ECE 147C
Solutions, Problem 6.2 (Noisy identification).

Contents

The Simulink block provided corresponds to a discrete-time system with transfer function

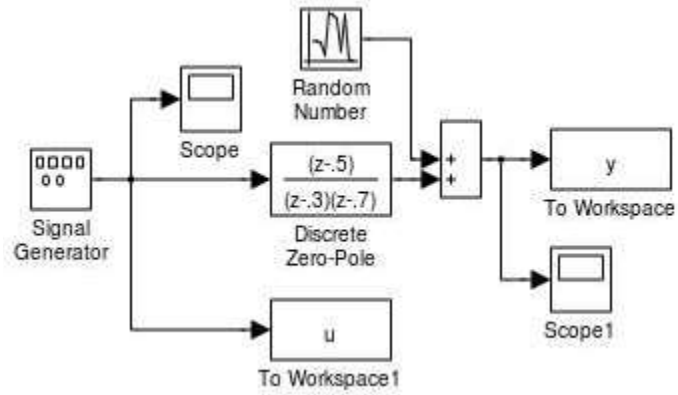
$$H(z) := \frac{\alpha_1 z + \alpha_0}{z^2 + \beta_1 z + \beta_0}$$

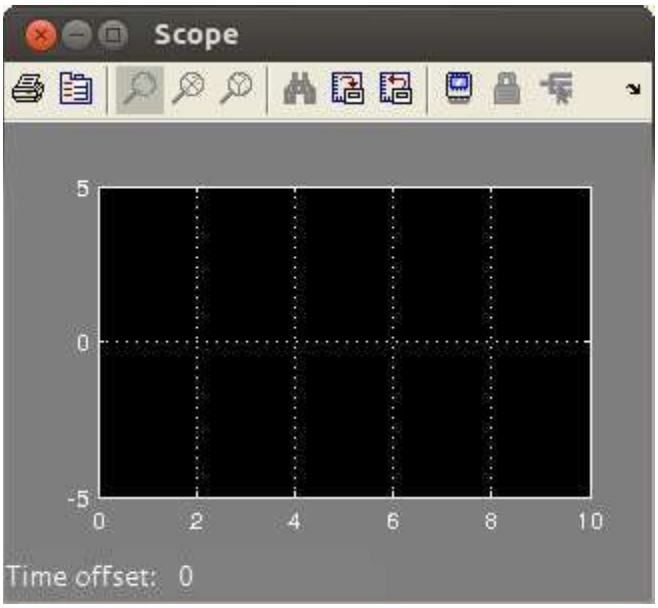
```
clc; clear all; close all
```

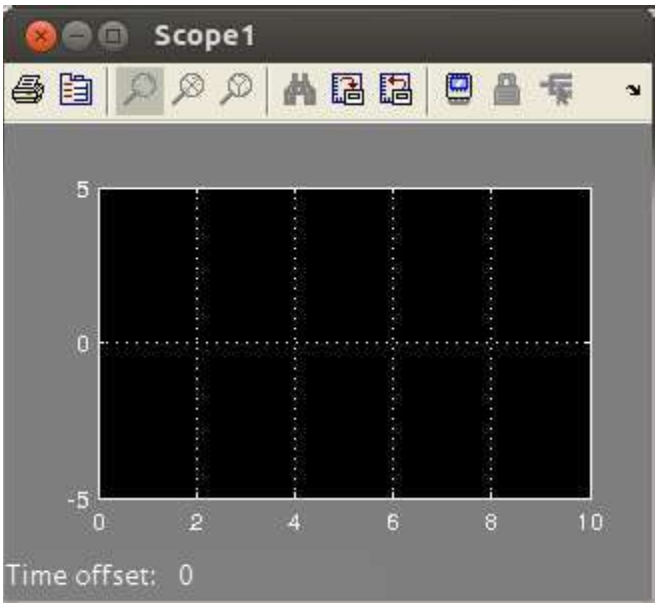
Part 1

Use least-squares to estimate the 4 coefficients of the transfer function. Repeat the identification experiment 25 times to obtain 25 estimates of the system's transfer functions.

```
T = 0.01;    % sampling period for the .mdl's disc-time zero-pole block
[sec/sample]
N=25;        % # times to run model

open robust_p2_mod
```







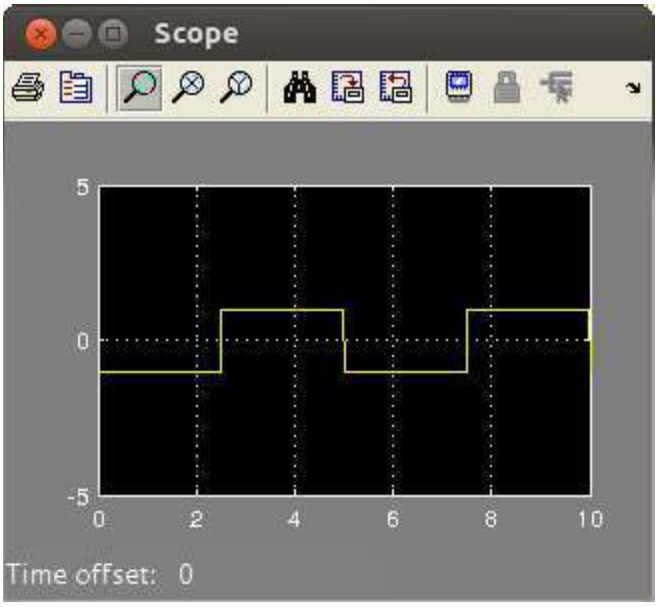
The given H has order:

