Today: The Z Transform

Lecture 3
ECE 147B
Digital Control

Reading: FPW 6.3.1, 4.6.1, 4.2.1
or: FV 2.3, 2.5, 2.8, 6.3.3

Topics:
- Definition of Z Transform (Review from Lecture 3)
- Some classic (and important) examples
- Block Diagram Algebra is the same (G(z) vs G(s))
- Properties of the Z Transform
- Tustin with Pre-Warp

Next Class: Time response: s-plane vs z-plane.
- FPW: 4.4, 7.3.1

or: - FV: 2.6

teaching the 'Z-TRANSFORM'...

Definition of Z Transform

Given a sequence of samples, x_k:

$$X(z) \equiv \mathcal{Z}\{x(k)\} \equiv \sum_{-\infty}^{\infty} x_k z^{-k}$$

Example:

$$G_1(z) = \frac{Y(z)}{X(z)} = \frac{4z^3 + 2z^2 + 5z + 3}{z^3}$$

Impulse Response of G(z): Inputs Z-xform is 1

Same Example:

$$G_1(z) = \frac{Y(z)}{X(z)} = \frac{4z^3 + 2z^2 + 5z + 3}{z^3}$$

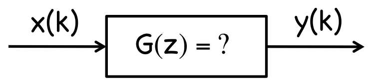
$$X(z) \equiv \mathcal{Z}\{x(k)\} \equiv \sum_{-\infty}^{\infty} x_k z^{-k}$$

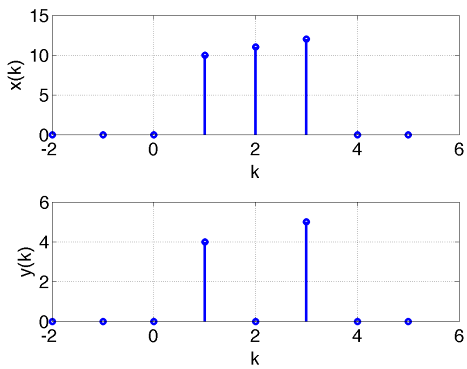
Imagine some u(k) for which $\boxed{\mathcal{Z}(x(z))=z^3}$ ...What is y(k)?

Imagine some u(k) for which $\boxed{\mathcal{Z}(x(z))=1}$ ...What is y(k)?

Examples: Find Z xforms for x(k) and y(k)

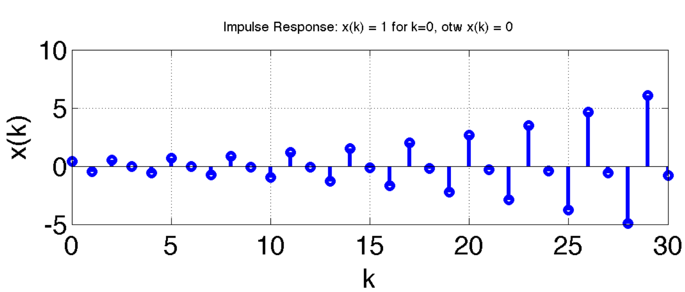
1. Apply the definition to signal x(k).
2. Apply the definition to signal y(k).
3. Divide 2 by 1 to get the TRANSFER FUNCTION.





Examples: Find Z xforms for x(k) and y(k)

Now, does this **impulse response** below for this G(z) make sense?



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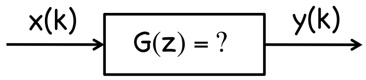

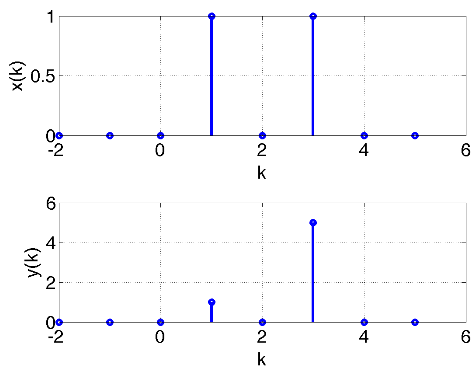

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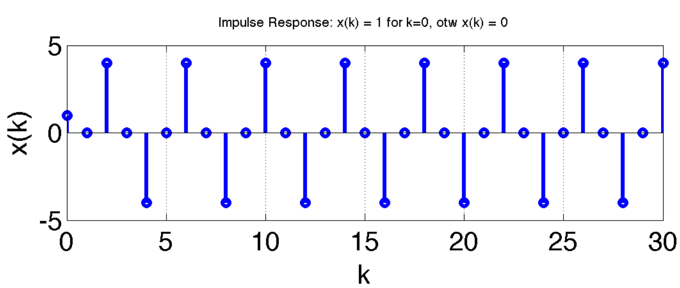

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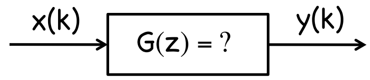

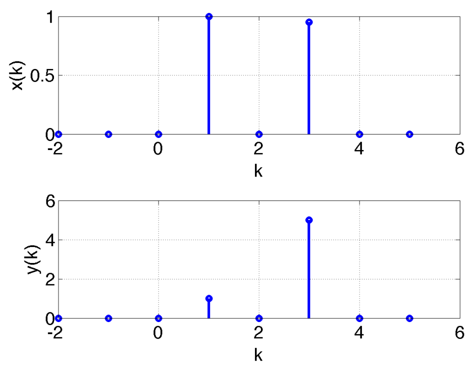

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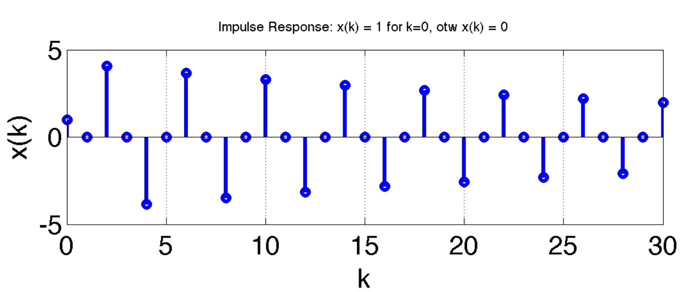

Example: impulse response is a UNIT STEP

$$X(z) \equiv \mathcal{Z}\{x(k)\} \equiv \sum_{-\infty}^{\infty} x_k z^{-k}$$

Output y(k) are SAMPLES of the IMPULSE response for:	$G(s) = \frac{1}{s}$

Example: impulse response is a UNIT STEP

Ex.: imp resp is a DECAYING EXPONENTIAL

$$X(z) \equiv \mathcal{Z}\{x(k)\} \equiv \sum_{-\infty}^{\infty} x_k z^{-k}$$

Output y(k) are SAMPLES of the IMPULSE response for: $G(s) = \dfrac{1}{\tau s + 1}$

Ex.: imp resp is a DECAYING EXPONENTIAL

Properties of the Z Transform

1. Linearity

2. Convolution of time sequences <-> Multiplication of xforms

3. Time Shift: $Z\{f(k+n)\}=z^{n}F(z)$

4. Scaling on the z-plane: $Z\{r^{-k}f(k)\}=F(rz)$

5. Final Value Theorem:

Tustin with Pre-warp (to be continued...)