ECE/ME 179D

Lagrangian EOM – Part 2

Lecture 12

1. Review Lagrangian Method (for EOM) :

$$\boxed{\mathcal{L} = T^* - V}$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i}\right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i}$$

2. Examples

3. Use of Relative vs Absolute Coordinates

4. Non-conservative (damping) term “tricks”

Notices:

- Midterm November 8 (next class).
- **Lab 4 dates**: **WED, Nov. 14 and MON, Nov. 19**
- **HW 4 Problem 1:** Design trajectory for Lab 4 (Omnibot path following, while pointing a laser at a target.)

Lagrangian Approach: Recall

Little "xi" is the set of DOF. Since we assume system is holonomic, there are the same number of GCs and DOFs.

ξ_j (j = 1, 2, 3, …)

Big "Xi" represents all non-conservative forces, for each DOF. This includes both the ACTUATION and LOSSES (due to damping, most typically).

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i}\right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i$$

Fancy "L" is the "Lagrangian"

V is potential energy

$$\mathcal{L} = T^* - V$$

T* is kinetic CO-ENERGY

Lagrangian Approach: Procedure

0. *Assume* system is **holonomic**:

$$\# GC = \xi_j = \delta\xi_k = \# DOF \longleftrightarrow \text{Holonomic}$$

1. Pick **generalized coordinates** (DOFs) that are:
   - **Complete**
   - **Independent**
     (j = 1, 2, 3, …)

$\boxed{\xi_j} \leftarrow$ Little xi: GCs (gen. coord's)

- Possible GC set(s) ***might not be unique***!
- **Any** independent and complete set is OK.
- Either **absolute or relative** coords is OK.

Lagrangian Approach: Procedure

2. Define kinetic (co-)energy, T. (*Technically*, T*)

Kinetic energy includes the sum of "one-half mass time velocity-squared" for every particle:

$$\boxed{T = \sum_i \frac{1}{2} m_i v_i^2}$$

For a **rigid rotating body**, we can use moment of inertia, J:

$$\boxed{T = \frac{1}{2} J \dot{\theta}^2}$$

(Note: We are **ignoring** electrical kinetic or potential energy here, for now and focusing on **MECHANICAL energy**.)

Lagrangian Approach: Procedure

3. Define potential energy, V.

Potential energy can be stored in a **spring**:

$$\boxed{V_{spring} = \frac{1}{2}k\left(x - x_o\right)^2}$$

…and also includes any "mgh" of particles in a gravitational field:

$$\boxed{V_{gravity} = mgh}$$

Note: "h" is relative to any arbitrary (but fixed) inertial reference frame.

Lagrangian Approach: Procedure

4. Lagrangian is **kinetic minus potential** energy:

$$\boxed{\mathcal{L} = T^* - V}$$

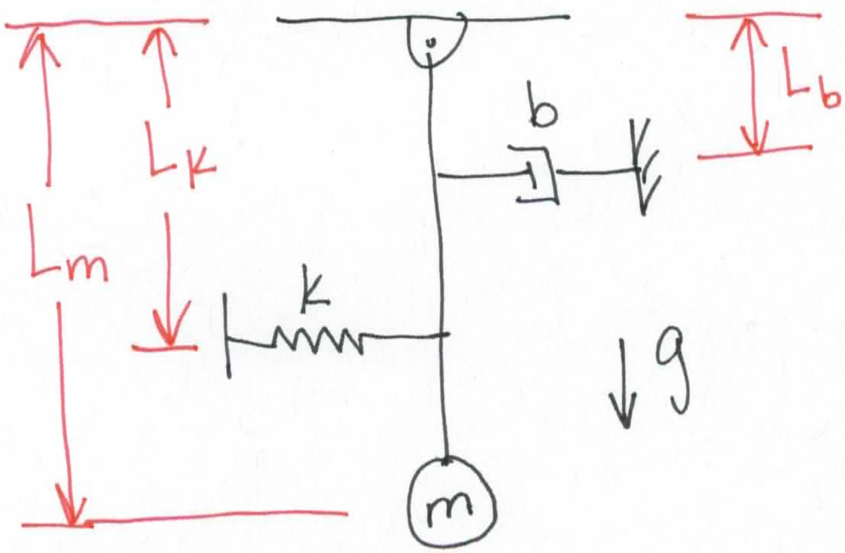
5. For each GC (DOF), solve lefthand side for an equation of motion:

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i}\right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i}$$

Note: A system with **N generalized coordinates** (GCs) will have **N equations of motion.**

6. For each GC (DOF), determine non-conservative forces (righthand side).

Spring-Damper Pendulum Example



One DOF.
One EOM.