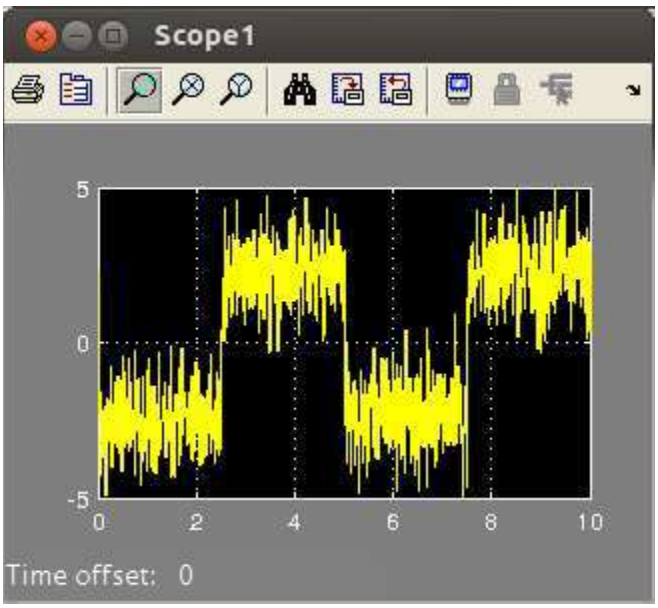
```
n=2;
m=1;
```

Simulate the system 25 times and estimate the TF. Please see the my_id function for details about its implementation.

```
% Store identification data here:
tfs = struct();

for i = 1:N
    sim('robust_p2_mod',10);
    [num,den,SSE,MnSV1,MxSV1] = my_id(u,y,2,1);
    tfs(i).num = num;
    tfs(i).den = den;
    tfs(i).SSE = SSE;
    tfs(i).MnSV1 = MnSV1;
    tfs(i).MxSV1 = MxSV1;
end

whos tfs

clearvars -EXCEPT tfs N T
```

```
  Name      Size             Bytes  Class     Attributes

  tfs       1x25             15920  struct
```





Part 2

Convert the discrete-time transfer functions so obtained to continuous-time using the Tustin transformation.

Hint: Use the MATLAB command d2c.

```
for i = 1:N
    num = tfs(i).num;
    den = tfs(i).den;

    P{i} = d2c(tf(num,den,T),'tustin');
end
```

```
clearvars -EXCEPT P N T
```

Part 3

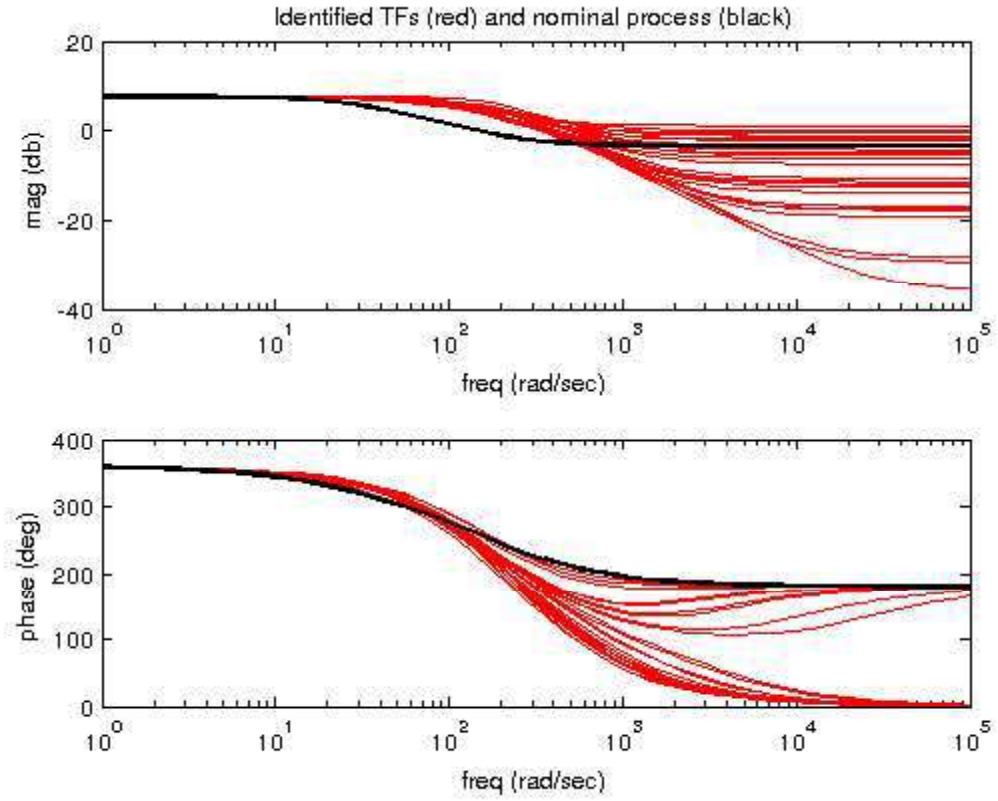
Select a nominal model and compute the corresponding additive and multiplicative uncertainty bounds.

Nominal Continuous Plant:

```
P0 = d2c(tf([1 -0.5],[1 -1 0.21],T),'tustin');
```

Plot to see if it looks representative of the estimated actual plants.

```
figure(1); clf;
w = logspace(0,5);
P{N+1} = P0;  % hack
for i=1:N+1
    if i==N+1; sty = {'k-','linewidth',2}; else; sty = {'r-'}; end;

    [mag,phase] = bode(P{i},w);
    mag = squeeze(mag); phase = squeeze(phase);
    magdb = 20*log10(mag);
    figure(1); subplot(2,1,1);
    semilogx(w,magdb,sty{:}); hold on;
    subplot(2,1,2);
    semilogx(w,phase,sty{:}); hold on;
end
subplot(2,1,1); ylabel('mag (db)'); xlabel('freq (rad/sec)')
title('Identified TFs (red) and nominal process (black)')
subplot(2,1,2); ylabel('phase (deg)'); xlabel('freq (rad/sec)')

P = P(1:end-1); % undo hack
clearvars -EXCEPT P0 N P
```



