Today: # Frequency Response: s- vs z-plane

Lecture 6
ECE 147B
Digital Control

Reading: FPW 4.5, 7.5
or: FV 2.8, 41-4.6, 6.7

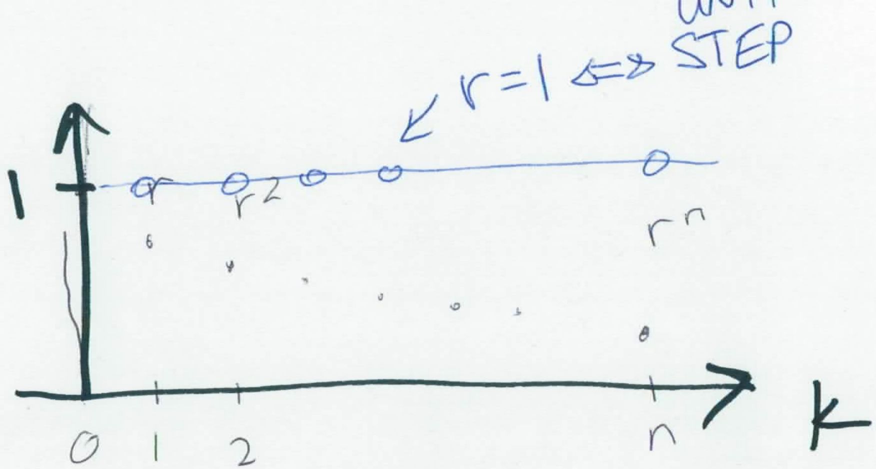
Topics:
- Time response (Finish from Lecture 5)
- Intuition for z-plane poles
- (Freq. Resp.?)

Next Class: Intro to state space (ss).
- FPW: 4.3.3

or: - FV: 7.1, 7.2

HW2 is DUE MONDAY
(AND a Prelab...)

Exponential: $y(k) = r^k$



$$\sum_{0}^{\infty} r^k z^{-k} = \sum_{0}^{\infty} \left(\frac{r}{z}\right)^k$$

$$\frac{\cancel{\left(1-\frac{r}{z}\right)}}{\left(1-\frac{r}{z}\right)}\left(1+\frac{r}{z}+\left(\frac{r}{z}\right)^2+\dots\right) = \frac{1}{\left(1-\frac{r}{z}\right)} = \frac{z}{z-r}$$

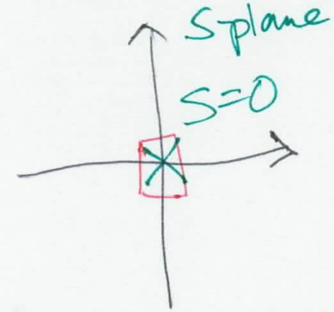
1

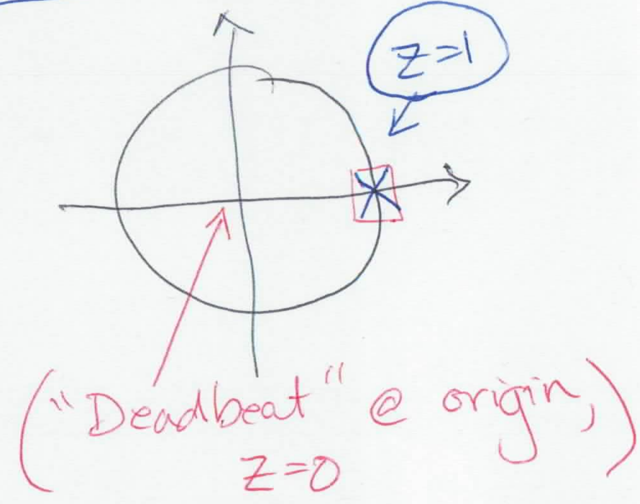
$$\left(-\frac{r}{z}-\left(\frac{r}{z}\right)^2\dots\right)$$

