Lecture 7
ECE 147B
Digital Control

Today: "Matched" pole/zero method

Reading: FPW 5.5, 4.5 (7.4), 6.2, 7.3, 7.5
or: FV 6.7, 2.8 (6.5), 6.3.2, 6.1, 6.6

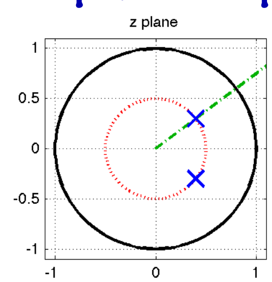
Topics:
- Time response: Ripple
- Frequency Response
- Pole-zero matching
- Root locus
- Direct design

Next Class: Intro to state space (ss).
- FPW: 4.3.3

or: - FV: 7.1, 7.2

Time Response: "Ripple"

$z \leftrightarrow e^{sT}$



$z_1 = 0.4 + 0.3j$

$z_2 = 0.4 - 0.3j$

$s_1 = \dfrac{\ln(r) + \theta j}{T}$

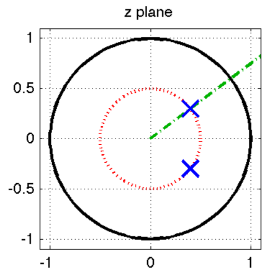
$s_2 = \dfrac{\ln(r) - \theta j}{T}$

$$G(z) = \frac{1.8}{(z-(0.4+0.3j))(z-(0.4-0.3j))} = \frac{1.8}{z^2 - 0.8z + 0.25}$$

r = ______________________ θ = ________________________

s = ______________________

Time Response: "Ripple"



$$G(z) = \frac{1.8}{z^2 - 0.8z + 0.25}$$

Unit step response

k	u(k)	y(k)
-1	0	0
0	1	0
1	1	0
2	1	1.8
n	1	$y(n) = K_{fv} + c_1 z_1^n + c_2 z_2^n$
∞	1	4

K_{fv} = ______________________________

$$y(k=0) = K_{fv} + c_1 z_1^0 + c_2 z_2^0$$

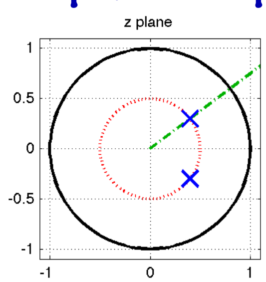
$$y(k=1) = K_{fv} + c_1 z_1^1 + c_2 z_2^1$$

$$Ax = b$$

$$\begin{bmatrix} 1 & 1 \\ z_1 & z_2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} y(0) - K_{fv} \\ y(1) - K_{fv} \end{bmatrix}$$

```
x = A\b
```

Time Response: "Ripple"



Use MATLAB at your desk!!