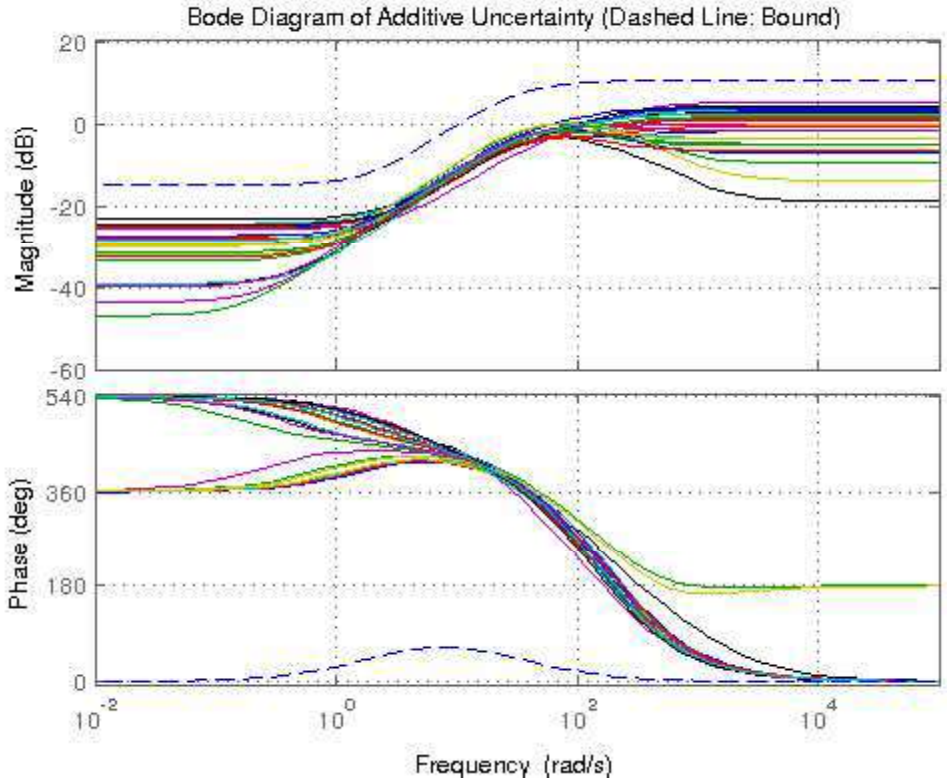
Looks like a pretty good compromise.

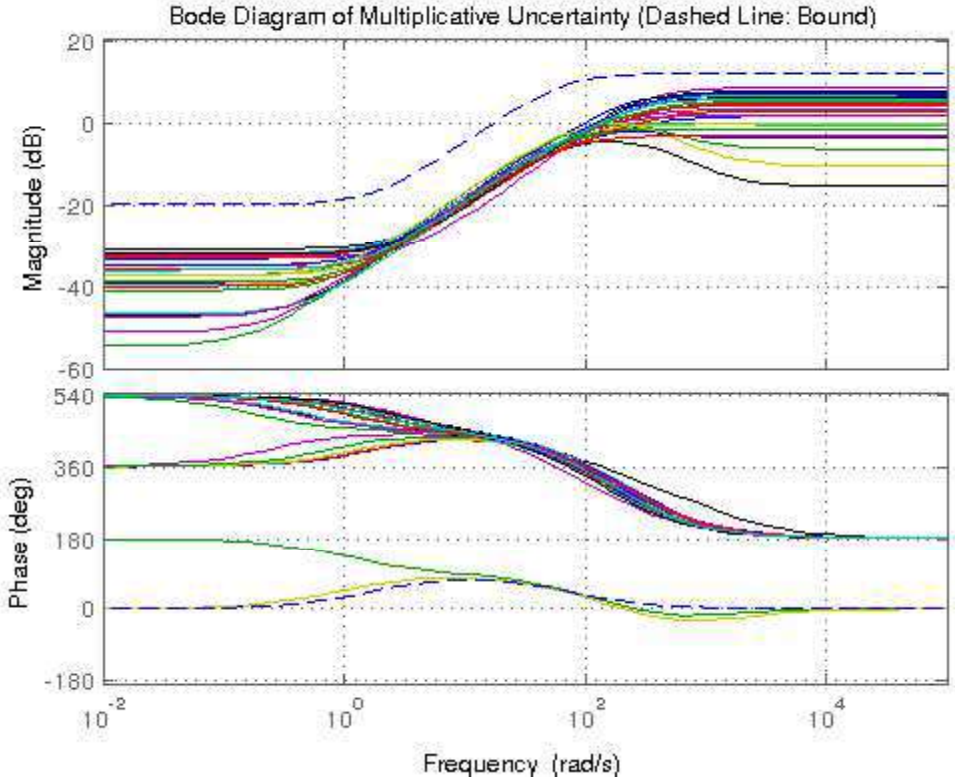
Compute uncertainty.

```
for i = 1:N
    % Additive Uncertainty
    AU{i} = P{i} - P0;

    % Multiplicative Uncertainty
    MU{i} = AU{i} / P0;
end
```

Plot.

```
figure(1); clf; bode(AU{:}); grid on;
title('Bode Diagram of Additive Uncertainty (Dashed Line: Bound)');
la = tf([3.3 6],[1 34]);
hold on; bode(la,'--');
figure(2); bode(MU{:}); grid on;
title('Bode Diagram of Multiplicative Uncertainty (Dashed Line: Bound)');
lm = tf([4 7],[1 70]);
hold on; bode(lm,'--');
```


Bode Diagram of Additive Uncertainty (Dashed Line: Bound)
Magnitude (dB)
20
0
-20
-40
-60
Phase (deg)
540
360
180
0
10^{-2}
10^{0}
10^{2}
10^{4}
Frequency (rad/s)


Bode Diagram of Multiplicative Uncertainty (Dashed Line: Bound)
Magnitude (dB)
20
0
-20
-40
-60
Phase (deg)
540
360
180
0
-180
10^{-2}
10^{0}
10^{2}
10^{4}
Frequency (rad/s)

ECE 147C
Solutions, Problem 6.3 (Small gain).

