Lagrangian EOM – Part 1

Lecture 11

1. Review Pt II, for Midterm (Q&A)

2. Derivation of Lagrangian equations of motion (EOMs):

- **Review of holonomic / nonholonomic definitions**
- **The "Lagrangian":** $\mathcal{L} = T^* - V$
  - **Kinetic co-energy**
  - **Potential energy**
- **Lagrange's equations**

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = \Xi_i$$

Notices:

- Midterm date: Tuesday, Nov. 6 (in class)
- Office hours Monday in 3120A (lab), 3:30-4:30pm
- One-sided single sheet of notes (8.5 x 11 inches) for midterm
- HW 3 due Friday at 5pm; Solutions on web soon after! (e.g., 6pm…)

Review for Midterm

Lecture 11

Topics for particular focus:

1. **Terminology**: manipulator, end effector, redundancy, singularity, ...
2. **Workspace**: Reachable vs Dexterous
3. **Kinematics**: Forward and
4. **The Jacobian matrix**: velocity kinematics
5. **Mechanical impedance**: Definition of; inertia; reflected inertia
6. **Work balance**: gear ratios; motor coefficients
7. **Motor dynamics model**: standard equations and block diagram
8. **Block diagrams**: understanding, creating and solving algebraically
9. **First- and second-order time response**
10. **Friction vs damping**: effect o … timating parameters
11. **SISO control**: P, PD, PID; PID … edforward
12. **Control law partitioning**: "cancel out" all dynamics except mass (or inertia)
13. **Wheeled vehicles**: mobility, steerability, maneuv.; wheel types; ICR; ...
14. **Holonomic vs nonholo** … C's
15. **Rotation matrices**: body frame vs inertial reference frame velocities

$\boxed{\dot{\xi} = J(q)\dot{q}}$

Geometric calculation of J…

$\boxed{N^2}$ $\boxed{\frac{1}{N^2}}$

$\boxed{s^2 + 2\zeta\omega_n s + \omega_n^2}$

Geometric calculation: of J…, and of ICR…

Notices:

- HW 3 (practice quiz) due: Tomorrow (Nov. 2)
- Midterm date: Tuesday, Nov. 6 (in class)

Holonomic / Nonholomic

- **Nonholonomic** when:
  - Accessible configuration space has higher dimension than the accessible velocity space.

i.e.,

  - **More generalized variables** required to describe the long-term configurations that are achievable…
  - **…than Differential Degrees of Freedom** (DDOFs) for local motion, instantaneously

$$\# GC = \xi_j > \delta\xi_k = \# DOF \longleftrightarrow \text{Nonholonomic}$$

$$\# GC = \xi_j = \delta\xi_k = \# DOF \longleftrightarrow \text{Holonomic}$$

Wheeled Robots

Common terminology differs from pure definition:

1. "kinematic posture": x, y, φ for body.
   - If we produce appropriate control laws, we can create an "apparent" holonomic system
     - The internal variables (wheel rotations) have not gone away
     - BUT, they are effectively "hidden" within a set of feedback laws…
   - 3 GC's (longterm);
   - 3 "DDOF" (instantaneous DOF, which we will just call "DOF")

