

Topics include:

1. **Terminology:** manipulator, end effector, redundancy, singularity, ...
2. **Workspace:** Reachable vs Dexterous
3. **Kinematics:** Forward and Inverse problems
4. **The Jacobian matrix:** velocity kinematics
5. **Mechanical impedance:** Definition of; inertia; reflected inertia
6. **Work balance:** gear ratios; motor coefficients
7. **Motor dynamics model:** standard equations and block diagram
8. **Block diagrams:** understanding, creating and solving algebraically
9. **First- and second-order time response:** estimating parameters (Lab 2)
10. **Friction vs damping:** effect of time response; estimating parameters
11. **SISO control:** P, PD, PID; PID tuning methods; feedforward
12. **Control law partitioning:** “cancel out” all dynamics except mass (or inertia)
13. **Wheeled vehicles:** mobility, steerability, maneuver; wheel types; ICR; ...
14. **Holonomic vs nonholonomic:** instantaneous DOFs vs long-term GC's
15. **Rotation matrices:** body frame vs inertial reference frame velocities

Notices:

- HW 3 (practice quiz) due FRIDAY (5pm).
- Midterm date: Tuesday, Nov. 6 (in class).
- No lab next week; No assignments due next week.

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1. Terminology

- **Manipulator:** a multi-link arm
- **End Effector (ee):** The point(s) of the robot designed to interact w/ environment; e.g., the tool tip
- **Configuration:** specification of all points, geometrically, on a robot
- **Configuration space:** set of all possible configs
- **Kinematics:** geometric relationships, e.g., between joints and end effector
- **Degree of Freedom (DOF):** independent rotational or translational direction of motion that is presently possible
- **Revolute Joint:** rotation DOF
- **Prismatic Joint:** translational DOF
- **Pitch (θ_y), roll (ϕ_x), yaw (ψ_z):** rotation about y, x, and z axes, respectively
- **Joint variables, q :** DOF for joints
- **Joint space:** set of all possible joint variable combinations
- **State space:** geometry AND dynamics; i.e., positions and velocities possible
- **Workspace:** Points reachable by ee, i.e., “reachable workspace” (next slide)
- **Kinematic redundancy:** when multiple joint solutions exist to put an end effector at a particular position and orientation
- **Accuracy:** How close to a desired target? (mean of the error wrt some goal)
- **Repeatability:** How close to other trials? (variance in error, trial after trial)

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2. Workspace

- The **workspace** is the set of all points ("volume" or "area") reachable by the end effector. When used alone, "workspace" generally means "reachable workspace".
- Reachable workspace**: see above
- Dexterous workspace**: Reach at an arbitrary orientation. For a planar mechanism (in the x-y plane), this means at any angle ψ_z . In the more general 3D robot case, orientation includes all 3 rotational DOF, θ_y , ϕ_x , ψ_z .

Planar case:

- To achieve an arbitrary combination of (x, y, ψ_z) [3 DOF of end effector] requires at least how many joint DOFs?

3D case:

- To achieve an arbitrary combination of **6 DOF** (3 positions, 3 rotations) for an end effector requires at least how many joint DOFs?

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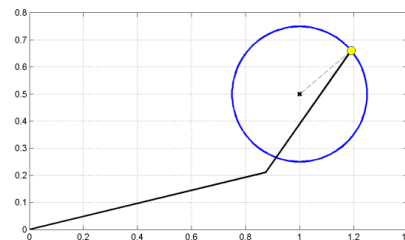
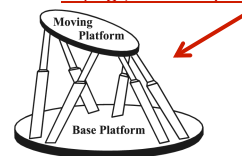
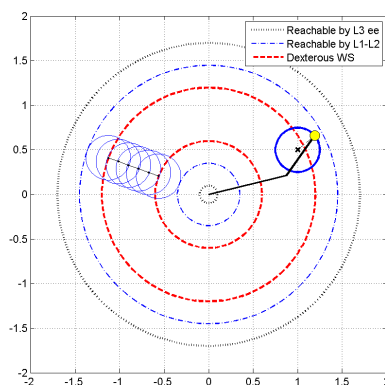
2. Workspace

Planar case:

- To achieve an arbitrary combination of (x, y, ψ_z) [3 DOF of end effector] requires at least how many joint DOFs? **3 (e.g., 3-link arm)**

3D case:

- To achieve an arbitrary combination of **6 DOF** (3 positions, 3 rotations) for an end effector requires at least how many joint DOFs? **6 (e.g., Stewart platform)**



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3. Kinematics

- Forward kinematics: Given a set of joint positions, q , there exists one solution for the end effector
- Inverse kinematics (IK): Given a desired end effector position, there may be multiple solutions (redundancy) or there may be no solution (infeasible).