Contents

For the nominal process model and multiplicative uncertainty that you obtained in Exercise 6.2, use the small gain condition to verify if the following controllers achieve stability for all admissible process models:

```
C{1} = tf(0.3,1);
C{2} = tf(2,[1 0.1]);
C{3} = tf(20,[1 -30]);
C{4} = zpk(-1,[-2 -3],5);
C{5} = zpk(-1,[-0.3 -0.3],2);
```

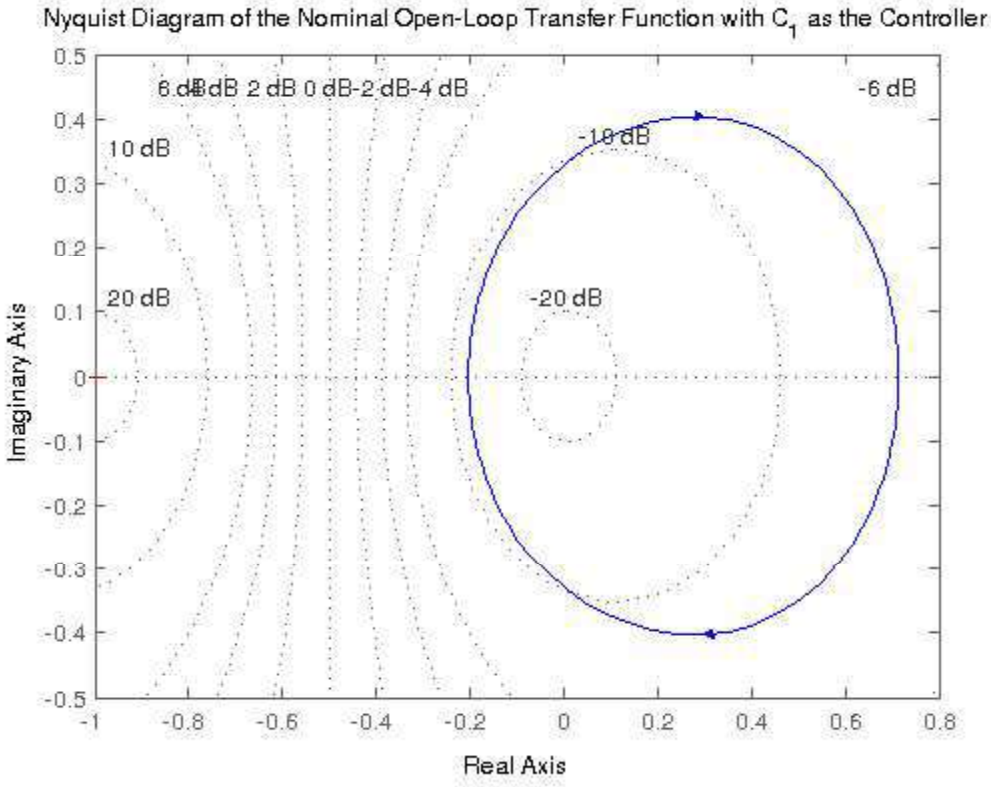
Justify your answers with appropriate Bode plots.

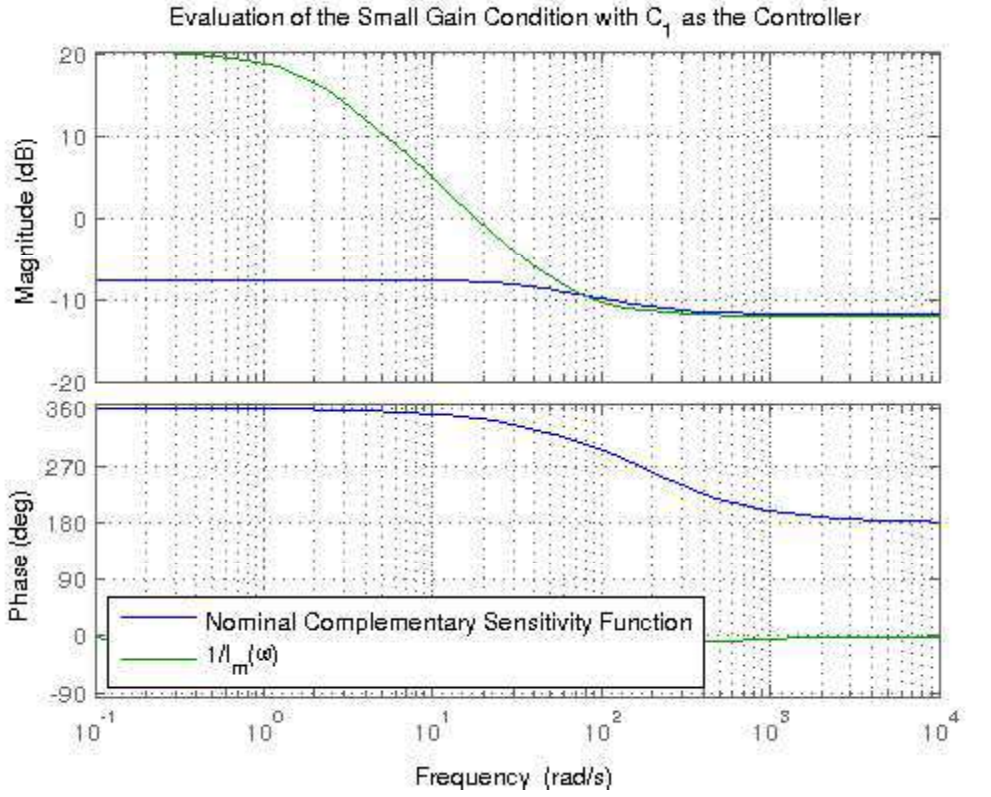
Load the process model and multiplicative uncertainty:

```
load P6_2_results P0 lm C
```
```
Warning: Variable 'C' not found.
```