$Ax = b$

```
x = A\b
```

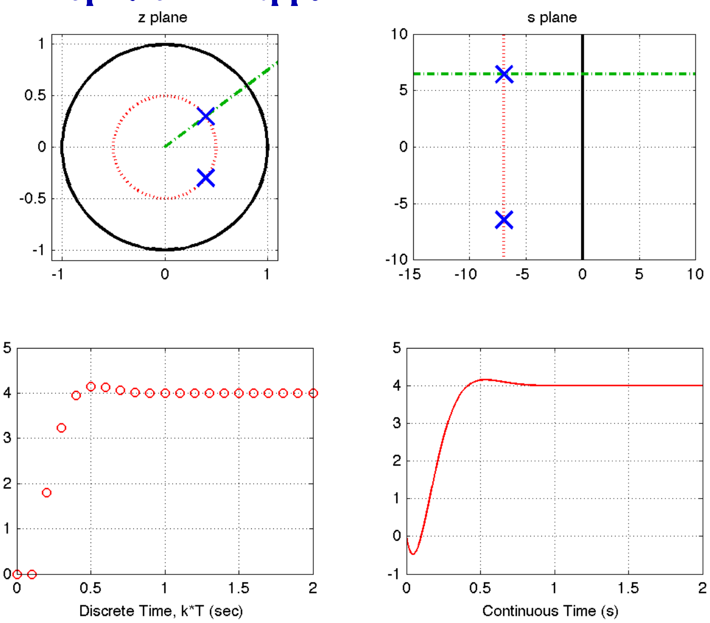
c_1 = ______________________________

c_2 = ______________________________

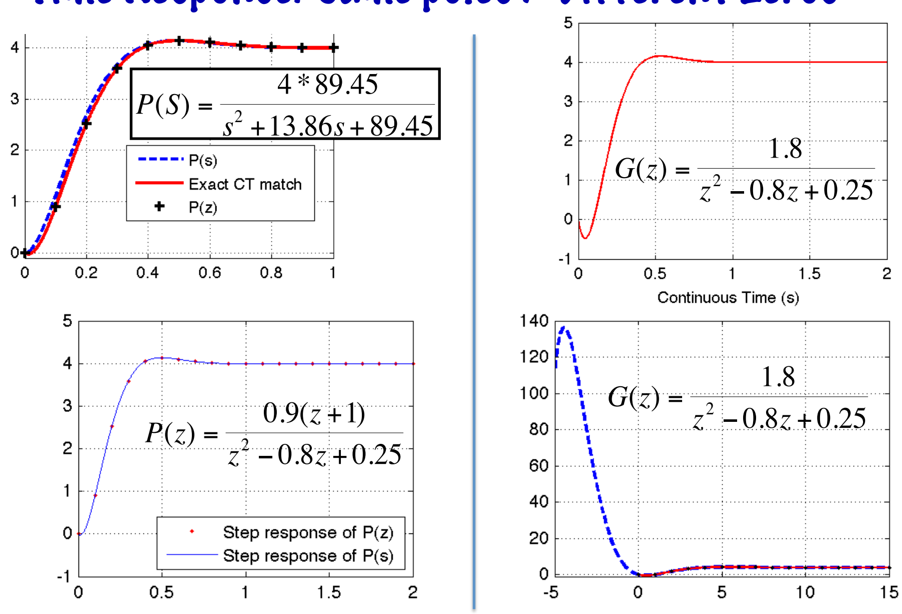
$$y(n) = K_{fv} + c_1 z_1^n + c_2 z_2^n$$

k	u(k)	y(k)
3	0	____
4	1	____
5	1	____
6	1	____
7	1	____
∞	1	4

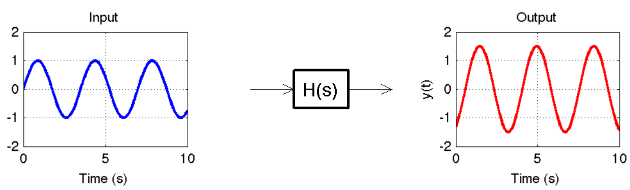
Time Response: “Ripple”



Time Response: Same poles / Different zeros



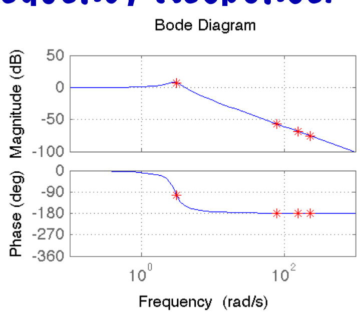
Frequency Response:

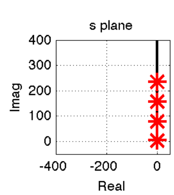


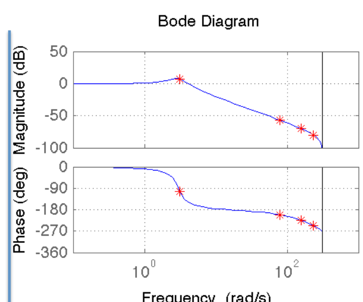
Continuous Time (CT)

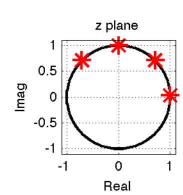
Discrete Time (DT)