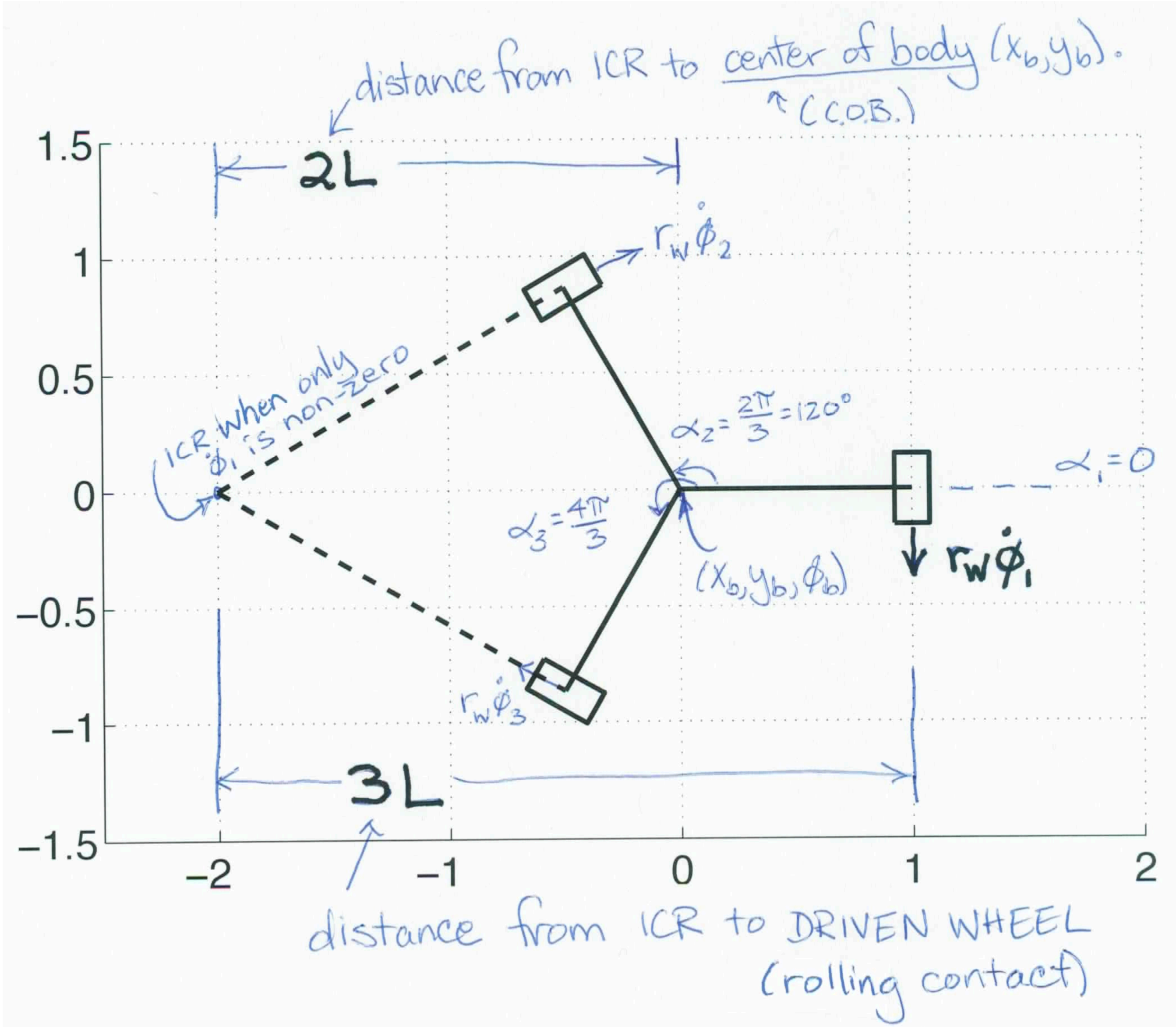
If we care about actual wheel positions, then:

2. "configuration posture": x, y, φ, q_1, q_2, q_3 (6 GC's)
   - Omnibot from lab has an integrable constraint:

$$\phi_b = \frac{-r_w}{3L}\left(q_1 + q_2 + q_3\right)$$

   - 5 GC's (longterm), since φ is a known function (above) of q1, q2, and q3
   - 3 DOF instantaneous (dq1/dt, dq2/dt, dq3/dt)

Omnibot: Jacobian (review at board)



Omnibot: Jacobian (review at board)

