Today: **Time Response: s-plane vs z-plane**

Lecture 5
ECE 147B
Digital Control

Reading: FPW 4.4, 7.3.1
or: FV 2.6

Topics:
- Classic examples (Review from Lecture 4
- Properties of the Z Transform
- Tustin with Pre-Warp
- Time response calculation

Next Class: Frequency response. Direct design; deadbeat control.
- FPW: 4.5, 7.5
or: - FV: 2.8, 4.1-4.6, 6.7

Definition of Z Transform

Recall: A given sequence of samples, x_k, has a corresponding Z Transform, defined as:

$$X(z) \equiv \mathcal{Z}\{x(k)\} \equiv \sum_{-\infty}^{\infty} x_k z^{-k}$$

Example:

$$G_1(z) = \frac{Y(z)}{X(z)} = \frac{4z^3 + 2z^2 + 5z + 3}{z^3}$$

Example: impulse response is a UNIT STEP

$$X(z) \equiv \mathcal{Z}\{x(k)\} \equiv \sum_{-\infty}^{\infty} x_k z^{-k}$$

Output y(k) are SAMPLES of the IMPULSE response for: $G(s) = \frac{1}{s}$

Ex.: imp resp is a DECAYING EXPONENTIAL

$$X(z) \equiv \mathcal{Z}\{x(k)\} \equiv \sum_{-\infty}^{\infty} x_k z^{-k}$$

Output y(k) are SAMPLES of the IMPULSE response for: $G(s) = \frac{1}{\tau s + 1}$

Ex.: imp resp is a DECAYING EXPONENTIAL

Properties of the Z Transform

1. Linearity
2. Convolution of time sequences <-> Multiplication of xforms
3. Time Shift: $Z\{f(k+n)\}=z^nF(z)$
4. Scaling on the z-plane: $Z\{r^{-k}f(k)\}=F(rz)$
5. Final Value Theorem:
6. and…(drumroll)… Inversion, $X(z) \Leftrightarrow \sum_{-\infty}^{\infty} x_k z^{-k}$

Tustin with Pre-warp

Recall the standard Tustin transformation (aka "trapezoid rule").

$$s \leftrightarrow \frac{2}{T}\frac{(z-1)}{(z+1)}$$

"Tustin with pre-warp" is similar, except is it <u>scaled</u> to preserve MAGNITUDE and PHASE of C(z) at a "pre-warp" frequency, ω_o.

$$s \leftrightarrow \alpha \frac{(z-1)}{(z+1)} \qquad \alpha = \frac{\omega_0}{\tan\left(\frac{\omega_0 T}{2}\right)}$$

Tustin with Pre-warp

1. Let $z = e^{j\omega_o T}$ for some ω_o of interest, and evaluate Gain and Phase of: $s = \alpha \frac{(z-1)}{(z+1)}$

2. Goal: If we match gain and phase for "s" (in the expression above),

we will match gain and phase for ANY: $H(s) = \frac{b(s)}{a(s)}$

$$\alpha = \frac{\omega_0}{\tan\left(\frac{\omega_0 T}{2}\right)}$$

Tustin with Pre-warp

$$\alpha = \frac{\omega_0}{\tan\left(\frac{\omega_0 T}{2}\right)}$$

Solving for TIME RESPONSE, y(k) <-> Y(z)

Given some H(z) = Y(z)/X(z),

Steps:

1. Compute plant TF, H(z).
2. Compute input x(k)'s xform, X(z).

$$X(z) \Leftrightarrow \sum_{-\infty}^{\infty} x_k z^{-k}$$

3. Form product: Y(z)=X(z)H(z)
4. Invert Y(z) to get y(kT)

Solving for TIME RESPONSE, y(k) <-> Y(z)

Approaches:

1. By hand ("inversion")… $X(z) \Leftrightarrow \sum_{-\infty}^{\infty} x_k z^{-k}$
2. Learn known "classic" patterns!
   (better for intuition AND more practical)

Sinusoid: $y(k) = r^k \left(\frac{e^{jk\theta} - e^{-jk\theta}}{2} \right)$

Sinusoid: $y(k) = r^k\left(\dfrac{e^{jk\theta} - e^{-jk\theta}}{2}\right) \Leftrightarrow Y(z) = \left(\dfrac{z^2 - r(\cos\theta)z}{z^2 - 2r(\cos\theta)z + r^r}\right)$