$$-\frac{r}{z}\left(1+\frac{r}{z}+\dots\right)$$

$$\boxed{r=1 \Longleftrightarrow \frac{z}{z-1}}$$

$$\boxed{\frac{1}{s}}$$





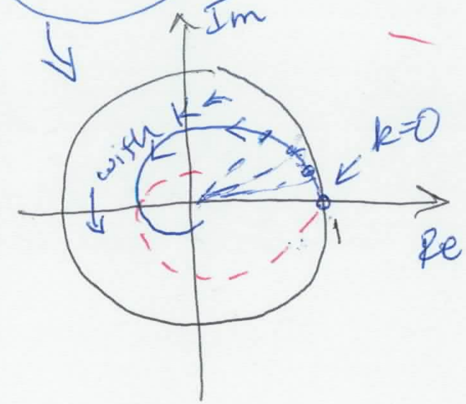
Sinusoid: $y(k) = r^k\left(\frac{e^{jk\theta} + e^{-jk\theta}}{2}\right) = \frac{(r)^k(e^{j\theta})^k}{2} + \frac{r^k e^{-jk\theta}}{2}$

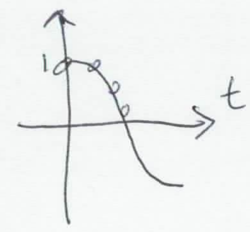
minus

r = 1, sine (not decaying)
(actually a cosine)

r < 1 decaying sinusoid

r > 1 unstable (growing)





Z-transform (definition)

$$\sum_{k=0}^{\infty} r^k\left(\frac{e^{jk\theta} + e^{jk\theta}}{2}\right) z^{-k}$$

Two Parts

$$\left(\frac{1}{2}\right)\left(\frac{1}{1-\frac{re^{j\theta}}{z}}\right)$$

$$\frac{1}{2}\sum r^k e^{j\theta k} z^{-k} = \sum\left(\boxed{\frac{re^{j\theta}}{z}}\right)^k * \frac{1}{2}$$

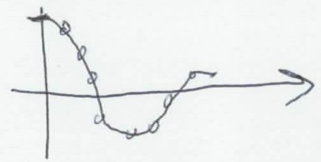
So, we can write this INFINITE sum

AS: $\boxed{\frac{1}{\left(1-\boxed{\frac{re^{j\theta}}{z}}\right)}} \boxed{*\frac{1}{2}}$ same trick

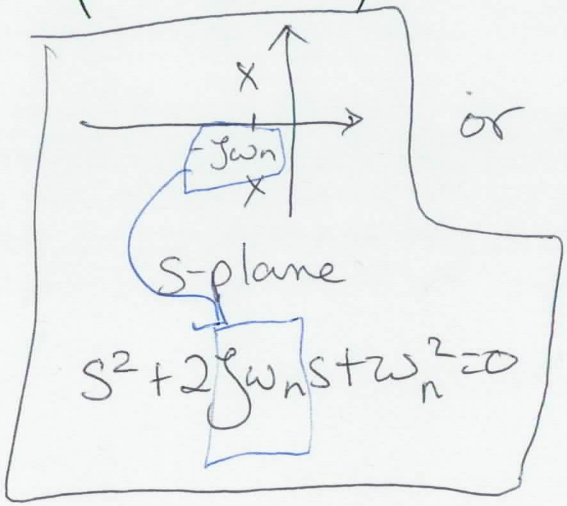
Last page ...

$$\sum_{0}^{\infty}\left(\boxed{\frac{r}{z}}\right)^k \Rightarrow \frac{1}{\left(1-\boxed{\frac{r}{z}}\right)}$$

Sinusoid: $y(k) = r^k\left(\dfrac{e^{jk\theta} + e^{-jk\theta}}{2}\right) \Leftrightarrow Y(z) = \left(\dfrac{z^2 - r(\cos\theta)z}{z^2 - 2r(\cos\theta)z + r^2}\right)$



$\Longleftrightarrow$



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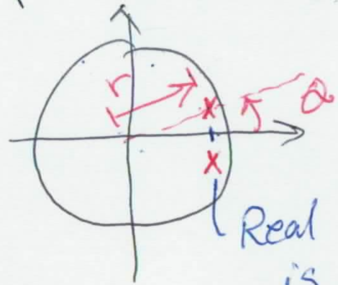


Combining terms from last page...

