

1. Review Lagrangian Method (for EOM) :

$$\mathcal{L} = T^* - V$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i} \right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i$$

2. Examples

3. Use of Relative vs Absolute Coordinates

4. Non-conservative (damping) term “tricks”

Notices:

- Midterm November 8 (next class).
- **Lab 4 dates: WED, Nov. 14 and MON, Nov. 19**
- **HW 4 Problem 1:** Design trajectory for Lab 4 (Omnibot path following, while pointing a laser at a target.)

Lagrangian Approach: Recall

Little “xi” is the set of DOF. Since we assume system is holonomic, there are the same number of GCs and DOFs.

ξ_j ($j = 1, 2, 3, \dots$)

Big “Xi” represents all non-conservative forces, for each DOF. This includes both the ACTUATION and LOSSES (due to damping, most typically).

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i} \right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i$$

Fancy “L” is the “Lagrangian”

$$\mathcal{L} = T^* - V$$

V is potential energy

T* is kinetic CO-ENERGY

Lagrangian Approach: Procedure

0. Assume system is **holonomic**:

$$\#GC = \xi_j = \delta\xi_k = \#DOF \longleftrightarrow \text{Holonomic}$$

1. Pick **generalized coordinates** (DOFs) that are:

– **Complete**

– **Independent**
($j = 1, 2, 3, \dots$)

$$\xi_j$$

Little xi: GCs
(gen. coord's)

- Possible GC set(s) **might not be unique!**
- **Any** independent and complete set is OK.
- Either **absolute** or **relative** coords is OK.

Lagrangian Approach: Procedure

2. Define kinetic (co-)energy, T . (*Technically*, T^*)

Kinetic energy includes the sum of “one-half mass time velocity-squared” for every particle:

$$T = \sum_i \frac{1}{2} m_i v_i^2$$

For a **rigid rotating body**, we can use moment of inertia, J :

$$T = \frac{1}{2} J \dot{\theta}^2$$

(Note: We are **ignoring** electrical kinetic or potential energy here, for now and focusing on **MECHANICAL energy**.)

Lagrangian Approach: Procedure

3. Define potential energy, V .

Potential energy can be stored in a **spring**:

$$V_{spring} = \frac{1}{2} k (x - x_o)^2$$

...and also includes any “mgh” of particles in a gravitational field:

$$V_{gravity} = mgh$$

Note: “h” is relative to any arbitrary (but fixed) inertial reference frame.

Lagrangian Approach: Procedure

4. Lagrangian is **kinetic minus potential** energy:

$$\mathcal{L} = T^* - V$$

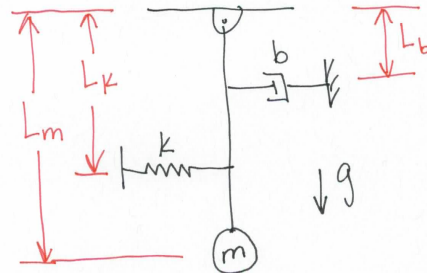
5. For each GC (DOF), solve lefthand side for an equation of motion:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i} \right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i$$

Note: A system with **N generalized coordinates** (GCs) will have **N equations of motion**.

6. For each GC (DOF), determine non-conservative forces (righthand side).

Spring-Damper Pendulum Example



One DOF.
One EOM.

Final answer is below,
and could also be found
via torque balance.

$\hat{\tau}_\theta$: non-conservative

$$mL_m^2 \ddot{\theta} + mgL_m \sin\theta + kL_k^2 \theta = -L_b^2 b \dot{\theta}$$

or, write as:

$$mL_m^2 \ddot{\theta} + bL_b^2 \dot{\theta} + kL_k^2 \theta + mgL_m \sin\theta = 0$$