Frequency Response:









Continuous Time (CT)

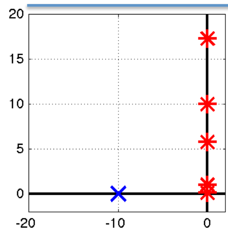
Discrete Time (DT)

Frequency Response: H(s)

$G(s) = s + 10$

$H(s) = \frac{10}{G(s)} = \frac{10}{s+10}$

$\omega = 0.1$
$\omega = 1$
$\omega = \frac{10}{\sqrt{3}}$
$\omega = 10$
$\omega = 10\sqrt{3}$



Consider some "comparable" H(z)...

$H(s) = \frac{10}{s+10}$

$s \to \frac{2}{T}\frac{(z-1)}{(z+1)}$

Tustin:

```
Hs = tf([10],[1, 10])
T = 0.05;
Hz = c2d(Hs,T,'tustin')
```

Consider some "comparable" H(z)...

$$H_2(z) = \frac{2}{5}\frac{1}{(z-0.6)}$$

Consider only the POLE first (same DC gain).

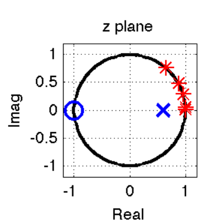
When is the phase -45°?

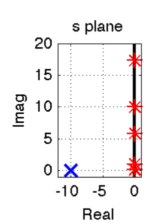
Frequency Response: H(z)

$$H(z) = \frac{1}{5}\left(\frac{z+1}{z-0.6}\right)$$

$$H(s) = \frac{10}{s+10}$$

	$\omega = 0.1$	
	$\omega = 1$	
	$\omega = \frac{10}{\sqrt{3}}$	
	$\omega = 10$	
	$\omega = 10\sqrt{3}$	





"Pole-Zero Matching" approximation

Heuristic method. This is another option on our "menu" to convert C(s) to C(z)... The idea is to use the same mapping between s and z planes for BOTH poles AND zeros:

$$z \leftrightarrow e^{sT}$$

Rules to convert some CTTF G(s) to a DTTF G(z):

1. Map poles to: $z_p = e^{s_p T}$
2. Map zeroes to: $z_z = e^{s_z T}$
3. Keep adding zeros @ "z=-1" to G(z) until the order of the numerator (of G(z)) is one less than the order of the denominator.
4. Match DC gains (for G(z), to match G(s)).

Usage in MATLAB (can go either way):

```
Gs = d2c(Gz,'matched')
```

```
Gz = c2d(Gs,T,'matched')
```

"Pole-Zero Matching" approximation $z \leftrightarrow e^{sT}$

```
T=1;
Gz = c2d(Gs,T,'matched')
```

```
Gs = d2c(Gz,'matched')
```

$$G(z) = \frac{(z+1)}{(z^2 - z + .5)} \leftrightarrow G(s) = \frac{4*0.737}{(s^2 + 0.6931s + 0.737)}$$

$$z_1 = 0.5 + 0.5j$$

$$s_1 = \ln(0.5 + 0.5j)/T = \text{-}0.3466 + 0.7854\text{j}$$

$$z_2 = 0.5 - 0.5j$$

$$s_1 = \ln(0.5 - 0.5j)/T = \text{-}0.3466 - 0.7854\text{j}$$

```
>> conv([1 -s1],[1 -s2])
ans =
   1.000      0.6931     0.737
```

Root Locus

The root locus rules for z-plane and s-plane are identical!

What is different is the meaning of the resulting pole locations, i.e., stable poles in the left-hand plane (CT) versus within the unit circle (DT).

$$P(s) = \frac{1}{(s+1)(s+3)}$$

$$G(z) = \frac{1}{(z-.4)(z-.8)}$$

Root Locus

The root locus rules for z-plane and s-plane are identical!

$$P(s) = \frac{1}{(s+0.2)^4}$$

$$G(z) = \frac{1}{(z+0.2)^4}$$