Final answer is below, and could also be found via torque balance.

$$mL_m^2\ddot{\theta} + mgL_m\sin\theta + kL_k^2\theta = \underbrace{-L_b^2 b\dot{\theta}}_{\Xi_\theta\text{ : non-conservative}}$$

or, write as:

$$mL_m^2\ddot{\theta} + bL_b^2\dot{\theta} + kL_k^2\theta + mgL_m\sin\theta = 0$$

Spring-Damper Pendulum: Derivation

1. GC(s) ?

2. T ?

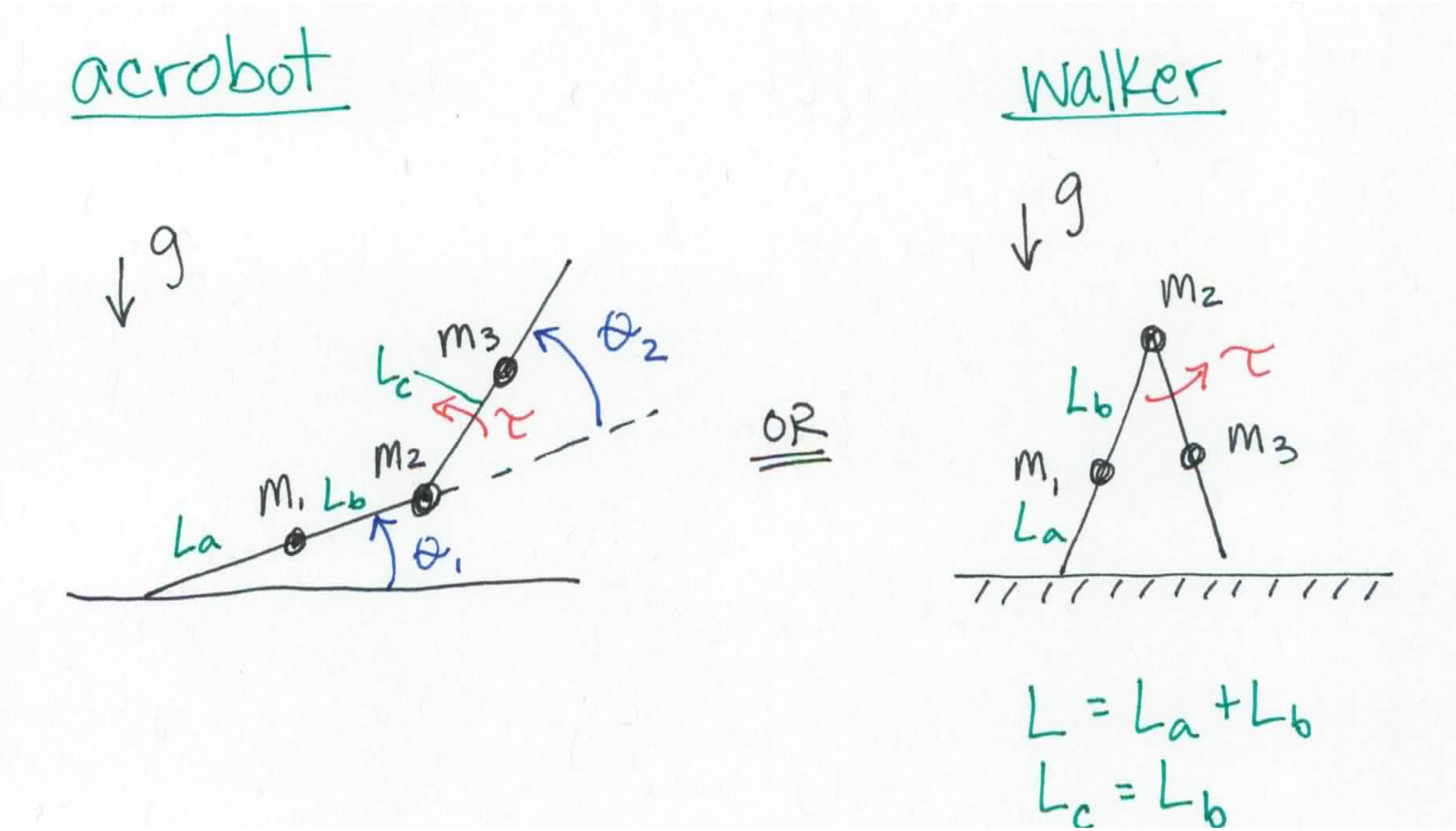
3. V ?

