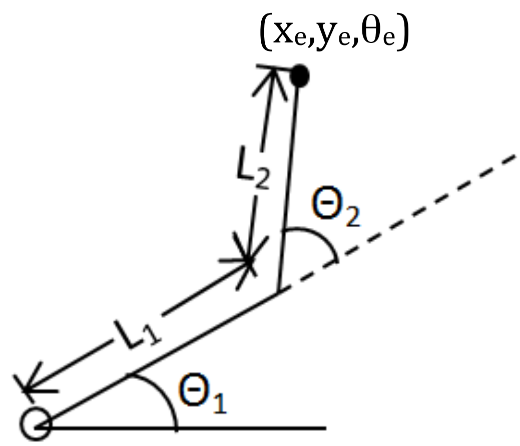
Lecture 2 Supplemental Notes:

1. Jacobian: A matrix relating instantaneous joint velocites to end effector velocities.

Example: 2-link, planar arm.

- Kinematics: Geometric relation between joints and end effector.



x_e = ________________________________

y_e = ________________________________

θ_e = ________________________________

- Velocity relationship. Just differentiate the above expressions, with respect to (wrt) time:

$\dot{x}_e$ =

$\dot{y}_e$ =

$\dot{\theta}_e$ =

Let Greek letter "xi", ξ, represent the task space coordinates of the end effector, and let q represent the joint space coordinates, which are the two joint angles:

$$\xi = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} \quad , \quad q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

The JACOBAN is the matrix that relates the velocities of the coordinates in xi to the velocities of the coordinates in q:

$$\dot{\xi} = J\dot{q}$$

QUESTION: Since xi is a (3x1) vector, and q is a (2x1) vector in this example, what size must matrix J be?

QUESTION: From the expressions we derived on page 1, what is J?

J =

QUESTION: If we can always control the velocities theta1 and theta2 independently, does this imply we can always control the velocities of coordinates xe and ye independently, too?

QUESTION: What happens to J if theta1 and theta2 are both zero?

QUESTION: How does this affect our ability to set the horizontal velocity, dx_e/dt, of the end effector?

2. Singularities: (all statements below are true)

- Loss of an independent DOF (direction of instantaneous motion) of the end effector always occurs at a singularity.

- Rank of J is reduced (i.e., less than its maximal possible value) at a configuration that is a singularity.

- When at the edge of the workspace (of the end effector), this is always a singularity; the converse is not necessarily true…

- i.e., singularities can also occur when the end effector is not at the boundary of the workspace.

- Near a singularity, matrix inversion techniques (e.g., to control the dynamics) can "blow up"! (If J is usually invertible, it becomes dangerous to do so as the system approaches a singularity…)

  e.g., if xi contains on only xe and ye (not theta_e), then J would be 2x2 and would be invertible when the system in NOT in a singularity:

  $\dot{\xi}_{ee} = J\dot{\theta}_q$, so we can invert the relationship: $\dot{\theta}_q = J^{-1}\dot{\xi}_{ee}$

- Mechanical advantage: Near a singularity, the system can sometimes have improved load bearing. For example, when you "lock your knees" to stand, your two-link leg systems are each at a singularity, which reduces the torque you need to supply to remain standing.

- The Jacobian must also describes the static force-torque relationship between actuator effort and output at the end effector:

  $$\tau_q = J^T F_{ee}$$

- Finally, remember that J is a function of the joint space configuration, q, so we should really always be writing J(q) above to indicate this as a reminder, instead of just "J", alone!

3. Work: force, displace, and energy

Terms

Force: a measure of “effort”.

Work: a measure of “accomplishment” (i.e., force time distance).

Energy: ability to do work. (Joules and Newton-meters are the same units!)

Power: Rate of doing work (work per unit time), and/or rate of using energy.

Inertia: Defined with respect to some point (such as center of mass), an inertia is the sum of “mass times radius squared” for all particles in a rigid object.

4. Impedance.

Impedance: The ratio of “Effort”/”Flow”. In general, this can be either a static relationship, such as a resistor value, R, or it can by a dynamic relationship, described by a differential equation.

Mechanical Impedance: Force/Velocity (Effort/Flow). $Z_m(s) = F(s)/(sX(s))$.
Examples: mass, damper, spring.

Electrical Impedance: Voltage/Current (Effort/Flow). $Z_e(s) = V(s)/I(s)$.
Examples: inductor, resistor, capacitor.