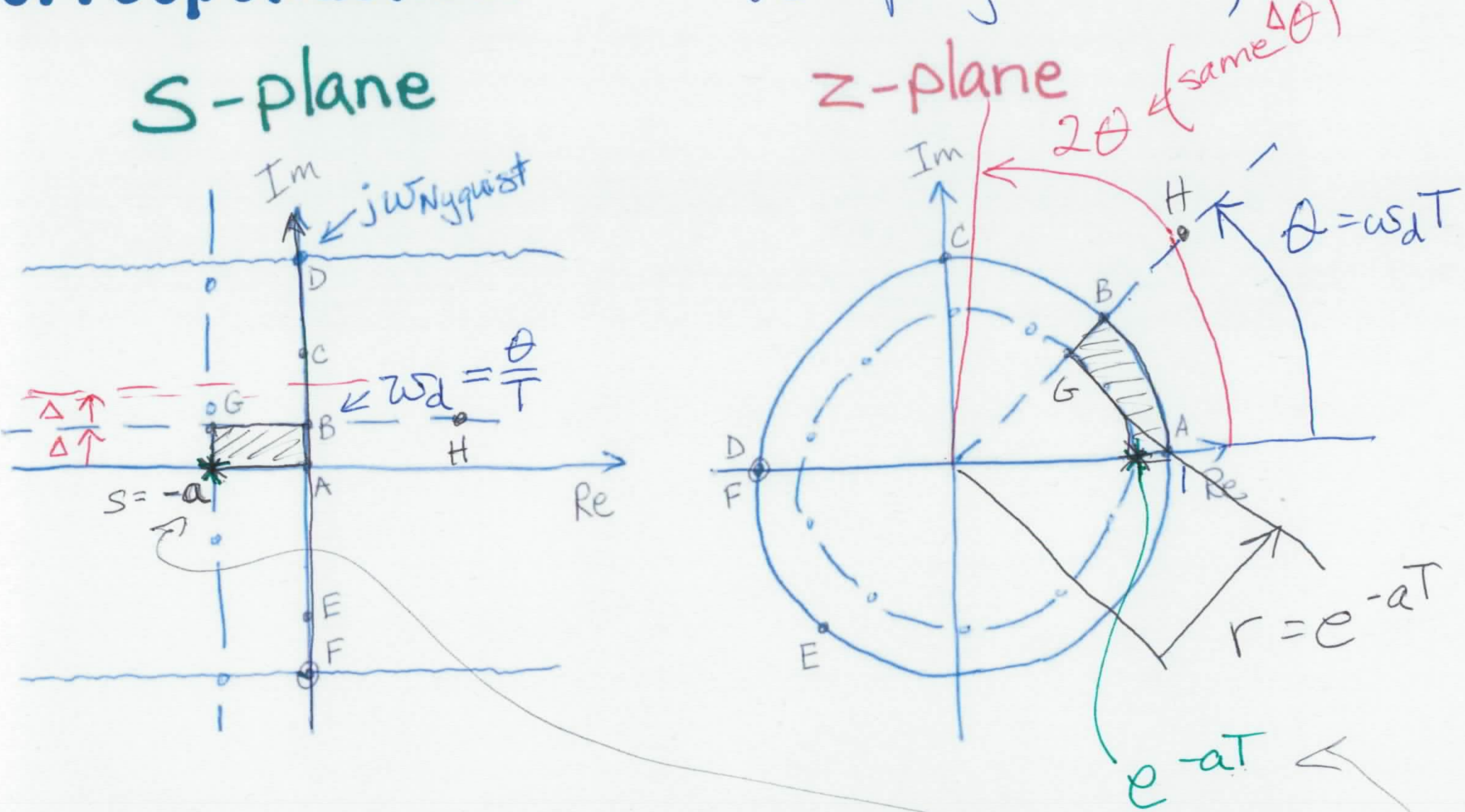
$$Y(z) = \left(\frac{1}{\left(1 - \frac{re^{j\theta}}{z}\right)} + \frac{1}{1 - \frac{re^{-j\theta}}{z}}\right)\frac{1}{2}$$