4. L=T-V ?

Spring-Damper Pendulum: Derivation

5. Now solve for EOM(*s*):

Two-Link Pendulum ("Walker") Example



Two-Link Pendulum (“Walker”) Example

1. GC(s) ? Let’s use ***relative*** **theta 2** here…

2. T ?

3. V ?

4. L=T-V ?

Two-Link Pendulum ("Walker") Example

5. Set-up form to solve EOM:

6. Be careful for <u>non-conservative term(s)</u>:
[One torque input, at "elbow"]

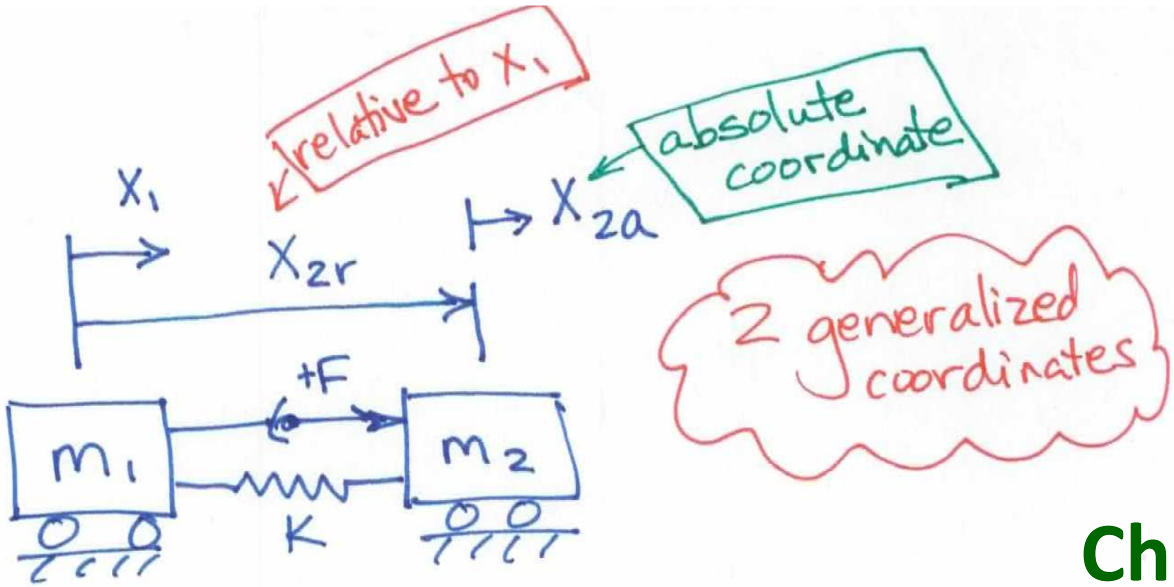
Relative vs Absolute Coordinates

- As mentioned earlier, **valid GCs can be either absolute** (wrt an inertial reference frame) **or relative.**

- The **non-conservative terms** must be written to be **compatible with the choice of absolute vs relative coordinates!**

- **Example:** For the acrobot **(relative coords), tau appears ONLY in the "theta 2" EOM!**

Example: Two-cart spring system



Absolute:

(A) $\boxed{\begin{aligned} \xi_1 &= x_1 \\ \xi_2 &= x_{2a} \end{aligned}}$

$x_1 = x_1$
$x_2 = x_{2a}$

Relative:

(R) $\boxed{\begin{aligned} \xi_1 &= x_1 \\ \xi_2 &= x_{2r} \end{aligned}}$

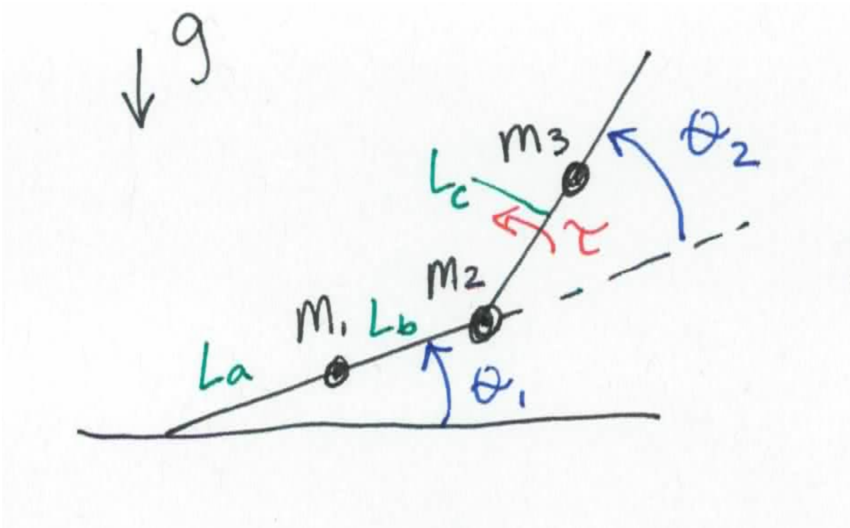
$x_1 = x_1$
$x_2 = x_1 + x_{2r}$

Choice of GCs is NOT UNIQUE. (Either choice here is OK!)

Example: Two-cart spring system

Example: Two-cart spring system

Example: Acrobot (Rel vs Abs)

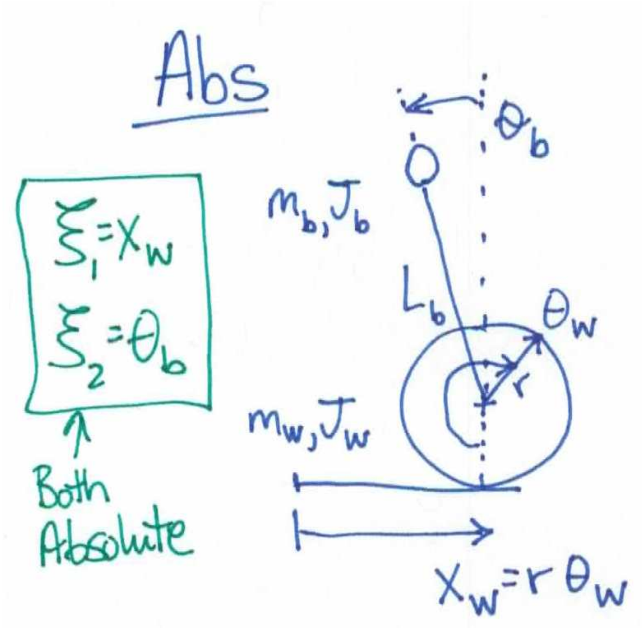


Theta 2 can be relative (as shown) OR absolute.

Example: Acrobot (Rel vs Abs)

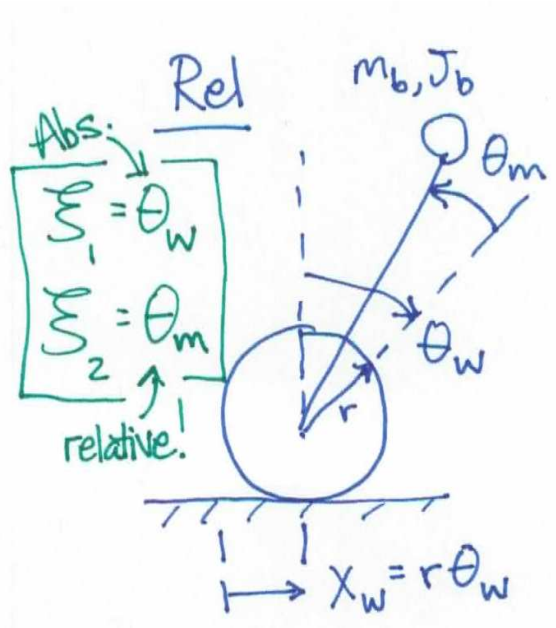
Example: Acrobot (Rel vs Abs)

Example: "Segway" style inverted pendulum



$$\boxed{\Xi_{1a} = -\tau / r}$$

$$\boxed{\Xi_{2a} = \tau}$$



$$\boxed{\Xi_{1r} = 0}$$

$$\boxed{\Xi_{2r} = \tau}$$

Example: “Segway” style inverted pendulum

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