Spring-Damper Pendulum: Derivation

1. GC(s) ?

2. T ?

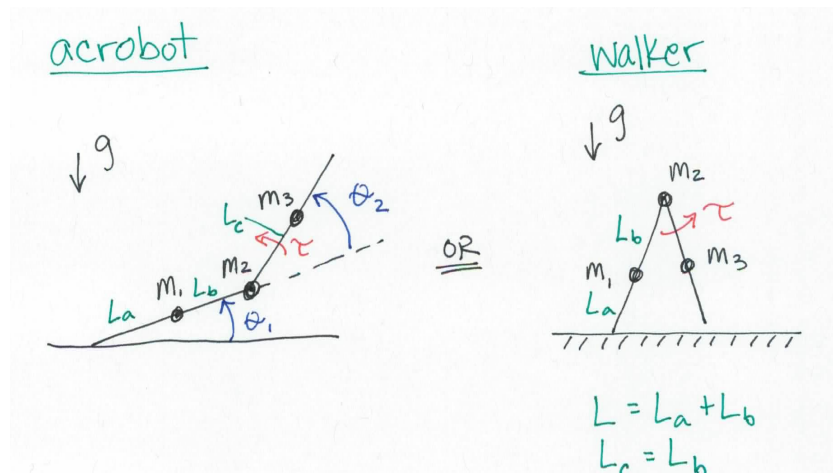
3. V ?

4. L=T-V ?

Spring-Damper Pendulum: Derivation

5. Now solve for EOM(s):

Two-Link Pendulum (“Walker”) Example



Two-Link Pendulum (“Walker”) Example

1. GC(s) ? Let's use *relative* theta 2 here...
2. T ?
3. V ?
4. $L=T-V$?

Two-Link Pendulum (“Walker”) Example

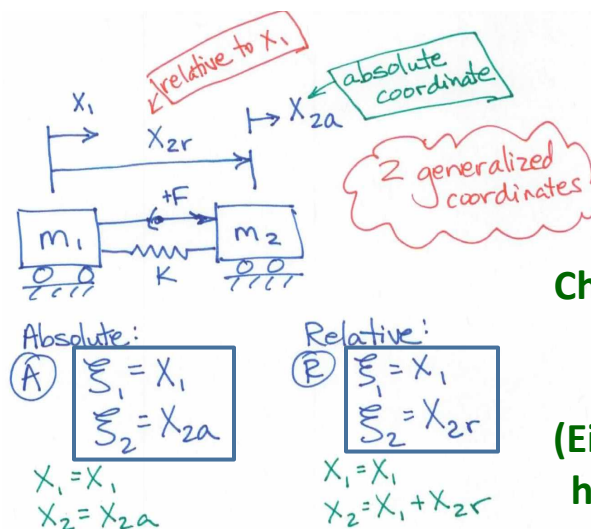
5. Set-up form to solve EOM:
6. Be careful for non-conservative term(s):
[One torque input, at “elbow”]

Relative vs Absolute Coordinates

- As mentioned earlier, **valid GCs can be either absolute** (wrt an inertial reference frame) or **relative**.
- The **non-conservative terms** must be written to be **compatible** with the choice of absolute vs relative coordinates!
- Example:** For the acrobot (relative coords), tau appears **ONLY** in the “theta 2” EOM!

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Example: Two-cart spring system



**Choice of GCs
is NOT
UNIQUE.
(Either choice
here is OK!)**

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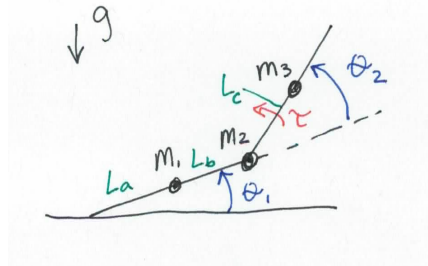
Example: Two-cart spring system

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Example: Two-cart spring system

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Example: Acrobot (Rel vs Abs)



Theta 2 can be
relative (as shown)
OR **absolute**.

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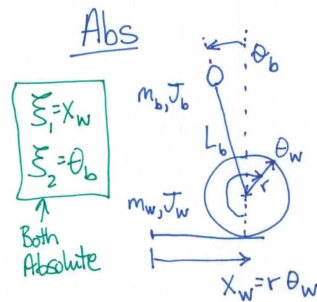
Example: Acrobot (Rel vs Abs)

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Example: Acrobot (Rel vs Abs)

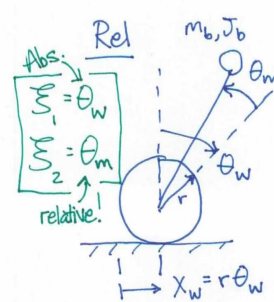
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Example: "Segway" style inverted pendulum



$$\Xi_{1a} = -\tau/r$$

$$\Xi_{2a} = \tau$$



$$\Xi_{1r} = 0$$

$$\Xi_{2r} = \tau$$

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Example: “Segway” style inverted pendulum



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Example: “Segway” style inverted pendulum



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