$$= \left(\frac{z(z - re^{-j\theta})}{(z - re^{j\theta})(z - re^{-j\theta})} + \frac{(z)\ldots}{(z - re^{-j\theta})\ldots}\right)\frac{1}{2}$$

$$= \frac{1}{2}\cdot\frac{(z^2 - zre^{-j\theta}) + (z^2 - zre^{j\theta})}{z^2 - \left[zre^{j\theta} + zre^{-j\theta}\right] + r^2 e^{(j\theta - j\theta)}} = \frac{2z^2 - 2r(\cos\theta)z}{(\ \cdots\ )2}$$

Pole Correspondence:

(Sampling time T)



$s = -\underline{a} + \omega_d j$

$\tau = \frac{1}{a}$, decay env.

$\omega_d \rightarrow$ damped nat. freq.

$z = e^{(-a+\omega_d j)T} = \underbrace{e^{-aT}}_{= r} \underbrace{e^{\omega_d j T}}_{\text{tells "}\theta\text{"}}$

$\rightarrow$ on z-plane, "pole-zero matching"

e.g. $s = bj$

$\updownarrow$

$z = e^{bjT}$

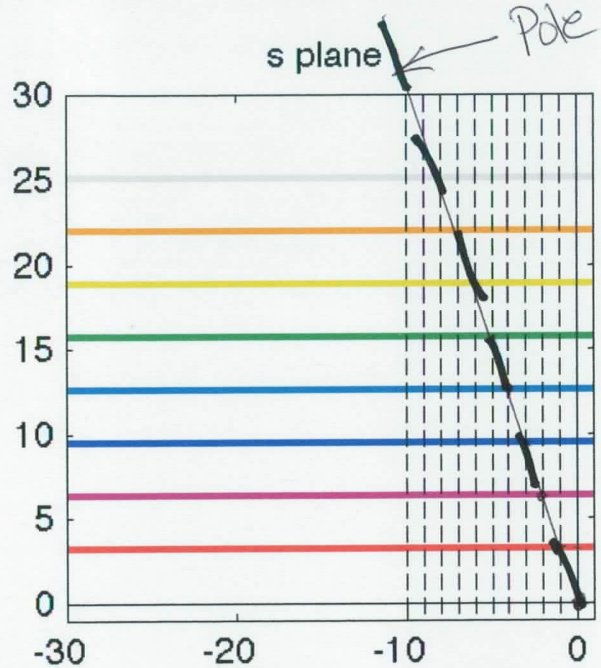
$$\boxed{z = e^{sT}}$$

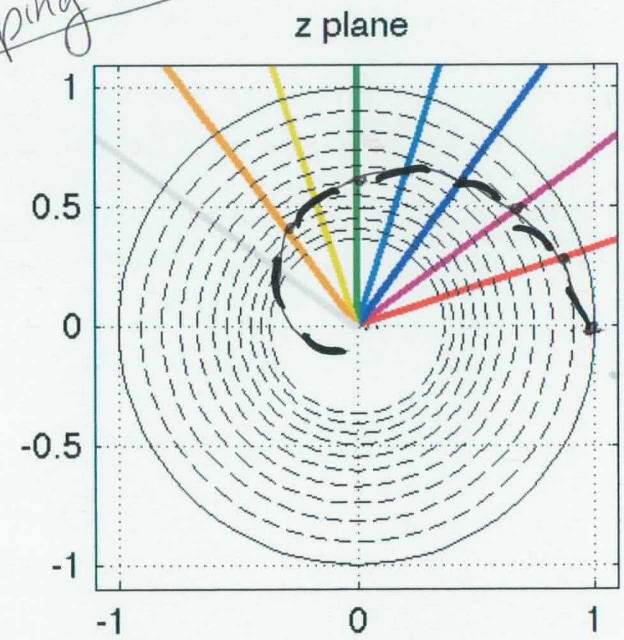
$s = -a,$

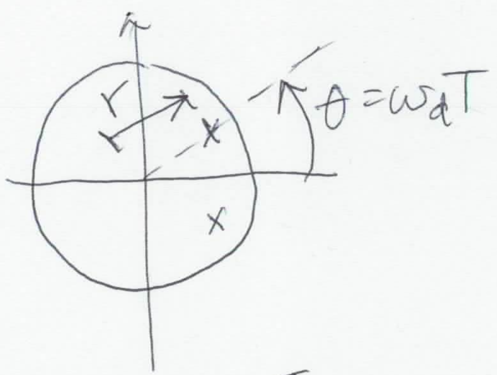
$z = e^{-aT}$

Pole Correspondence:

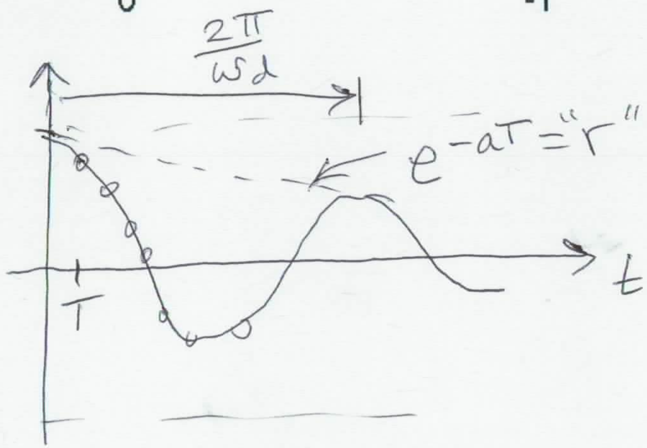
Pole locations w/ same $\zeta \leftarrow$ Damping Ratio

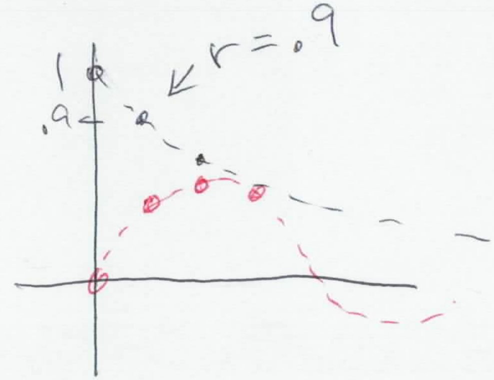






$r = e^{-aT}$





Example: $G(z) = Y(z)/Z(z) = z/(z^2-z+0.5)$

What would a step response look like?

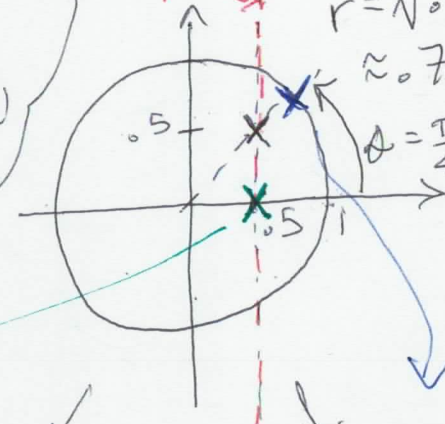
$$\frac{z}{\underbrace{z^2 - z}_{\text{negative } 2\times \text{ the real part...}} + \underbrace{.5}_{r^2 \text{ (like } \omega_n^2)}}$$

Final value to a unit step in?

