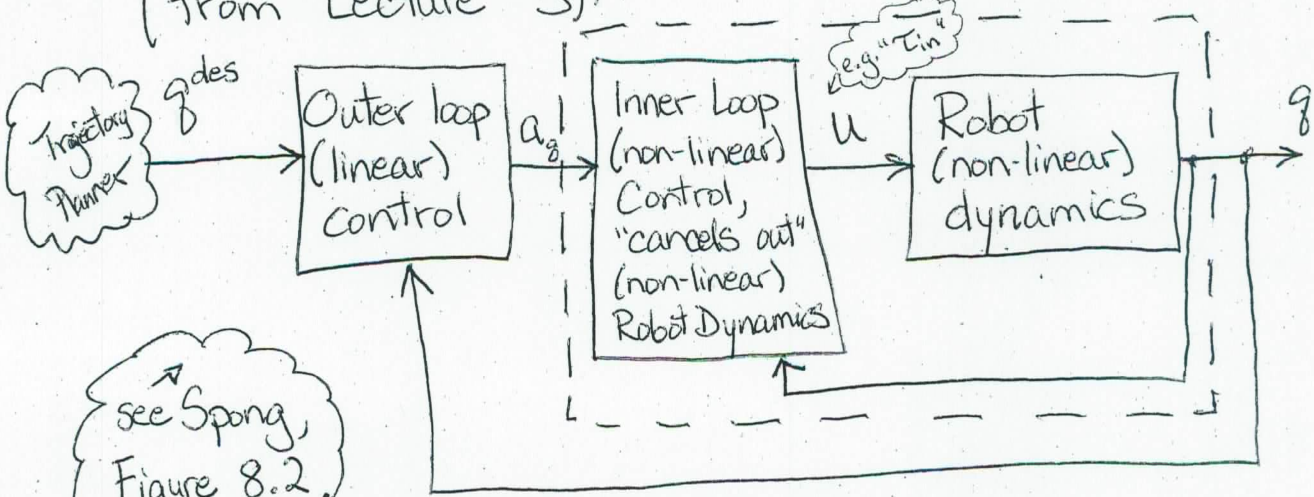
Lecture 16

ECE/ME 179D

• Inverse Dynamics
(Inner loop/outer loop)
(Spong 8.2, Craig 10.4)

• Recall the idea of "Control Law Partioning", (from Lecture 5):



see Spong, Figure 8.2

* Example: Nonlinear Robot Dynamics of pendulum with friction (from Lecture 5):

$$\text{EOM:}\quad u = \underbrace{mL^2\ddot{\theta}}_{J\ddot{\theta}} + \underbrace{B\dot{\theta}}_{\text{viscous (linear) damping}} + \underbrace{C\,\mathrm{sign}(\dot{\theta})}_{\text{nonlinear friction}} + \underbrace{mgL\cos\theta}_{\text{nonlinear moment term due to "cos"}}$$

$u = $ "τ_{in}"

Inner Loop:
$$\begin{cases} \tau_{cancel} = B\dot{\theta} + C\,\mathrm{sign}(\dot{\theta}) + mgL\cos\theta \\ \tau_{add} = mL^2 a_q, \end{cases}$$
where "a_q" is the desired $\ddot{\theta}$, set by outer loop.

$$\boxed{u = \tau_{in} = \tau_{add} + \tau_{cancel}}$$

Inner Loop, cont'd:

$$u = (\tau_{add}) + (\tau_{cancel}) = (mL^2\ddot{\theta}) + (B\dot{\theta} + C\,\mathrm{sign}(\dot{\theta}) + mgL\cos\theta)$$

$$\boxed{(mL^2 a_q) + \cancel{(B\dot{\theta} + C\,\mathrm{sign}(\dot{\theta}) + mgL\cos\theta)} = (mL^2\ddot{\theta})} + \cancel{(B\dot{\theta} + C\,\mathrm{sign}(\dot{\theta}) + mgL\cos\theta)}$$