1. Imagine moving only one wheel, with the other two fixed: $\dot{\phi}_2 = \dot{\phi}_3 = 0$
   $\dot{\phi}_1 = 1$
2. Calculate ICR location.
3. Calculate $\dot{\phi}_b$.
4. Given x and y distance between ICR and Center of Body (COB) of robot, calculate $\dot{x}_b$ and $\dot{y}_b$, each as a function of $\dot{\phi}_b$:
5. Generalize for all $\dot{\phi}_i = \dot{q}_i$, and build the Jacobian:

Block Diagrams

- Feedforward:

- Gear Ratios (and other transformations):

Lagrangian Approach

1. Define generalized coordinates that are:
   - Complete
   - Independent

   ξ_j (j = 1, 2, 3, …)

   (*For now, we will also assume system is holonomic*…)

2. Force-dynamics relationship requires:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i}\right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \boldsymbol{\Xi}_i$$

   where:

$$\mathcal{L} = T^* - V$$

Lagrangian Approach

ξ_j (j = 1, 2, 3, …)

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i}\right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i$$

$$\mathcal{L} = T^* - V$$

Lagrangian Approach

Little "xi" is the set of DOF. Since we assume system is holonomic, there are the same number of GCs and DOFs.

ξ_j (j = 1, 2, 3, …)

Big "Xi" represents all non-conservative forces, for each DOF. This includes both the ACTUATION and LOSSES (due to damping, most typically).

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i}\right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i$$

Fancy "L" is the "Lagrangian"