Controller 1

```
i=1;
plot_controller
```

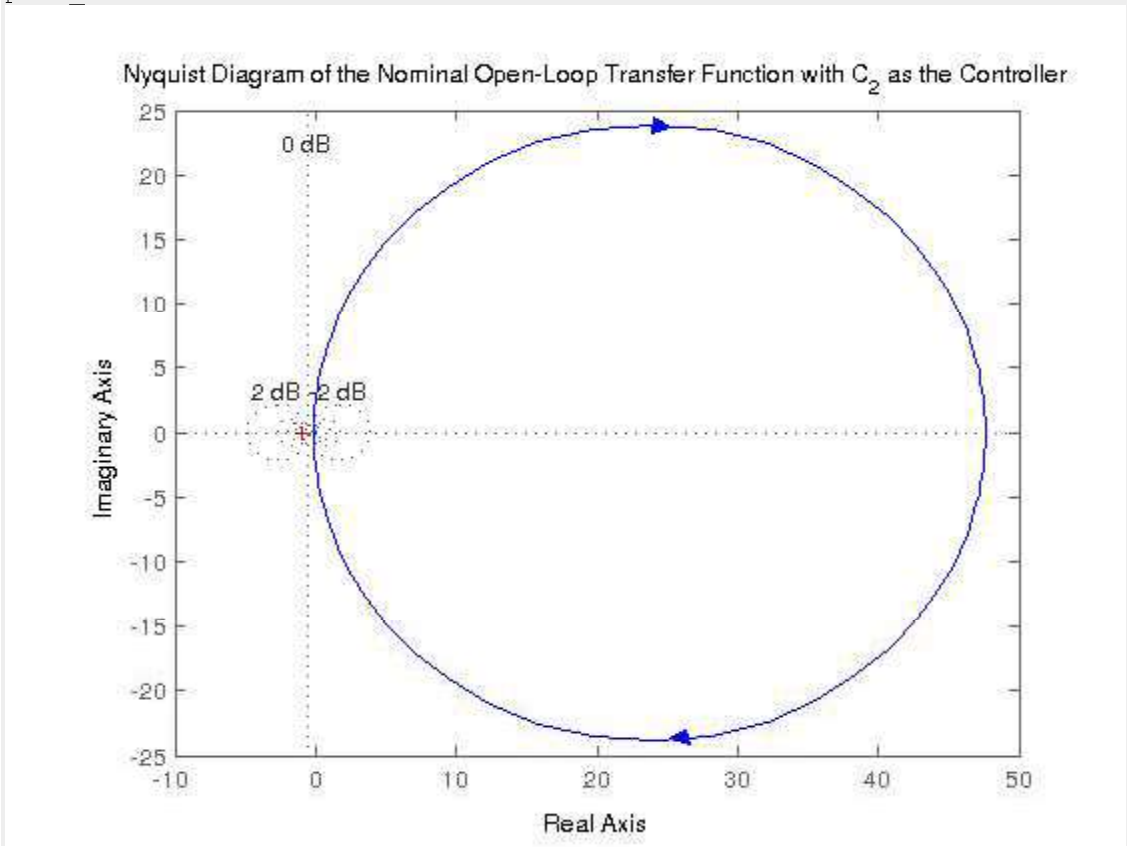

Nyquist Diagram of the Nominal Open-Loop Transfer Function with C1 as the Controller
6 dB
4 dB
2 dB
0 dB
-2 dB
-4 dB
-6 dB
10 dB
-10 dB
20 dB
-20 dB
Imaginary Axis
Real Axis
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8

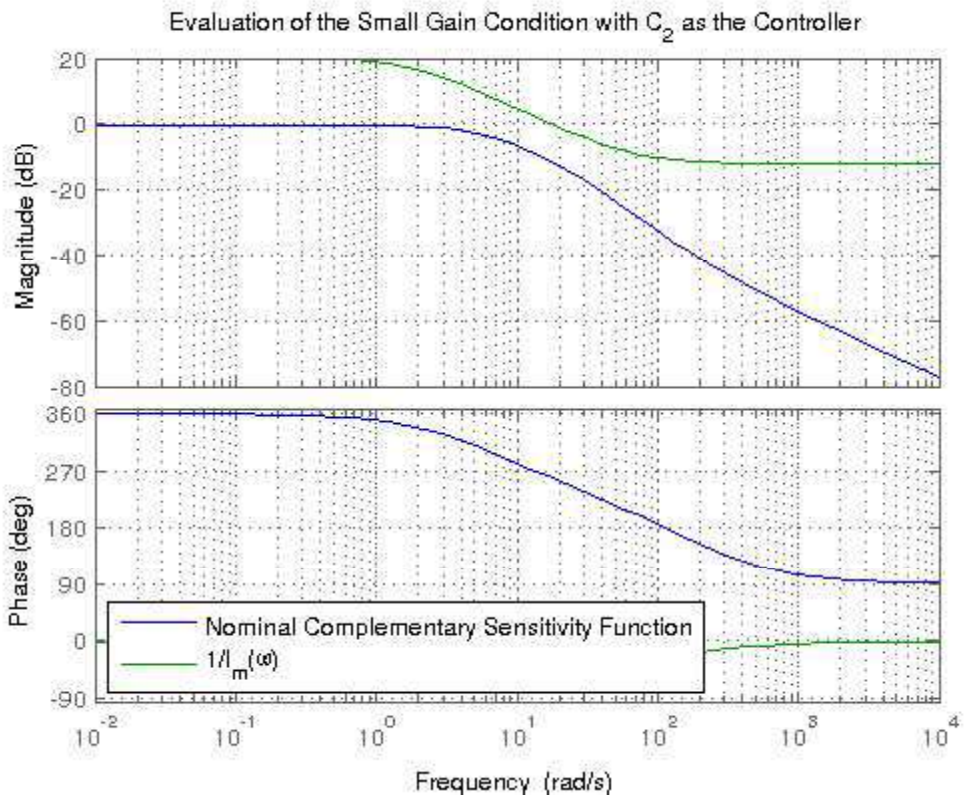

Evaluation of the Small Gain Condition with C1 as the Controller
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
Nominal Complementary Sensitivity Function
1/lm(ω)
20
10
0
-10
-20
360
270
180
90
0
-90
10-1
100
101
102
103
104

C1 stabilizes the nominal plant, but the small gain condition is not always satisfied. Not a Robust Controller.