If we know all parameters & the dynamics match,

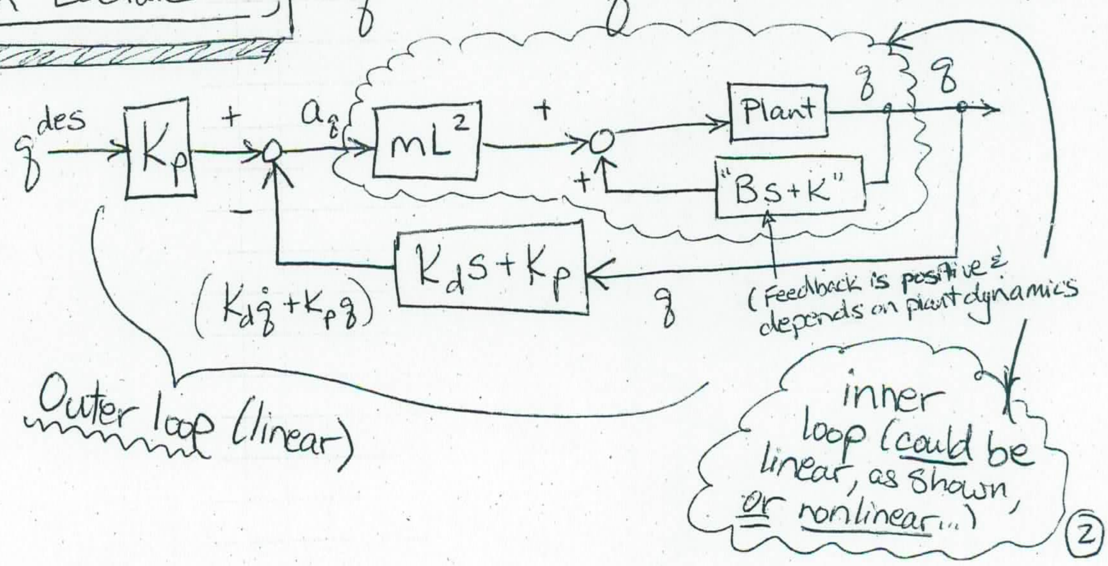
Inner Loop results in: $\boxed{\boxed{\ddot{\theta} = a_q}}$

Outer Loop: Assume we have a "Trajectory Planner" which calculates the desired angle(s) & first and second derivatives:

$$q^{des}, \quad \dot{q}^{des}, \quad \ddot{q}^{des}$$

Then, outer loop sets $\boxed{a_q = \ddot{q}^{des} + K_p(q^{des} - q) + K_d(\dot{q}^{des} - \dot{q})}$

$\boxed{\text{* In Lecture 5,}}$ $\dot{q}^{des} = 0$ & $\ddot{q}^{des} = 0$ were assumed, so:

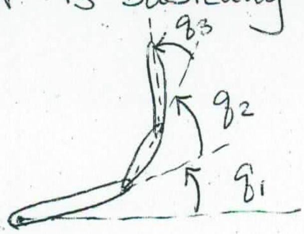


- This "control law partitioning" (from Lecture 5) is also known as an "inverse dynamics" approach.
  - The "inner loop" cancels out all natural plant dynamics, except the accelerations, $\ddot{q}$. Usually, this requires a nonlinear control law.
  - The "outer loop" is simply a set of linear, PD control laws, resulting in second-order (2-pole) closed-loop pole pair(s).

Robot Manipulators - EOM have the form:
(See Spong eqn 8.21; Craig 10.4)

$$\boxed{\underbrace{M(q)\ddot{q}}_{\text{Inertia (mass) matrix}} + \underbrace{C(q,\dot{q})\dot{q}}_{\text{Coriolis \& centrifugal torques}} + \underbrace{G(q)}_{\text{Gravity}} + \underbrace{F(q,\dot{q})}_{\text{Friction}} = \underbrace{u}_{\text{Torque inputs}}}$$

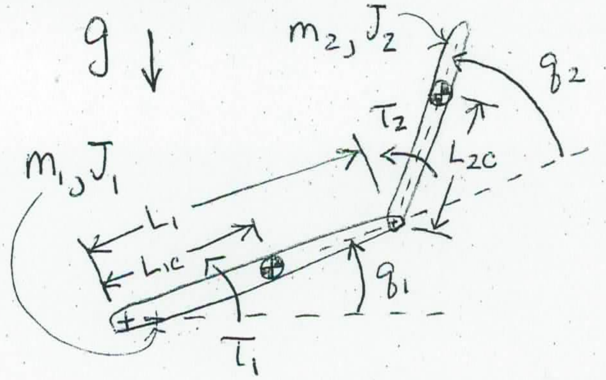
"Manipulator" is basically a multi-link arm, e.g.:



etc., up to q_n,

$$q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}$$

2-link Example:



Let:

$$\boxed{h \equiv m_2 L_1 L_{2c} \sin\theta_2}$$

2 Generalized Coordinates (GCs), q_1 & q_2.

$\therefore$ 2 EOMs:

①

$$\boxed{\begin{array}{ll} M_{11}\ddot{\theta}_1 & \rightarrow (m_1 L_{1c}^2 + J_1 + J_2 + m_2(L_1^2 + L_{2c}^2 + 2L_1 L_{2c}\cos\theta_2))\ddot{\theta}_1 + \dots \\ + M_{12}\ddot{\theta}_2 & \rightarrow (m_2(L_{2c}^2 + L_1 L_{2c}\cos\theta_2) + J_2)\ddot{\theta}_2 + \dots \\ + G_1 & \rightarrow (m_1 L_{1c}\cos\theta_1 + m_2 L_1 \cos\theta_1 + m_2 L_{2c}\cos(\theta_1 + \theta_2))\, g + \dots \\ + C_1\dot{\theta} & \rightarrow -2h\dot{\theta}_1\dot{\theta}_2 - h\dot{\theta}_2^2 = \dots \\ = \Xi_1 & \rightarrow \tau_1 \end{array}}$$

②

$$\boxed{\begin{array}{ll} M_{21}\ddot{\theta}_1 & \rightarrow (m_2(L_{2c}^2 + L_1 L_{2c}\cos\theta_2) + J_2)\ddot{\theta}_1 + \dots \\ + M_{22}\ddot{\theta}_2 & \rightarrow (m_2 L_{2c}^2 + J_2)\ddot{\theta}_2 + \dots \\ + G_2 & \rightarrow (m_2 L_{2c}\cos(\theta_1 + \theta_2))\, g + \dots \\ + C_2\dot{\theta} & \rightarrow h\dot{\theta}_1^2 = \dots \\ = \Xi_2 & \rightarrow \tau_2 \end{array}}$$

Matrix Form:

$$\overbrace{\begin{bmatrix} M_{11}, & M_{12} \\ M_{21}, & M_{22} \end{bmatrix}}^{M} \overbrace{\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}}^{\ddot{q}} + \overbrace{\begin{bmatrix} -h\dot{\theta}_2, & -h(\dot{\theta}_1+\dot{\theta}_2) \\ h\dot{\theta}_1, & 0 \end{bmatrix}}^{C} \overbrace{\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}}^{\dot{q}} + \overbrace{\begin{bmatrix} G_1 \\ G_2 \end{bmatrix} g}^{G} = \overbrace{\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}}^{u}$$

$M_{21} = M_{12}$

where G_1 & G_2 contain all terms except "g" itself in the gravity torques:

$$G_1 = m_1 L_{1c} \cos\theta_1 + m_2 L_1 \cos\theta_1 + m_2 L_{2c} \cos(\theta_1 + \theta_2)$$

$$G_2 = m_2 L_{2c} \cos(\theta_1 + \theta_2)$$

Note, $\boxed{M \text{ is invertible}} \rightarrow \ddot{q} = M^{-1}(u - C\dot{q} - G)$

Let's say our outer loop (linear) control law sets a "desired acceleration", a_q:

if friction exists in EOM, too...

Define $\boxed{u = M a_q + C\dot{q} + G \;(+F)}$

Nonlinear "inner loop" control law: "inverse dynamics" control

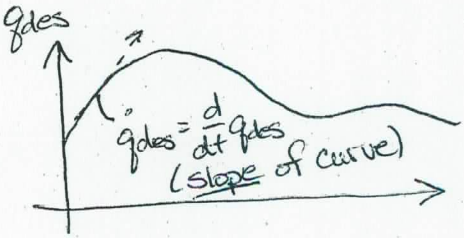
Solving for $\ddot{q}$: $\boxed{\ddot{q} = a_q}$ ← actual accelerations, $\ddot{q}$, should match desired accelerations, a_q.

$$a_q = M^{-1}(u - C\dot{q} - G)$$

Outer Loop?

The job of the outer loop is to set a_q, the desired acceleration.

How to set a_q?



- Start with desired joint trajectories, q_{des}.

- Differentiate to calculate $\dot{q}_{des}$ & $\ddot{q}_{des}$,

$$\boxed{\dot{q}_{des} = \frac{d}{dt} q_{des}} \quad \text{and} \quad \boxed{\ddot{q}_{des} = \frac{d^2}{dt^2} q_{des}}$$

- Doesn't a_q just equal $\ddot{q}_{des}$?

  Answer: No.

  Why: Because if there are errors in q or $\dot{q}$, i.e., $q \neq q_{des}$ and/or $\dot{q} \neq \dot{q}_{des}$, we need to "catch up" to the desired trajectory, and we will use "a_q" to do so!

- Therefore, we use a PD control law:

This is a simple, LINEAR controller.

$$\boxed{a_q = \underbrace{\ddot{q}_{des}}_{①} + \underbrace{\left(K_p(q_{des} - q) + K_d(\dot{q}_{des} - \dot{q})\right)}_{②}}$$

① $\ddot{q}_{des}$ needed for desired q_{des} trajectory at this time	PLUS	② Feedback (PD) for errors in pos. & vel. of trajectory.