V is potential energy

$$\mathcal{L} = T^* - V$$

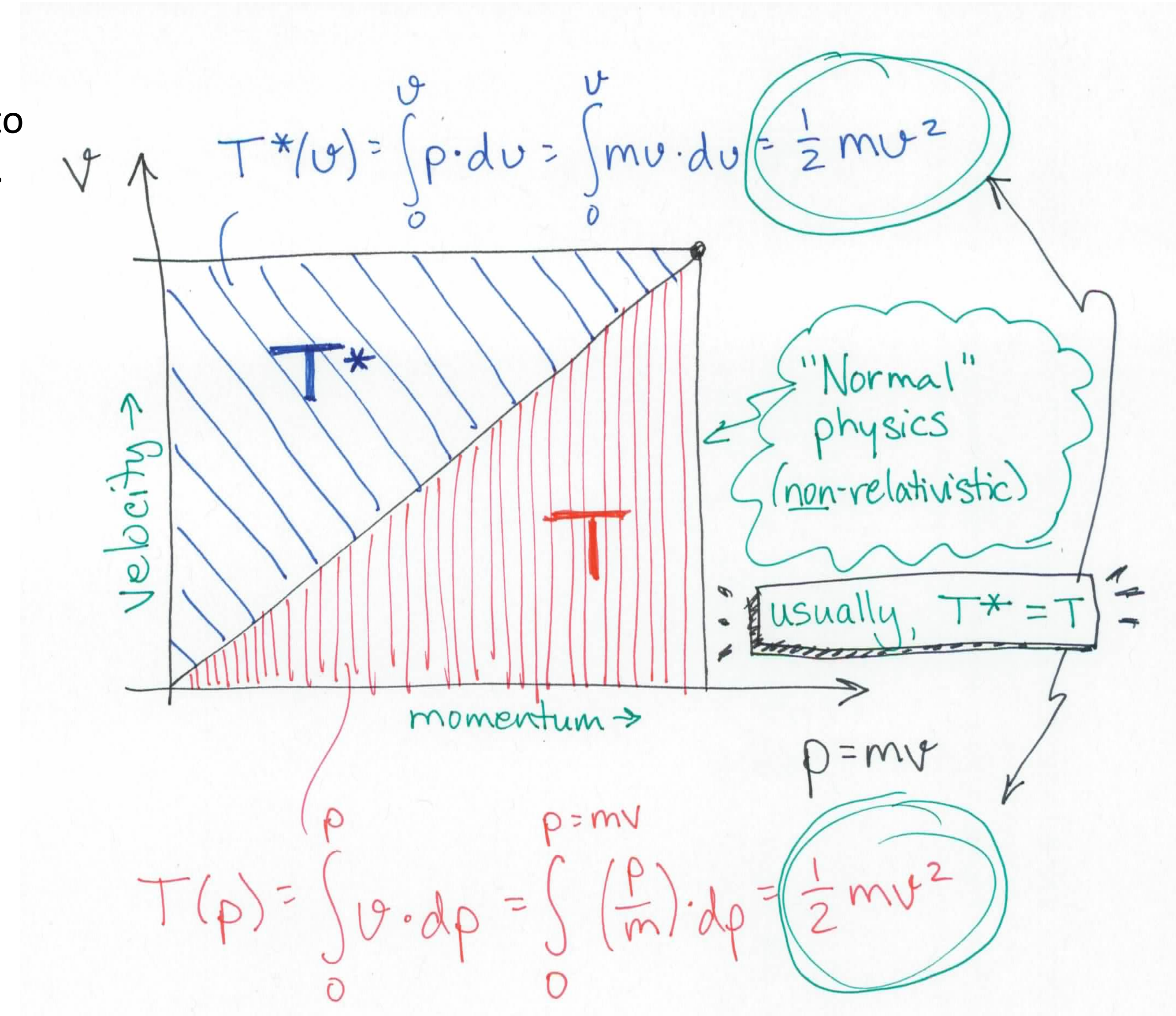
T* is kinetic CO-ENERGY

Kinetic Co-energy

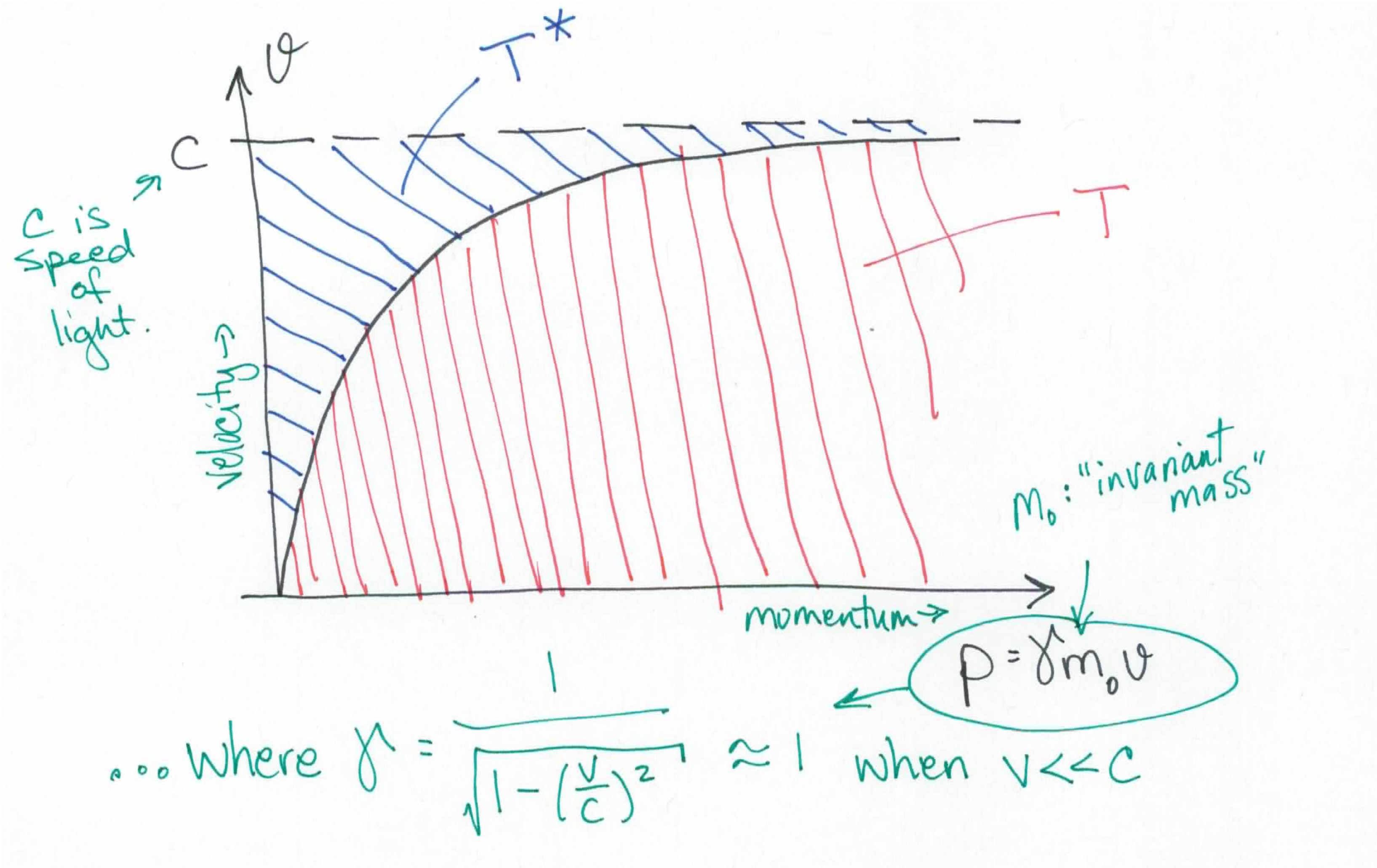
Usually, kinetic energy is identical to kinetic "co-energy".