Controller 2

```
i=2;
plot_controller
```

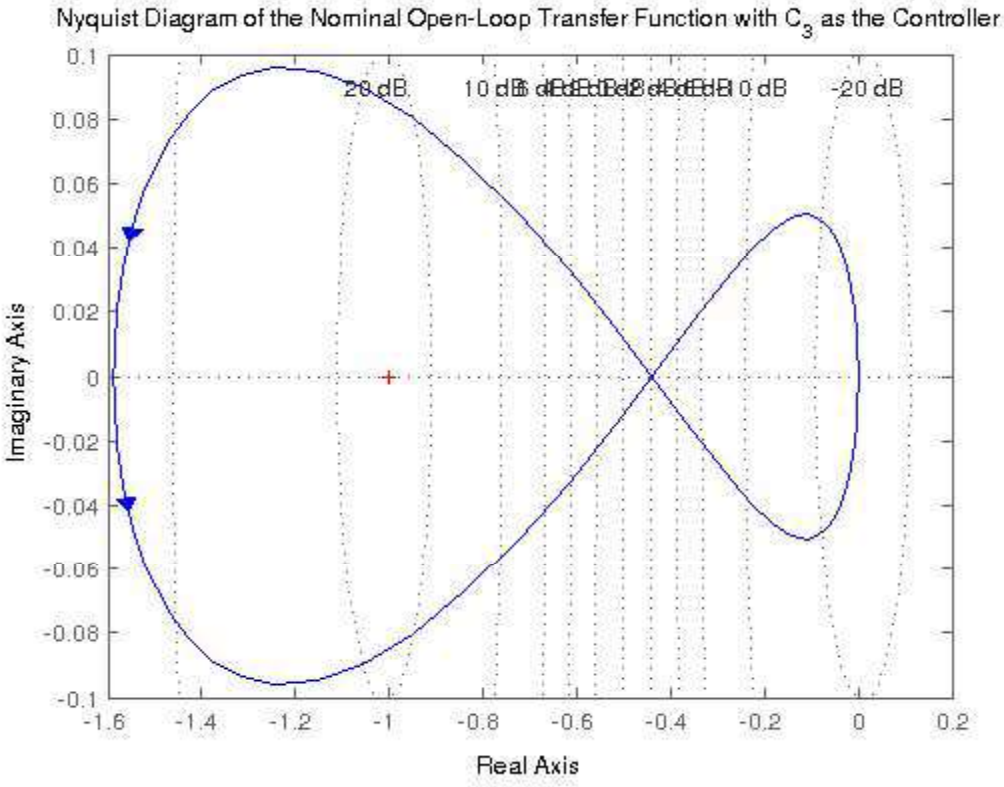


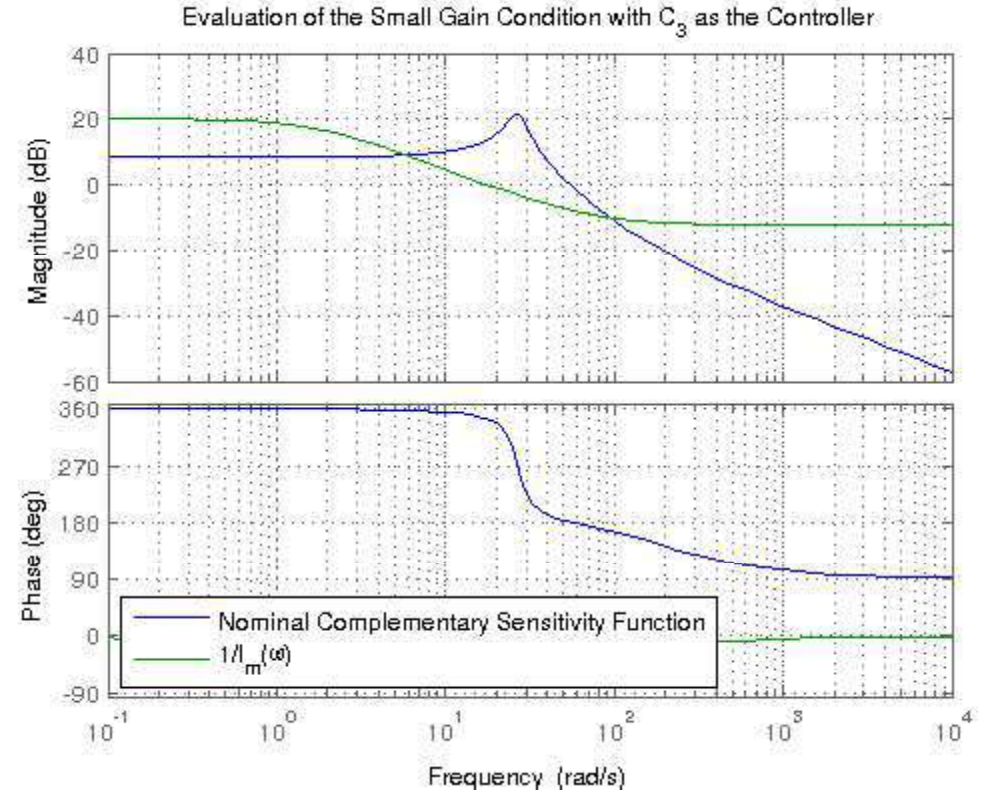


C2 stabilizes the nominal plant and the small gain condition is also satisfied. A Robust Controller.

