frac{1}{z^2} + \dots$$

$$= \frac{\left(1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} \dots\right)}{\underbrace{\left(1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} \dots\right)\left(1 - \frac{1}{z}\right)}}$$

$$\frac{1}{\left(1 - \frac{1}{z}\right)} = \frac{1}{\left(\frac{z-1}{z}\right)} = \frac{z}{z-1}$$

Want we want is in the numerator (below)

So, we need the denominator to be 1

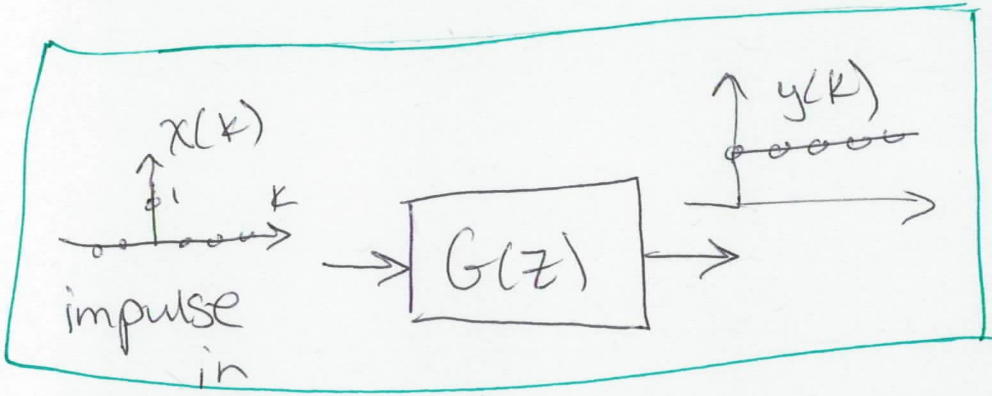
$$1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} \dots$$

$$- \frac{1}{z} - \frac{1}{z^2} - \frac{1}{z^3} \dots$$

$1 + 0$ ✓ SO LONG AS $|z| > 1$ (Region of Convergence... RoC, TBC'd...)

Example: impulse response is a UNIT STEP

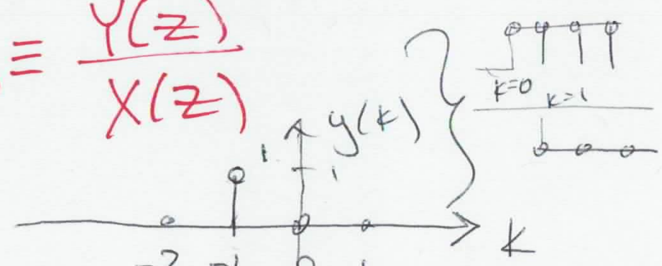
$$\text{So, } \frac{\cancel{(1+\frac{1}{z}\cdots)}}{\cancel{(1+\frac{1}{z}\cdots)}(1-\frac{1}{z})} = \frac{1}{(1-\frac{1}{z})} = \boxed{\frac{z}{z-1} = G(z)}$$



Correspondence.

$$\frac{1}{s} = G(s)$$

So, $G(z) = \frac{z}{z-1}$, and $G(z) \equiv \frac{Y(z)}{X(z)}$



$$\left(\frac{1}{1-\frac{1}{z}}\right) = \frac{z}{z-1}$$

← y is ∅ except for,

← x is ∅ except for