$$T = \sum_i \tfrac{1}{2} m_i v_i^2$$

For a rigid rotating body: $T = \tfrac{1}{2} J \dot{\theta}^2$



Relativistic (and other unusual) Physics



Note: That said, <u>just assume T*=T in ECE/ME 179d…</u>

Relativistic (and other unusual) Physics

Case 2: Inductor

- Linear Inductor: $\Psi = Li$, L is a constant.

$$v = L\frac{di}{dt} \leftarrow \text{electrical impedance "}Ls\text{"}$$

kinetic energy

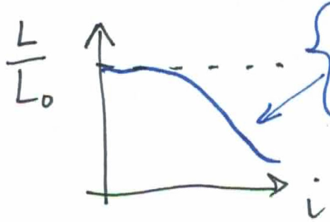
$$T = \int_0^{\Psi_e} i(\Psi)\,d\Psi = \int_0^{\Psi_e} \frac{\Psi}{L}\,d\Psi = \frac{\Psi^2}{2L} = \frac{(Li)^2}{2L} = \frac{1}{2}Li^2$$

kinetic co-energy

$$T^* = \int_0^{i_e} \Psi(i)\,di = \int_0^{i} Li\,di = \frac{Li^2}{2} = \frac{1}{2}Li^2$$

$\boxed{T^* = T}$ ← same value

- Non-linear Inductor: L is a function of i...

