

7.2 From the free electron theory [eqn 6.16], $E_F = \frac{h^2}{2m^*} \left(\frac{3N}{8\pi} \right)^{2/3}$ (note that we replace the actual mass m by the effective mass m^*).

Now N = no. of electrons available for conduction per unit volume
 = density of atoms $\times$ no. of free electrons contributed by each atom

$$= \left(\frac{530}{6.94} \times 6.02 \times 10^{23} \right) \times 1 = 4.60 \times 10^{28} \text{ m}^{-3}$$

and $E_F = 4.2 \text{ eV}$

$$\text{Hence } m^* = \frac{h^2}{2E_F} \left(\frac{3N}{8\pi} \right)^{2/3} = 1.01 \times 10^{-30} \text{ kg or } \underline{1.11 m_0}$$

7.4 The group velocity $V_g = \frac{1}{h} \frac{\partial E}{\partial k} = \frac{1}{h} \frac{\partial}{\partial k} \left(\frac{h^2 \alpha^2}{2m} \right)$ (from [eqn 7.5])

$$= \frac{h\alpha}{a^2 m} \frac{\partial \alpha}{\partial k} \text{ where } \alpha = \alpha a$$

Differentiate [eqn 7.3] with respect to k

$$\therefore -a \sin ka = P \frac{y \cos y - \sin y}{y^2} \frac{dy}{dk} - \sin y \frac{dy}{dk}$$

$$\text{Hence at } k = n\pi/a, \frac{dy}{dk} = \frac{-a \sin ka}{P \frac{y \cos y - \sin y}{y^2} - \sin y} \bigg|_{k=n\pi/a} = 0$$

$\therefore$ the group velocity of the electron is zero.

7.5 In three dimension, $\left(\frac{1}{m^*}\right)_{ij} = \frac{1}{\hbar^2} \frac{\partial^2 E}{\partial k_i \partial k_j}$

Hence the equation of motion $\frac{d\mathbf{v}}{dt} = \frac{1}{m} \mathbf{F}$ should be modified to

$$\begin{bmatrix} \frac{dv_x}{dt} \\ \frac{dv_y}{dt} \\ \frac{dv_z}{dt} \end{bmatrix} = \frac{1}{\hbar^2} \begin{bmatrix} \frac{\partial^2 E}{\partial k_x^2} & \frac{\partial^2 E}{\partial k_x \partial k_y} & \frac{\partial^2 E}{\partial k_x \partial k_z} \\ \frac{\partial^2 E}{\partial k_y \partial k_x} & \frac{\partial^2 E}{\partial k_y^2} & \frac{\partial^2 E}{\partial k_y \partial k_z} \\ \frac{\partial^2 E}{\partial k_z \partial k_x} & \frac{\partial^2 E}{\partial k_z \partial k_y} & \frac{\partial^2 E}{\partial k_z^2} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$

7.7 The potential energies of the electrons in the forward- and backward-travelling wave functions are given by [eqn 7.19]:

$V_{\pm} = \pm \frac{1}{a} \int_0^a \cos 2kx V(x) dx$ where $V(x)$ is the actual potential of the lattice.

For $V(x)$ as shown in [Fig. 7.2] and with $2w = a$,

$$\begin{aligned} V_{\pm} &= \pm \frac{2}{a} \left\{ \int_0^{a/4} -\frac{V_0}{2} \cos 2kx dx + \int_{a/4}^{a/2} \frac{V_0}{2} \cos 2kx dx \right\} \\ &= \pm \frac{V_0}{2ka} \left(\sin \frac{3ka}{2} - \sin \frac{ka}{2} - \frac{1}{2} \sin 2ka \right) \end{aligned}$$

In the first energy band, $k = \pi/a$ (from [eqn 7.15]). Therefore $V_{\pm} = V_0/\pi$.

The total energies of the electrons = kinetic energy + potential energy

$$= \hbar^2 k^2 / 2m \pm V_0/\pi$$

$\therefore$ the width of the first forbidden band is $2V_0/\pi$.