Controller 3

```
i=3;
plot_controller
```

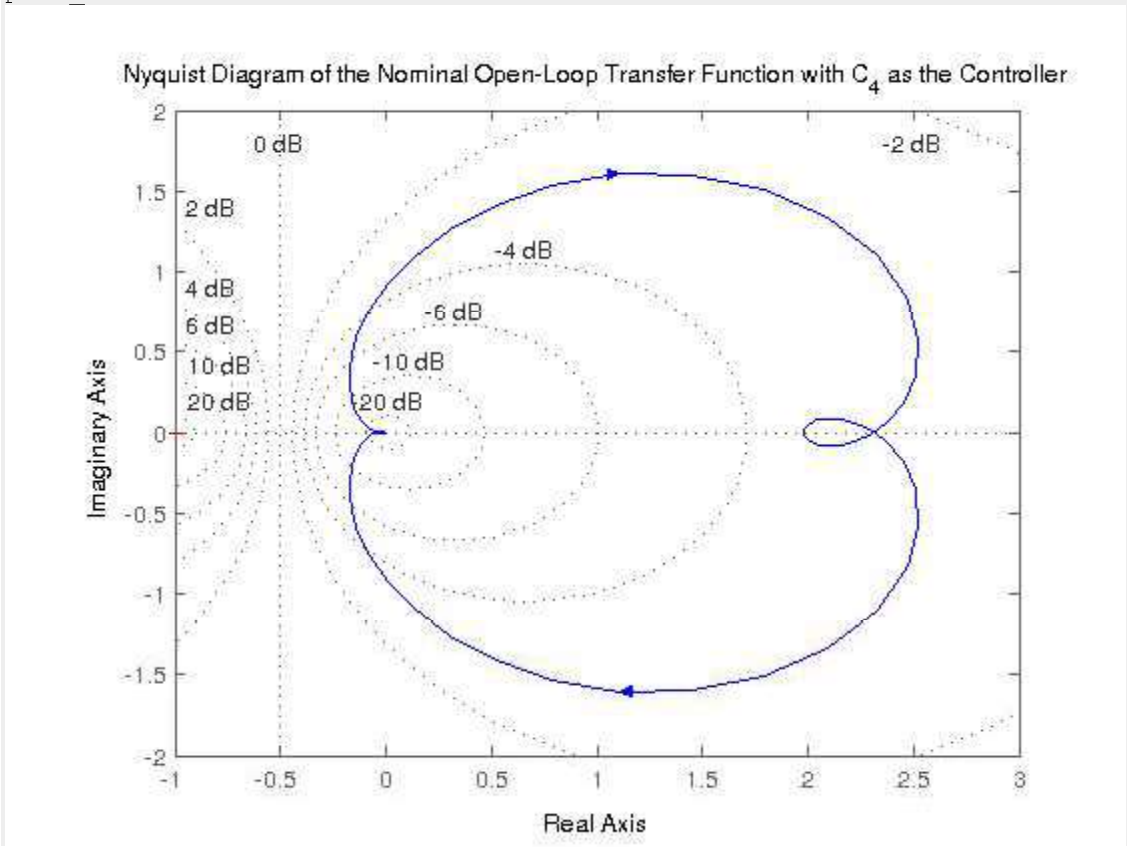

Nyquist Diagram of the Nominal Open-Loop Transfer Function with C_3 as the Controller
20 dB
10 dB
0 dB
-20 dB
Imaginary Axis
Real Axis
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-1.6
-1.4
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2

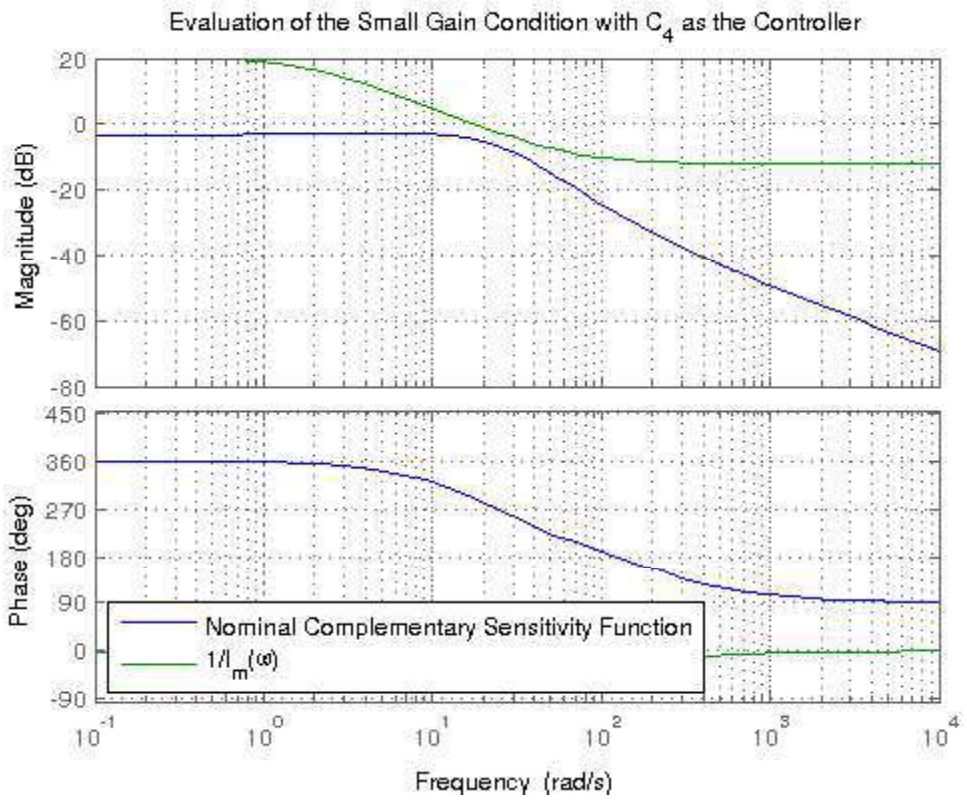

Evaluation of the Small Gain Condition with C_3 as the Controller
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
Nominal Complementary Sensitivity Function
$1/l_m(\omega)$
40
20
0
-20
-40
-60
360
270
180
90
0
-90
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}

With C3 as the controller, the small gain condition is not always satisfied. Not a Robust Controller

