ECE/ME 179D

Review Pt I, for Midterm

Lecture 10

Topics include:

1. **Terminology**: manipulator, end effector, redundancy, singularity, …
2. **Workspace**: Reachable vs Dexterous
3. **Kinematics**: Forward and Inverse problems
4. **The Jacobian matrix**: velocity kinematics
5. **Mechanical impedance**: Definition of; inertia; reflected inertia
6. **Work balance**: gear ratios; motor coefficients
7. **Motor dynamics model**: standard equations and block diagram
8. **Block diagrams**: understanding, creating and solving algebraically
9. **First- and second-order time response**: estimating parameters (Lab 2)
10. **Friction vs damping**: effect of time response; estimating parameters
11. **SISO control**: P, PD, PID; PID tuning methods; feedforward
12. **Control law partitioning**: "cancel out" all dynamics except mass (or inertia)
13. **Wheeled vehicles**: mobility, steerability, maneuv.; wheel types; ICR; …
14. **Holonomic vs nonholonomic**: instantaneous DOFs vs long-term GC's
15. **Rotation matrices**: body frame vs inertial reference frame velocities

Notices:

- HW 3 (practice quiz) due FRIDAY (5pm).
- Midterm date: Tuesday, Nov. 6 (in class).
- No lab next week; No assignments due next week.

1. Terminology

- **Manipulator**: a multi-link arm
- **End Effector** (ee): The point(s) of the robot designed to interact w/ environment; e.g., the tool tip
- **Configuration**: specification of all points, geometrically, on a robot
- **Configuration space**: set of all possible configs
- **Kinematics**: geometric relationships, e.g., between joints and end effector
- **Degree of Freedom** (DOF): independent rotational or translational direction of motion that is presently possible
- **Revolute Joint**: rotation DOF
- **Prismatic Joint**: translational DOF
- **Pitch** (θ_y), **roll** (ϕ_x), **yaw** (ψ_z): rotation about y, x, and z axes, respectively
- **Joint variables, q**: DOF for joints
- **Joint space**: set of all possible joint variable combinations
- **State space**: geometry AND dynamics; i.e., positions and velocities possible
- **Workspace**: Points reachable by ee, i.e., "reachable workspace" (next slide)
- **Kinematic redundancy**: when multiple joint solutions exist to put an end effector at a particular position and orientation
- **Accuracy**: How close to a desired target? (mean of the error wrt some goal)
- **Repeatability**: How close to other trials? (variance in error, trial after trial)

2. Workspace

- The **workspace** is the set of all points ("volume" or "area") reachable by the end effector. When used alone, "workspace" generally means "reachable workspace".
- **Reachable workspace**: see above
- **Dexterous workspace**: Reach at an arbitrary orientation. For a planar mechanism (in the x-y place), this means at any angle ψ_z . In the more general 3D robot case, orientation includes all 3 rotational DOF, θ_y , ϕ_x , ψ_z .

Planar case:

- To achieve an arbitrary combination of (x, y, ψ_z) [**3 DOF** of end effector] requires at least how many joint DOFs?

3D case:

- To achieve an arbitrary combination of **6 DOF** (3 positions, 3 rotations) for an end effector requires at least how many joint DOFs?