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4. The Jacobian matrix

- Velocity kinematics
- Matrix equations that relate velocities of the joints, dq/dt , to velocities at the end effector: $\dot{\xi} = J(q)\dot{q}$ via Spong convention, or as: $\dot{\xi} = J(q)\dot{q}$
...because most books define “xi” as position, rather than as velocity.
- Can solve for analytically, differentiating relationship for position (kinematics).
- But can ALSO often solve via geometry... especially for planar geometry.

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5. Mechanical Impedance

- Effort: Force (analogous to voltage, usually the “source”)
- Flow: Velocity, dx/dt (analogous to current)
- Capacitance (Spring): $Z_e = 1/(Cs)$, $Z_m = k/s$ (or K/s , rotational system)
- Resistance (Damping): $Z_e=R$, $Z_m=b$ (or B)
- Inductance (Mass): $Z_e=Js$, $Z_m=ms$ (or Js)
- Understand “reflected inertia”

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6. Work Balance

- Work is the integral of “ $F \cdot dx$ ” (or “ $\tau \cdot d\theta$ ”). Energy and work both have units of Joules.
- Rate of work is POWER (J/s):
 - Force times velocity
 - Voltage times current
- Gear ratios and electro-mechanical coefficients (e.g., motor torque constant) describe a work balance

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7. Motor Dynamics Model

- Motor constant
- Resistance (and inductance)
- Effect of back emf (adds damping)

Block diagram(s):

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8. Block Diagrams

- Be able to write equations for block diagrams
- Be able to write block diagrams from equations
- Be able to REDUCE a block diagram, to derive a desired input-output relationship. e.g., from x_{desired} to x_{actual} , or from a disturbance input, "D", to the error, " $e = x_{\text{des}} - x_{\text{act}}$ ".
- Know that gear ratios and transformation coefficients (such as the motor torque and back emf coefficient) typically appear TWICE in a loop. i.e., same N (not $1/N$), and $K_{\text{tau}} = K_{\text{back-emf}}$
- Know whether an impedance or "inverse impedance" goes in an empty block.

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9. 1st- and 2nd-order responses

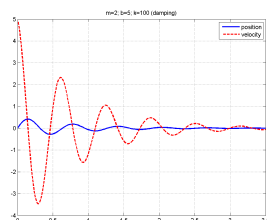
$$\frac{K}{Ts + 1}$$

$$\frac{A\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

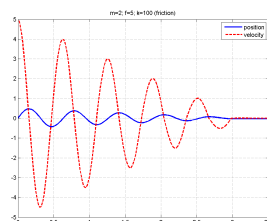
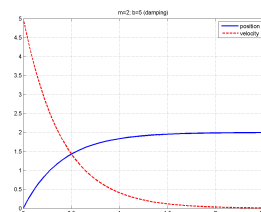
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10. Friction vs Damping

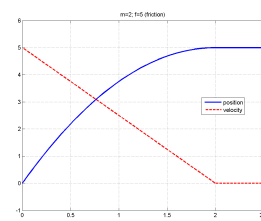
- Nonlinear Coulomb friction: $F = -f \cdot \text{sign}(dx/dt)$
- Linear viscous damping: $F = -b \cdot dx/dt$
- Friction non-linearity can result in steady state errors and make it difficult to predict the final value of a time response.



Damping:
Exponential decay



Friction:
Linear decay



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11. Single-input single-output (SISO) control

- The poles are roots of the denominator of the closed-loop transfer fn (TF).
- Proportional (P): A gain time difference in position. Acts like “stiffness”.
- Proportional-derivative (PD): K_p times error in position (“stiffness”), and K_d times negative velocity (or times error in velocity) (“damping”). The “ K_d*s ” term may appear in either the forward or feedback paths: this affects numerator (zeros) of transfer function, but not denominator (poles). PD is similar in function to a LEAD controller, adding phase to the open-loop TF (transfer fn) at crossover frequency.
- Proportional-integral-derivative (PID): Compared with PD, PID now also includes a gain times the integral of error. The integrator increases gain of the open-loop TF at low frequencies, and it is typically used with the purpose of eliminating steady-state error.
- Proportional-integral (PI): Similar to PID, but no derivative term. Recall the integral term can be useful in eliminating the effects of a disturbance, such as static friction or the weight of a mass being supported under gravity.