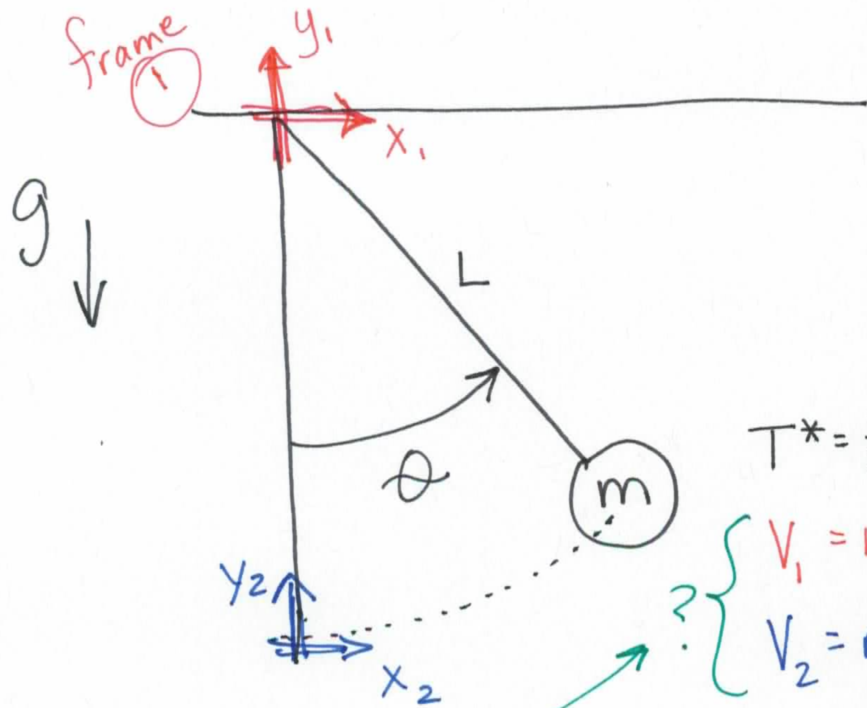


{not a constant, due to magnetic saturation of inductor core...}

$\boxed{T^* \neq T}$ ← Just be aware this is possible...

Note: That said,
just assume $T^*=T$ in ECE/ME 179d...

Simple, Passive Pendulum Example



$$\boxed{\begin{aligned} \xi &: \theta \\ \delta\xi &: \delta\theta \end{aligned}}$$

(holonomic)

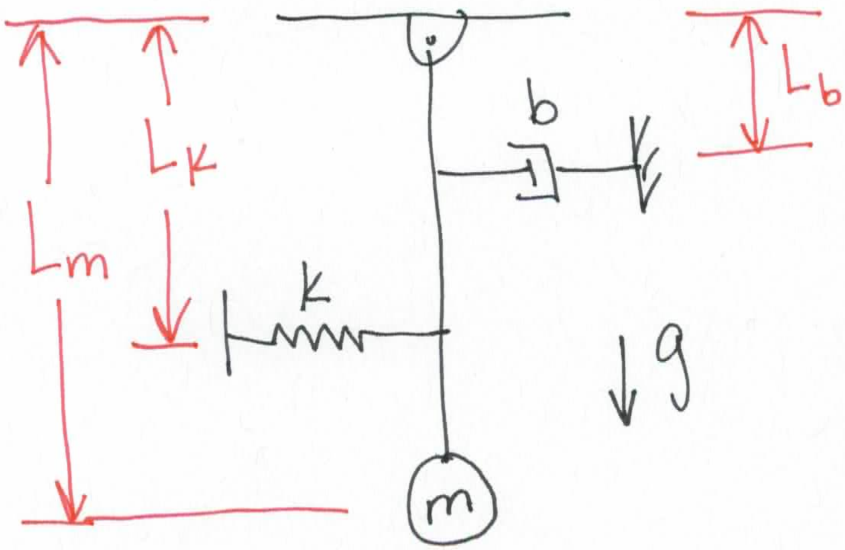
$$T^* = \tfrac{1}{2} m v \cdot v = \tfrac{1}{2} m (L\dot{\theta})^2$$

$$?\begin{cases} V_1 = mgh_1 = -mgL\cos\theta \\ V_2 = mgh_2 = mg(L - L\cos\theta) \end{cases}$$

? Does it matter where we define our fixed (inertial) reference frame?