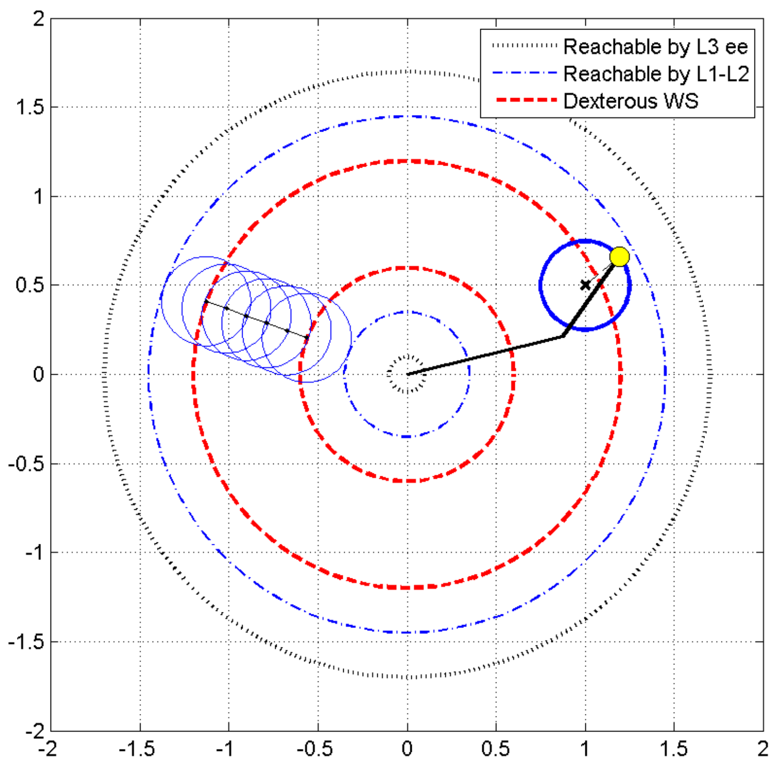
2. Workspace

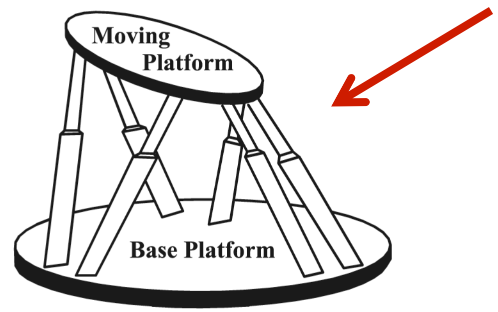
Planar case:

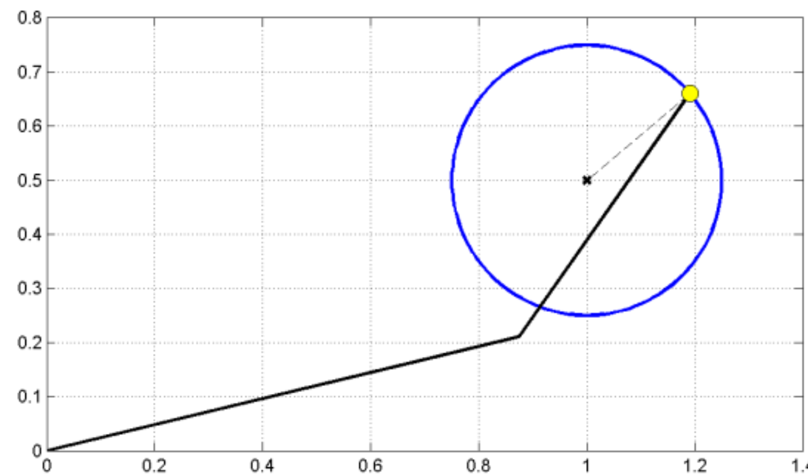
- To achieve an arbitrary combination of (x, y, ψ_z) [**3 DOF** of end effector] requires at least how many joint DOFs? **3 (e.g., 3-link arm)**

3D case:

- To achieve an arbitrary combination of **6 DOF** (3 positions, 3 rotations) for an end effector requires at least how many joint DOFs? **6** (e.g., Stewart platform)







3. Kinematics

- Forward kinematics: Given a set of joint positions, q, there exists one solution for the end effector
- Inverse kinematics (IK): Given a desired end effector position, there may be multiple solutions (redundancy) or there may be no solution (infeasible).

4. The Jacobian matrix

- Velocity kinematics
- Matrix equations that relate velocities of the joints, dq/dt, to velocities at the end effector: $\xi = J(q)\dot{q}$ via Spong convention, or as: $\dot{\xi} = J(q)\dot{q}$

  …because most books define “xi” as position, rather than as velocity.
- Can solve for analytically, differentiating relationship for position (kinematics).
- But can ALSO often solve via geometry… especially for planar geometry.

5. Mechanical Impedance

- Effort: Force (analogous to voltage, usually the “source”)
- Flow: Velocity, dx/dt (analogous to current)
- Capacitance (Spring): Ze = 1/(Cs), Zm = k/s (or K/s, rotational system)
- Resistance (Damping): Ze=R, Zm=b (or B)
- Inductantce (Mass): Ze=Ls, Zm=ms (or Js)
- Understand “reflected inertia”

6. Work Balance

- Work is the integral of “F*dx” (or “Tau*dtheta”). Energy and work both have units of Joules.
- Rate of work is POWER (J/s):
  - Force times velocity
  - Voltage times current
- Gear ratios and electro-mechanical coefficients (e.g., motor torque constant) describe a work balance