Controller 4

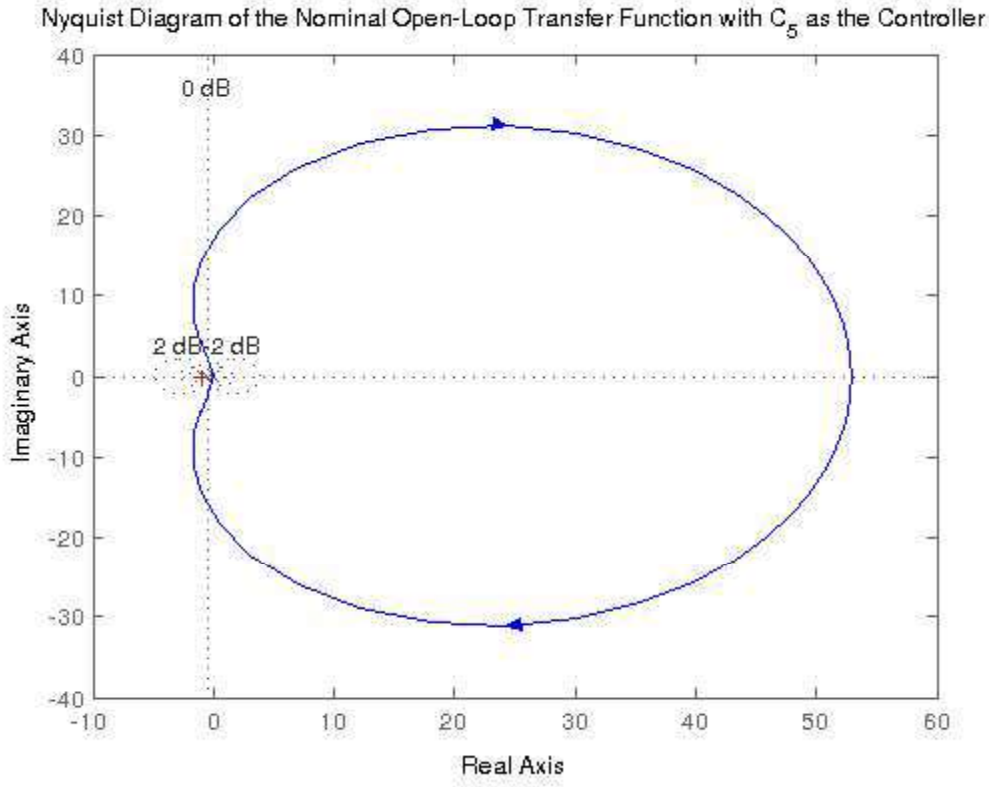
```
i=4;
plot_controller
```

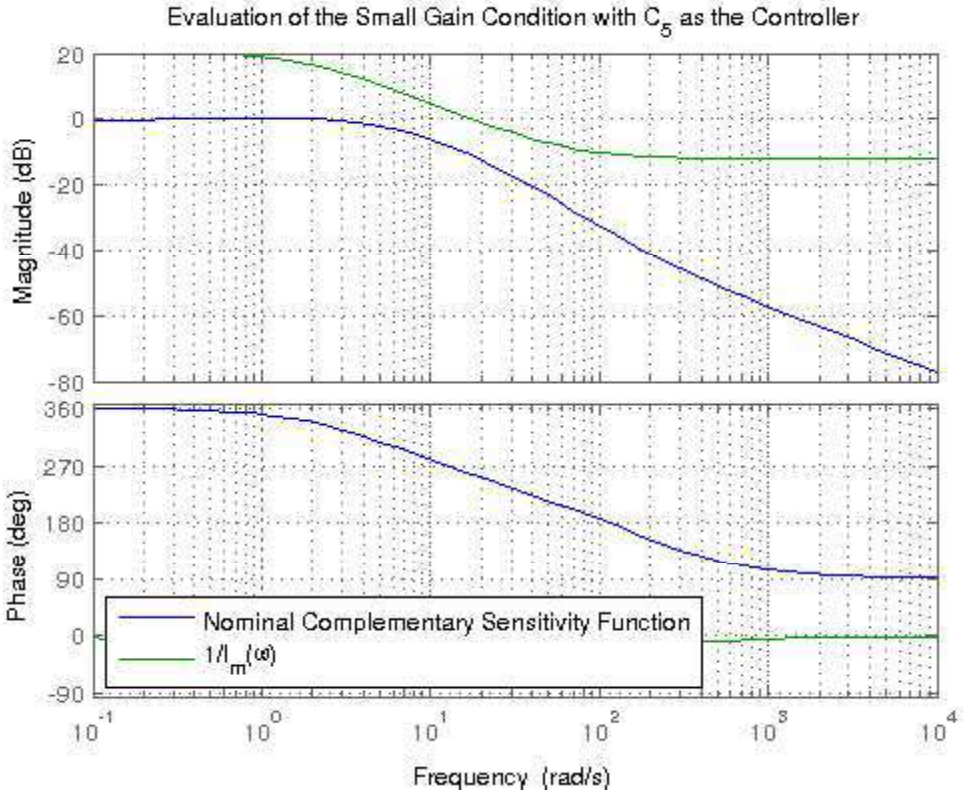




C4 stabilizes the nominal plant and the small gain condition is also satisfied. A Robust Controller.

```
i=5;
plot_controller
```


Nyquist Diagram of the Nominal Open-Loop Transfer Function with C_5 as the Controller
0 dB
2 dB
2 dB
Imaginary Axis
Real Axis
40
30
20
10
0
-10
-20
-30
-40
-10
0
10
20
30
40
50
60


Evaluation of the Small Gain Condition with C_5 as the Controller
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
Nominal Complementary Sensitivity Function
$1/l_m(\omega)$
20
0
-20
-40
-60
-80
360
270
180
90
0
-90
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}

C5 stabilizes the nominal plant and the small gain condition is also satisfied. A Robust Controller.