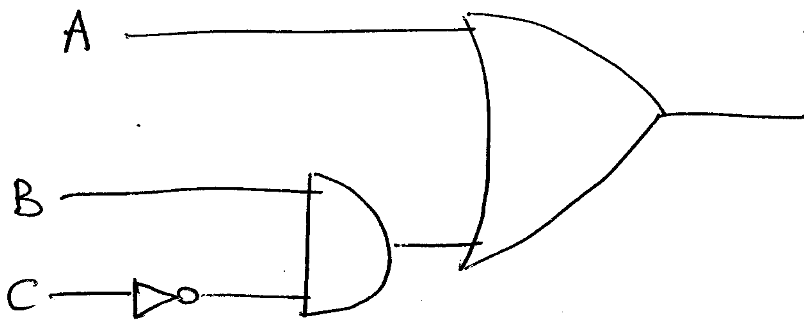
LECTURE #2 01/07/04

~~ECE 152A Lecture #2~~ [Reading: see syllabus]

- Boolean functions:

$$F = A + BC'$$

- Implementation:



schematic

- Truth table:

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	$\boxed{1}$
0	1	1	0
1	0	0	$\boxed{1}$
1	0	1	$\boxed{1}$
1	1	0	$\boxed{1}$
1	1	1	$\boxed{1}$

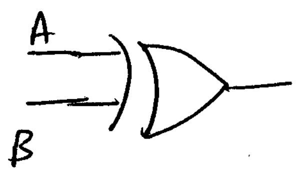
"off-terms"

"on-terms"

Shows how to go from ~~the~~ a Boolean expression to the truth table.

— Can we do the reverse?

Ex) **XOR**:



A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

$$F = \boxed{A'B + AB'}.$$

Ex2)

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

$$F = A'B'C' + A'B'C + A'BC + ABC'.$$

Review,

Associative: $A+(B+C) = (A+B)+C$

Commutative: $A\cdot B = BA$, $A+B = B+A$

Distributive: $A\cdot(B+C) = AB+AC$

$\boxed{A+BC = (A+B)(B+\ldots}$

Boolean simplification (Algebraic);

$$F = A'B'C' + A'B'C + A'BC + ABC'$$

$$= A'B'\underbrace{(C'+C)}_{1} + A'BC + ABC'$$

$$= A'B' + A'BC + ABC'$$

C \ AB	00	01	11	10
0	1	0	1	0
1	1	1	0	0

A'

$A'B' + A'C + ABC'$

$$= A'(B' + BC) + ABC'$$

(distributive law #2)

$$= A'\underbrace{(B'+B)}_{1}(B'+C) + ABC'$$

$$= \boxed{A'B' + A'C + ABC'}$$

Simplification Theorems ; (Divide board into 7 sections)

1) $XY + XY' = X(\underbrace{Y+Y'}_{1}) = \boxed{X}$

2) $X + XY = X(\underbrace{Y+1}_{1}) = \boxed{X}$

3) $(X+Y')Y = XY + \underbrace{YY'}_{0} = \boxed{XY}$

4) $(X+Y)(X+Y')$

$= XX + \underbrace{YY'}_{0} + YX + XY'$

$= X + YX + XY'$

$= X(Y+1) + XY'$

$= X + XY' = X(\underbrace{Y'+1}_{1}) = \boxed{X}$.

5) $X(X+Y)$

$= XX + XY = X + XY = X(\underbrace{Y+1}_{1}) = \boxed{X}$.

6) $XY' + Y$

$= Y + XY' = (\underbrace{Y+Y'}_{1})(Y+X) = \boxed{X+Y}$

BACK

7) <u>consensus theorem</u>

$$XY + X'Z + YZ$$

$$= XY + X'Z + (X + X')YZ$$

$$= XY + X'Z + XYZ + X'YZ$$

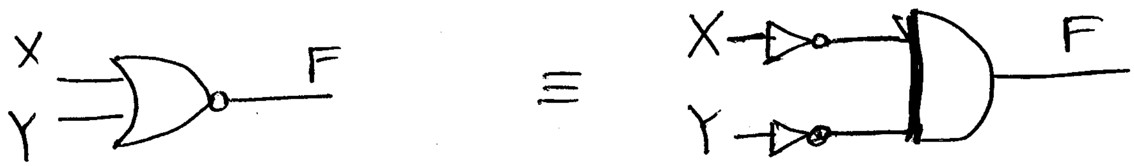
$$= XY(Z+1) + X'Z(Y+1)$$

$$= XY + X'Z$$

(Do this yourself)

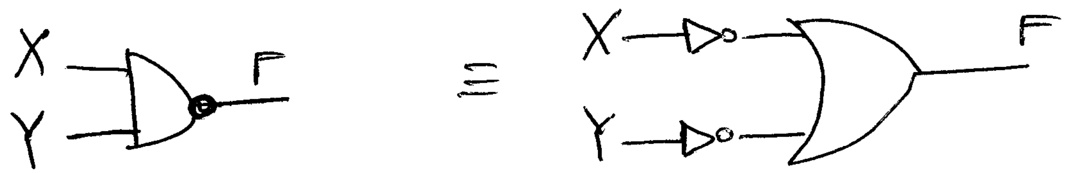
De Morgan's Laws:

① $(X+Y)' = X'Y'$.