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12. Control law partitioning (SISO)

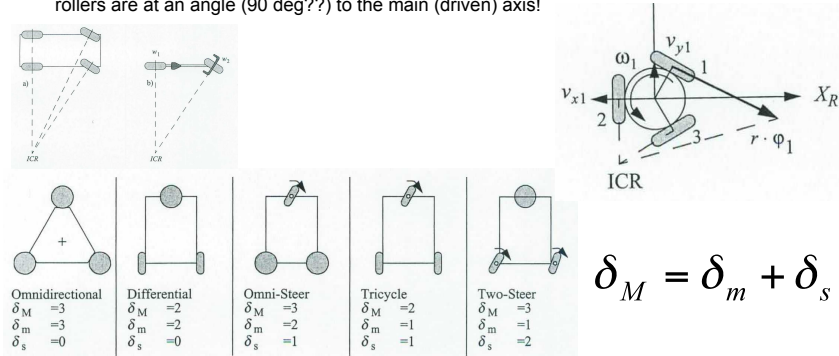
- Divide control law into two parts:
 1. First, feedback a force (or torque) that exactly cancels all terms EXCEPT mass time acceleration:
 2. Then, use simple PD control to set damping ratio (ζ) and natural frequency (ω_n) of the desired, 2nd-order closed loop response
- Note you can “cancel out” both linear terms AND nonlinear terms (such as gravity or Coulomb friction)

Example:

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13. Wheeled Vehicles

- At most 3 DOF for the CHASSIS: x, y, and phi
- **Mobility**: DDOF via wheel velocities, without steering (no reorienting wheels)
- **Steerability**: DOF for ICR, by steering, only. (At most equal to 2.)
- **Maneuverability**: at most 3. Sum of the other two. (See eqn at bottom.)
- **Instantaneous center of rotation** (ICR): What is allowed, given all no-slip constraints.
 - Note for the **OMNIBOT**, calculating ICR can be tricky. Remember the direction of the free rollers are at an angle (90 deg??) to the main (driven) axis!



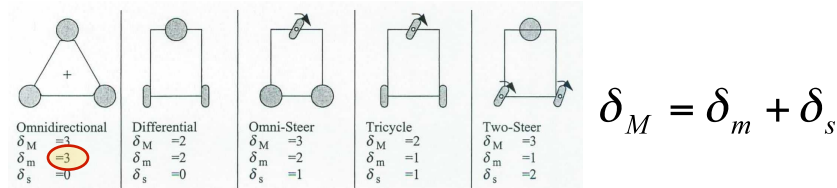
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14. Holonomic vs Nonholonomic

- Generalized variables
 - Long-term coordinates needed
- Admissible variations
 - Immediate DDOF allowed
- Independent and complete: know definitions

$$GC = \xi_j > \delta \xi_k = DOF \iff \text{Nonholonomic}$$

$$GC = \xi_j = \delta \xi_k = DOF \iff \text{Holonomic}$$



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15. Rotation matrices

- Rotation matrices are quite important in kinematics.
- However, we touch on only the basics in this class (since our focus is in dynamics and control).
- You should understand how to transform velocity in one coordinate frame (e.g., wrt omnibot body frame) to velocity in the GLOBAL frame
- For rotation in the x-y plane, $R^T = R^{-1}$.

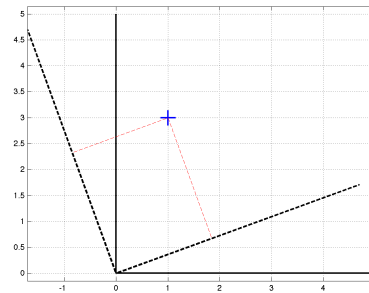
- $P_{xyz} = [1, 3, 5]^T$
- $R = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$P_{rot} = R * P_{xyz}$$

$$P_{xyz} = (R^{-1}) * P_{rot} = (R^T) * P_{rot}$$

Here, $\theta = 20 \text{ deg} = 0.3491$,

$$P_{rot} = [1.97, 2.48, 5]^T$$



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