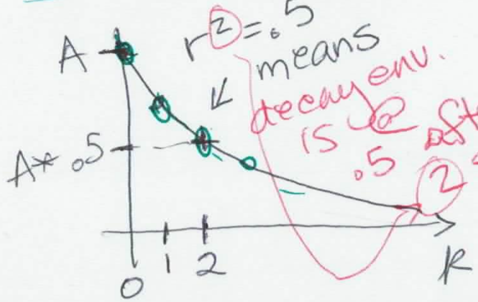
$$(z-1)\cdot\frac{z}{(z^2-z+.5)}\cdot\underbrace{\frac{z}{\cancel{(z-1)}}}_{\text{unit step in.}} \leftarrow \boxed{\text{limit as } z \to 1}$$

$$\frac{1}{1-1+.5} = 2 \quad \text{F.V.}$$

Thinking about poles ONLY first

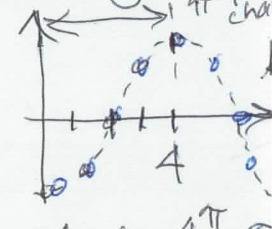


Real Part



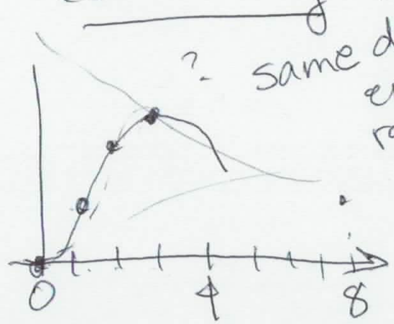
$r^2 = .5$ means decay env. is @ .5 after 2 steps

Imag Part



$4 * \theta = 4\frac{\pi}{4} = \pi$

Combining?



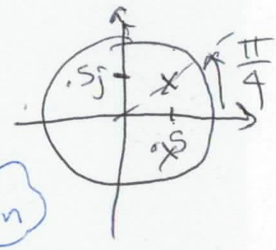
same decay env. rate

Example: $G(z) = Y(z)/X(z) = z/(z^2-z+0.5)$

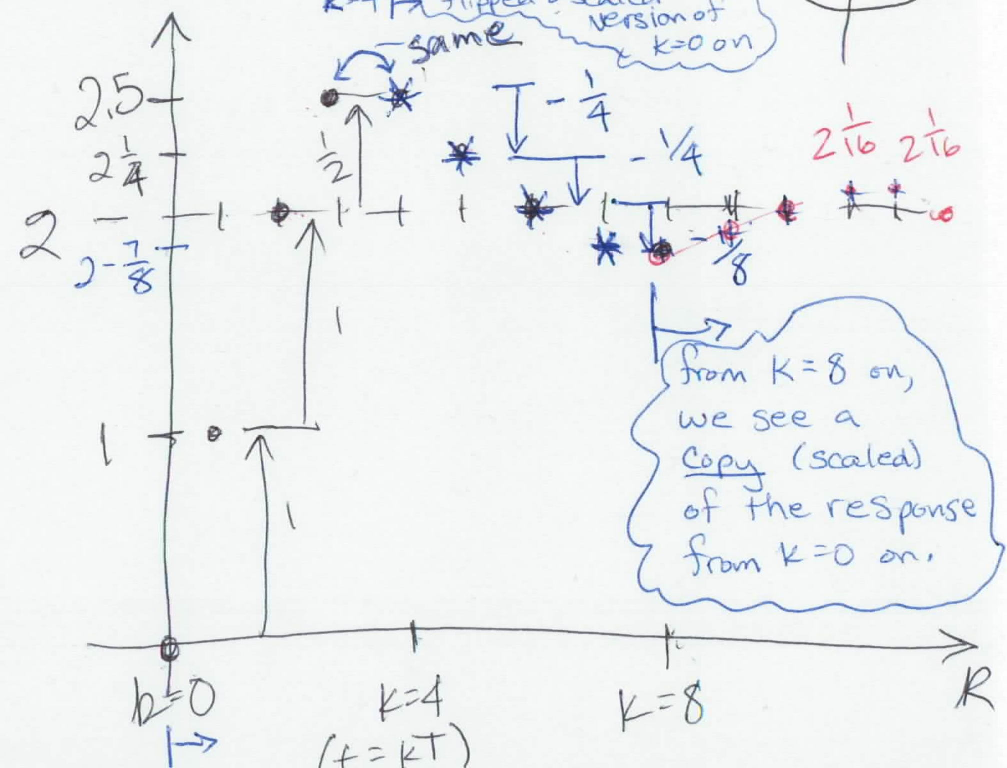
Second look, lets write the diff eq. $\rightarrow$ $\dfrac{Y}{X} = \dfrac{z}{z^2 - z + .5}$