7. Motor Dynamics Model

- Motor constant
- Resistance (and inductance)
- Effect of back emf (adds damping)

Block diagram(s):

8. Block Diagrams

- Be able to write equations for block diagrams
- Be able to write block diagrams from equations
- Be able to REDUCE a block diagram, to derive a desired input-output relationship. e.g., from x_desired to x_actual, or from a disturbance input, “D”, to the error, “e=x_des-x_act”.
- Know that gear ratios and transformation coefficients (such as the motor torque and back emf coefficient) typically appear TWICE in a loop. i.e., same N (not 1/N), and $K_{tau} = K_{back\text{-}emf}$
- Know whether an impedance or “inverse impedance” goes in an empty block.

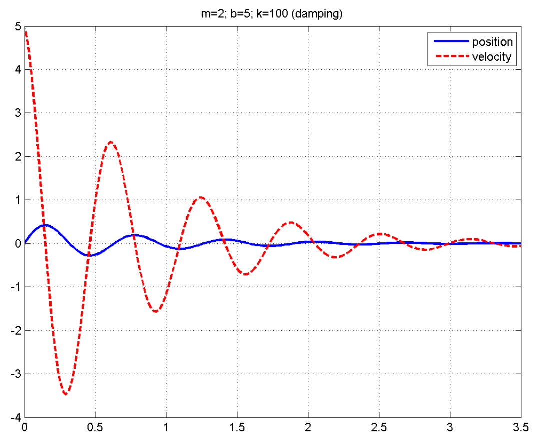
9. 1st- and 2nd-order responses

$$\frac{K}{Ts+1}$$

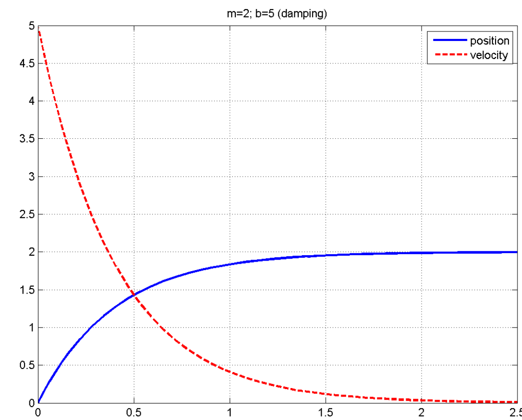
$$\frac{A\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

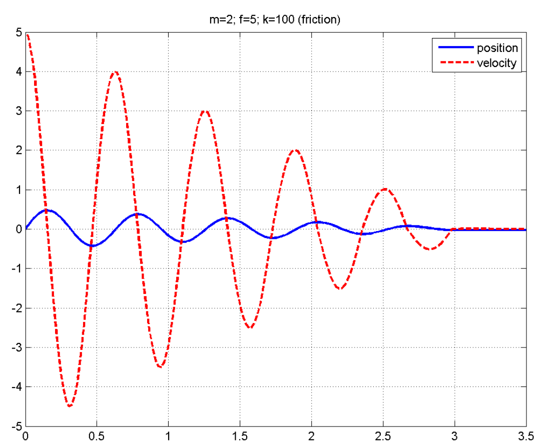
10. Friction vs Damping

- Nonlinear Coulomb friction: F = -f*sign(dx/dt)
- Linear viscous damping: F = -b*dx/dt
- Friction non-linearity can result in steady state errors and make it difficult to predict the final value of a time response.