Simple, Passive Pendulum Example

Spring-Damper Pendulum Example



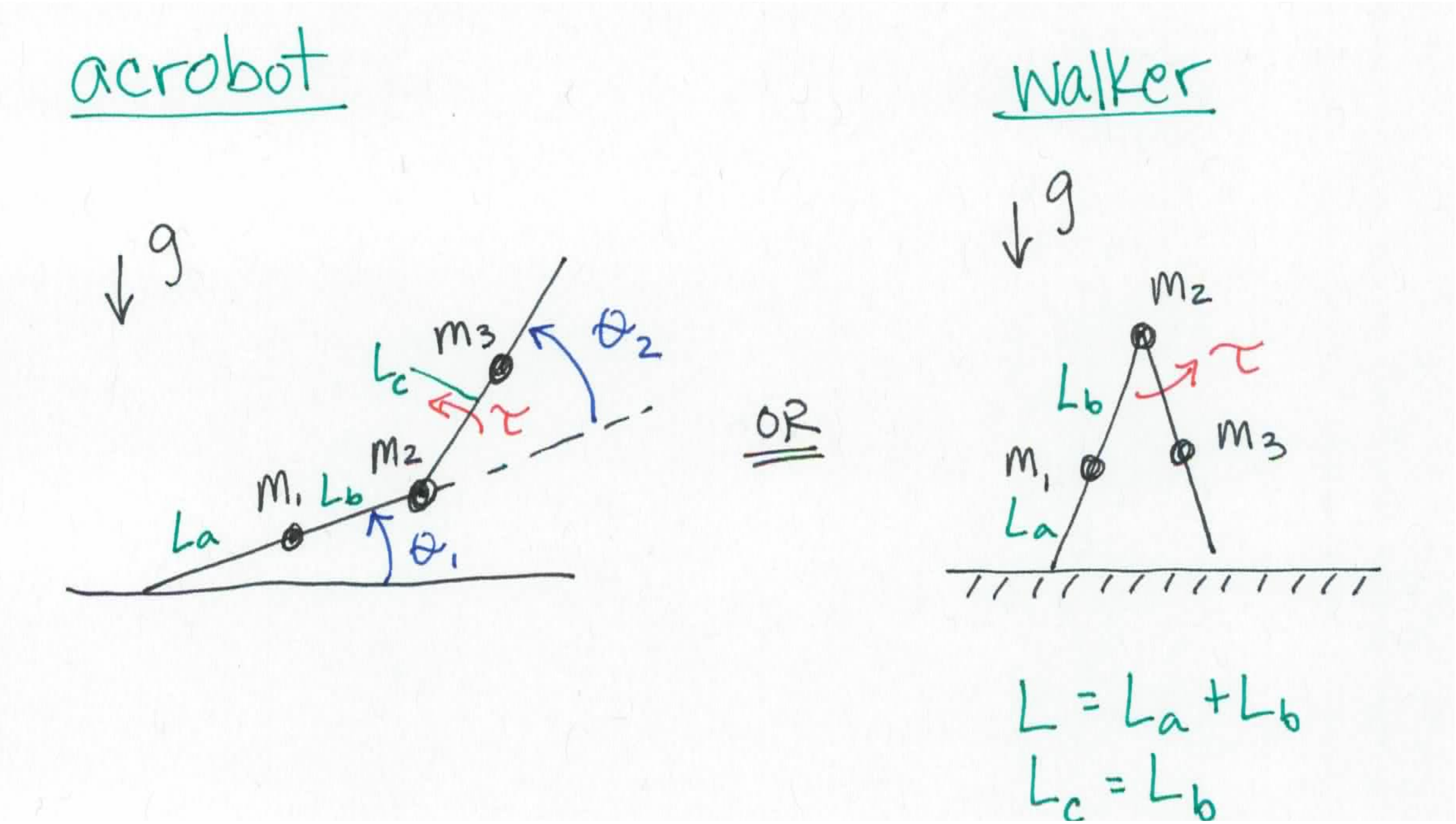
$$m L_m^2 \ddot{\theta} + m g L_m \sin\theta + k L_k^2 \theta = \overbrace{-L_b^2 b \dot{\theta}}^{\Xi_\theta \text{ : non-conservative}}$$

or, write as:

$$m L_m^2 \ddot{\theta} + b L_b^2 \dot{\theta} + k L_k^2 \theta + m g L_m \sin\theta = 0$$

Spring-Damper Pendulum Example

Two-Link Pendulum ("Walker") Example



Two-Link Pendulum ("Walker") Example