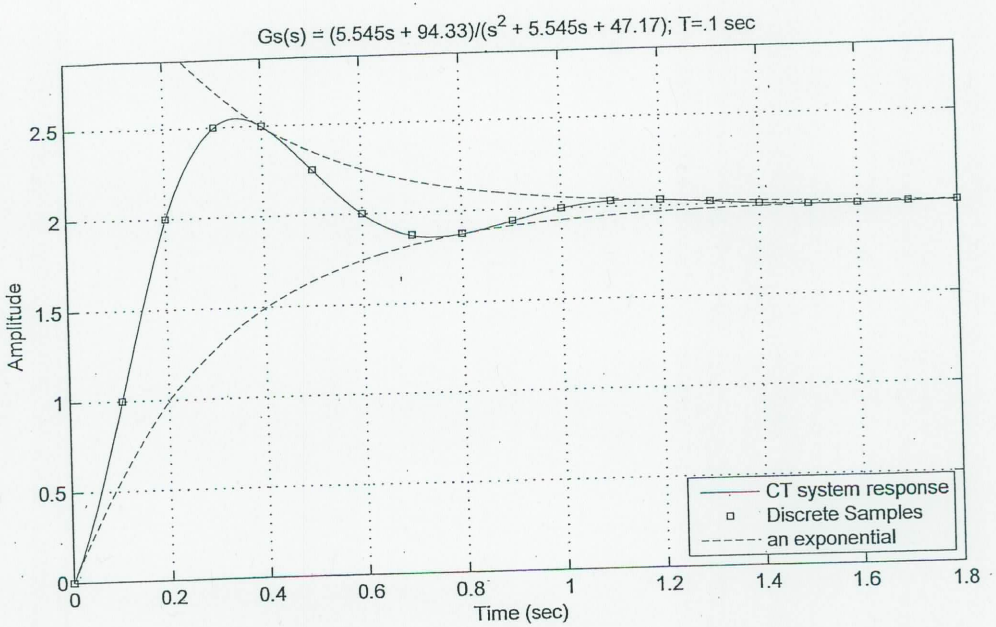
$$y_{k+2} - y_{k+1} + .5y_k = x_{k+1}$$

$$\boxed{y_k = y_{k-1} - .5y_{k-2} + x_{k-1}}$$



k	x	y	
-1	0	0	
0	1	0	
1	1	1	
2	1	2	$1+1-\frac{0}{2}$
3	1	2.5	$1+2-\frac{1}{2}$
4	1	2.5	$1+2.5-1$
5	1	2.25	$1+2.5-1.25$
6	1	2	$1+2.25-1.25$
7	1	$1\frac{7}{8}$	$1+2-1.125$
8	1	$1\frac{7}{8}$	$1+1\frac{7}{8}-1$