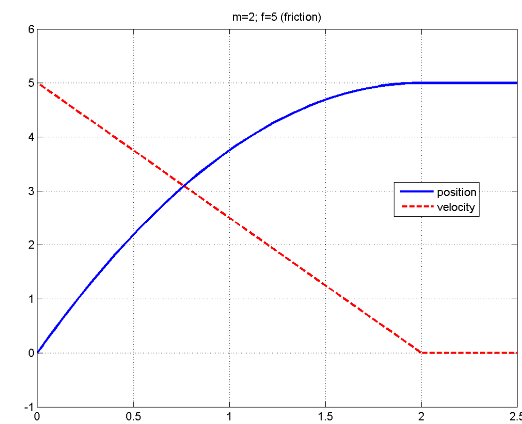


Damping:
Exponential decay





Friction:
Linear decay



11. Single-input single-output (SISO) control

- The poles are roots of the denominator of the closed-loop transfer fn (TF).
- Proportional (P): A gain time difference in position. Acts like “stiffness”.
- Proportional-derivative (PD): Kp times error in position (“stiffness”), and Kd times negative velocity (or times error in velocity) (“damping”). The “Kd*s” term may appear in either the forward or feedback paths: this affects numerator (zeros) of transfer function, but not denominator (poles). PD is similar in function to a LEAD controller, adding phase to the open-loop TF (transfer fn) at crossover frequency.
- Proportional-integral-derivative (PID): Compared with PD, PID now also includes a gain times the integral of error. The integrator increases gain of the open-loop TF at low frequencies, and it is typically used with the purpose of eliminating steady-state error.
- Proportional-integral (PI): Similar to PID, but no derivative term. Recall the integral term can be useful in eliminating the effects of a disturbance, such as static friction or the weight of a mass being supported under gravity.

12. Control law partitioning (SISO)

- Divide control law into two parts:

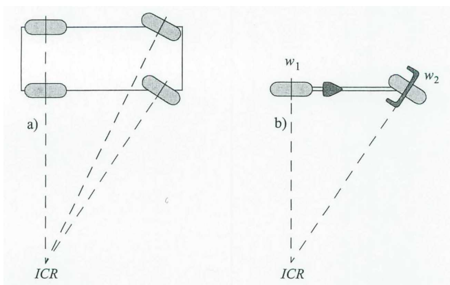
1. First, feedback a force (or torque) that exactly cancels all terms EXCEPT mass time acceration:
2. Then, use simple PD control to set damping ratio (zeta) and natural frequency (omega_n) of the desired, 2^{nd}-order closed loop response

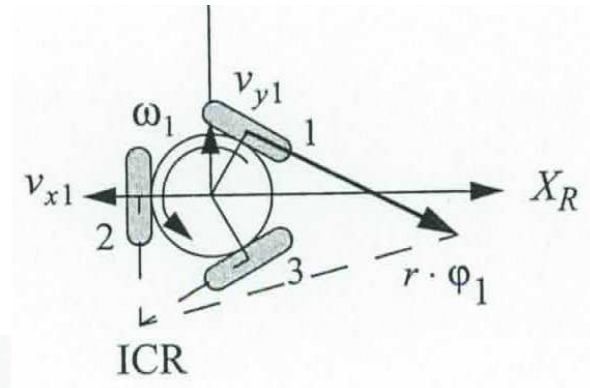
- Note you can "cancel out" both linear terms AND nonlinear terms (such as gravity or Coulomb friction)

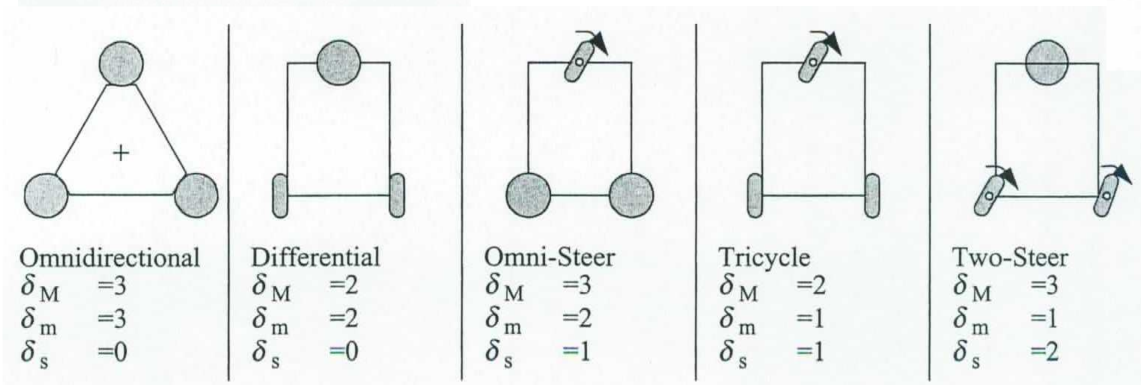
Example:

13. Wheeled Vehicles

- At most 3 DOF for the CHASSIS: x, y, and phi
- **Mobility**: DDOF via wheel velocities, without steering (no reorienting wheels)
- **Steerability**: DOF for ICR, by steering, only. (At most equal to 2.)
- **Maneuverability**: at most 3. Sum of the other two. (See eqn at bottom.)
- **Instantaneous center of rotation** (ICR): What is allowed, given all no-slip constraints.
  - Note for the **OMNIBOT**, calculating ICR can be tricky. Remember the direction of the free rollers are at an angle (90 deg??) to the main (driven) axis!