"Bubble-pushing".

② $(X \cdot Y)' = X' + Y'$



Ex1) $(A'+B)' = (A')' \cdot B' = A \cdot B'$

Ex2) $\left(\boxed{A'+B}\boxed{C'}\right)' = (A'+B)' + (C')'$

$= (A'+B)' + C$

$= \left[(A')' \cdot B'\right] + C$

$= A \cdot B' + C$.

Ex3 $\left(\left(AB' + C\right)D' + E\right)'$

$$= \underbrace{\left(\left(AB' + C\right)D'\right)'}_{\downarrow} \cdot E'$$

$$\left(AB' + C\right)' + D$$

$$\left(AB'\right)' \cdot C'$$

$$A' + B$$

$$= \left(\left(A' + B\right)C' + D\right)E'$$ $\longrightarrow$ Bubble-pushing

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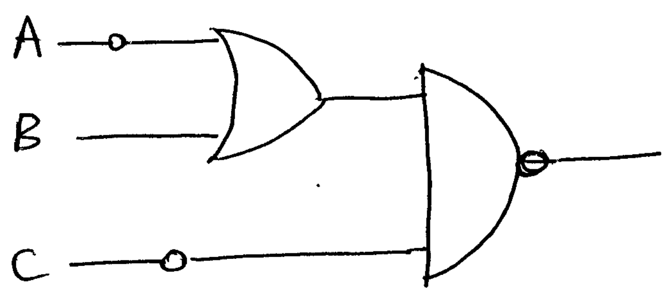
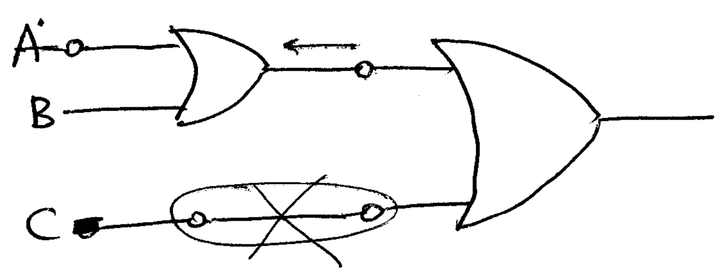
Dual of a Boolean expression

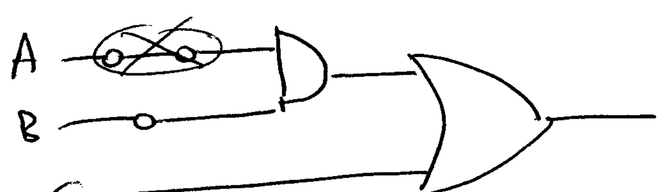
- AND $\longrightarrow$ OR
- OR $\longrightarrow$ AND
- $0 \longrightarrow 1$
- $1 \longrightarrow 0$.

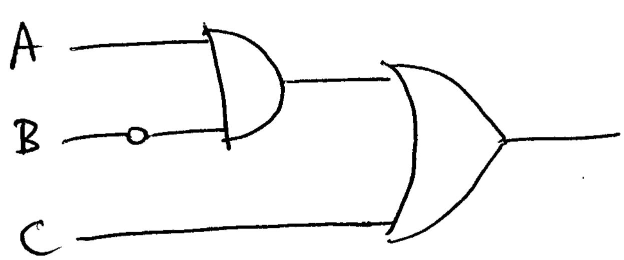
(Variables & complements are unchanged)

Ex2) $((A'+B)C')' =$









$\cancel{A+}$ $\boxed{(AB') + C}$.

Dual of a Boolean expression:

$$f^D(0, 1, A, B, C, \cdot, +, ') = f(1, 0, A, B, C, +, \cdot, ')$$

Ex) $(X+Y)^D = X \cdot Y$

$(AB' + C)^D = (A+B')\cdot C$

Ex3: Prove: $XY' + Y = X + Y$

$$XY' + Y = X+Y$$

$$\Big\downarrow \text{Dual}$$

$$(X+Y')\cdot Y = XY + \underbrace{YY'}_{0} = XY \xrightarrow{\text{Dual}} X+Y$$

1- $[(f^D)^D = f.]$

Thm: 2- $(f(x_1, \ldots, x_n))^D = f'(x_1', \ldots, x_n')$.

~~Homework: Solve all problems at the end of Ch2.~~
~~(2.1 - 2.22)~~