Gs(s) = (5.545s + 94.33)/(s2 + 5.545s + 47.17); T=.1 sec
Amplitude
Time (sec)
CT system response
Discrete Samples
an exponential

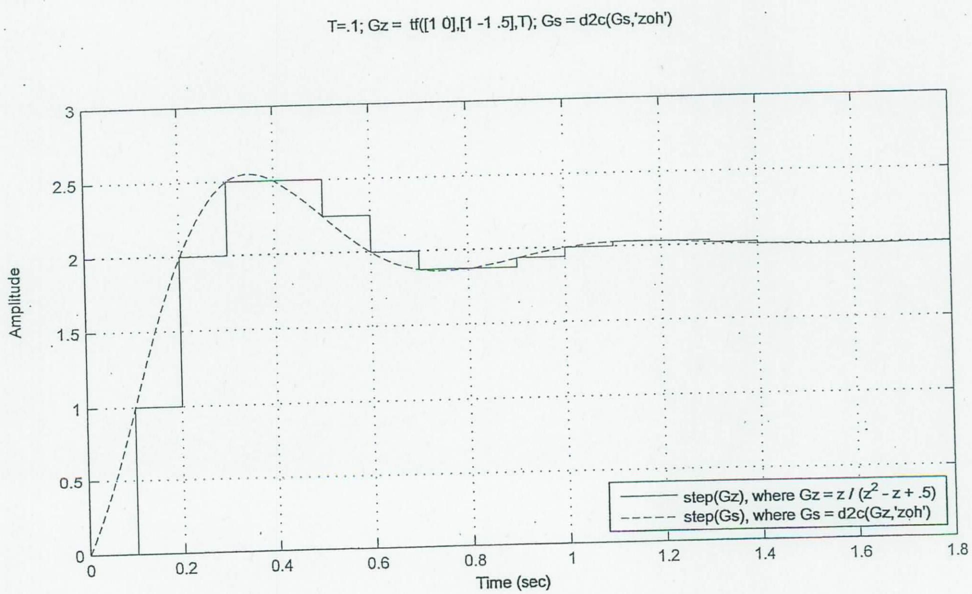

T=.1; Gz = tf([1 0],[1 -1 .5],T); Gs = d2c(Gs,'zoh')
Amplitude
Time (sec)
step(Gz), where Gz = z / (z2 - z + .5)
step(Gs), where Gs = d2c(Gz,'zoh')

Figure 4.26
Corresponding lines in the s-plane and the z-plane according to $z = e^{sT}$

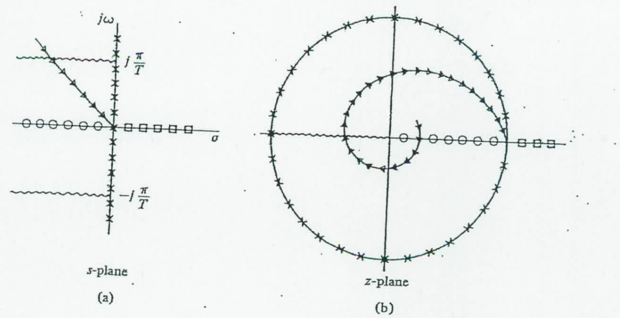


Figure 2.5
Time functions associated with points in the s-plane

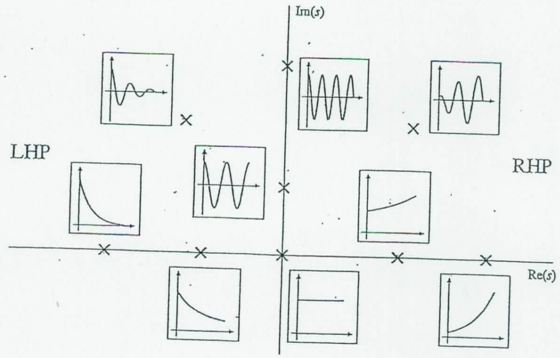


Figure 4.25
Time sequences associated with pole locations in the z-plane