$$\delta_M = \delta_m + \delta_s$$

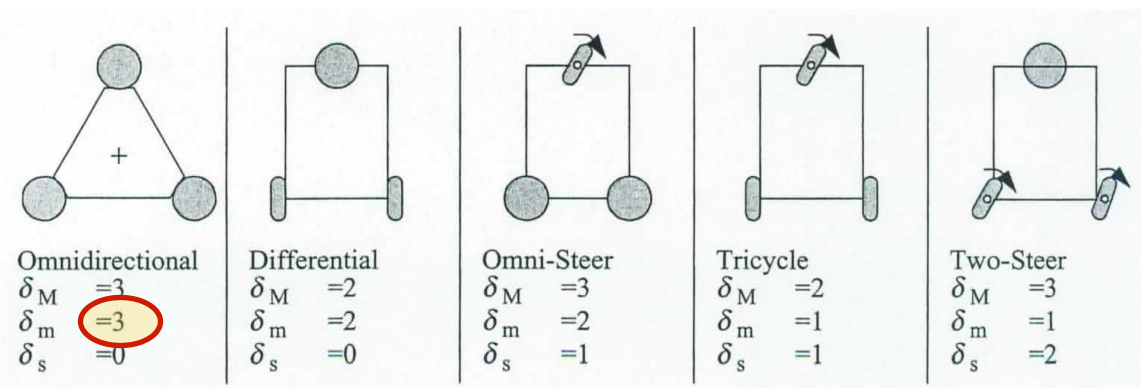
14. Holonomic vs Nonholonomic

- Generalized variables
  - Long-term coordinates needed
- Admissible variations
  - Immediate DDOF allowed
- Independent and complete: know definitions

$$DDOF = \delta_m \le \delta_M \le DOF$$

$$GC = \xi_j > \delta\xi_k = DOF \longleftrightarrow \text{Nonholonomic}$$

$$GC = \xi_j = \delta\xi_k = DOF \longleftrightarrow \text{Holonomic}$$



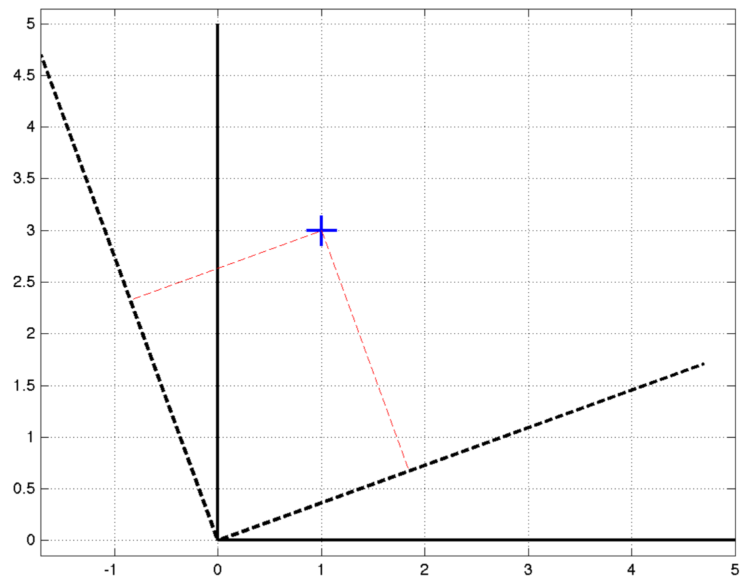
$$\delta_M = \delta_m + \delta_s$$

15. Rotation matrices

- Rotation matrices are quite important in kinematics.
- However, we touch on only the basics in this class (since our focus in dynamics and control).
- You should understand how to transform velocity in one coordinate frame (e.g., wrt omnibot body frame) to velocity in the GLOBAL frame

- For rotation in the x-y plane, $R^T = R^{-1}$.

- $P_{xyz} = [1, 3, 5]^T$
- R = [cos(theta), sin(theta), 0
  -sin(theta), cos(theta), 0
  0 , 0 , , 1]



$P_{rot} = R*P_{xyz}$

$P_{xyz} = (R^{-1})*P_{rot} = (R^T)*P_{rot}$

Here, theta = 20 deg = 0.3491,

$P_{rot} = [1.97, 2.48, 5]^T$