In Lecture 5, we introduced a technique called "Control Law Partitioning". This involves separating a control law into two parts. The first part "cancels out" the natural dynamics of a system, and the second part adds in control forces (or torques) that result in a desired (user-defined) 2nd-order linear system dynamics for the controlled system. The natural dynamics can be either linear or nonlinear; they may also be either time invariant or time-varying. Example of nonlinear dynamics include Coulombic friction and configuration-dependent gravity terms.

The notes below are from pp. 273-275 in Craig. (See syllabus for book info.)

9.5 CONTROL-LAW PARTITIONING

In preparation for designing control laws for more complicated systems, let us consider a slightly different controller structure for the sample problem of Fig. 9.6. In this method, we will partition the controller into a **model-based portion** and a **servo portion**. The result is that the system's parameters (i.e., m, b, and k, in this case) appear only in the model-based portion and that the servo portion is independent of these parameters. At the moment, this distinction might not seem important, but it will become more obviously important as we consider nonlinear systems in Chapter 10. We will adopt this **control-law partitioning** approach throughout the book.

The open-loop equation of motion for the system is

$$m\ddot{x} + b\dot{x} + kx = f. \tag{9.40}$$

We wish to decompose the controller for this system into two parts. In this case, the model-based portion of the control law will make use of supposed knowledge of m, b, and k. This portion of the control law is set up such that it *reduces the system so that it appears to be a unit mass*. This will become clear when we do Example 9.5. The second part of the control law makes use of feedback to modify the behavior of the system. The model-based portion of the control law has the effect of making the system appear as a unit mass, so the design of the servo portion is very simple—gains are chosen to control a system composed of a single unit mass (i.e., no friction, no stiffness).

The model-based portion of the control appears in a control law of the form

$$f = \alpha f' + \beta, \tag{9.41}$$

where α and β are functions or constants and are chosen so that, if f' is taken as the *new input* to the system, *the system appears to be a unit mass*. With this structure of the control law, the system equation (the result of combining (9.40) and (9.41)) is

$$m\ddot{x} + b\dot{x} + kx = \alpha f' + \beta. \tag{9.42}$$

Clearly, in order to make the system appear as a unit mass from the f' input, for this particular system we should choose α and β as follows:

$$\alpha = m,$$
$$\beta = b\dot{x} + kx. \tag{9.43}$$

Making these assignments and plugging them into (9.42), we have the system equation

$$\ddot{x} = f'. \tag{9.44}$$

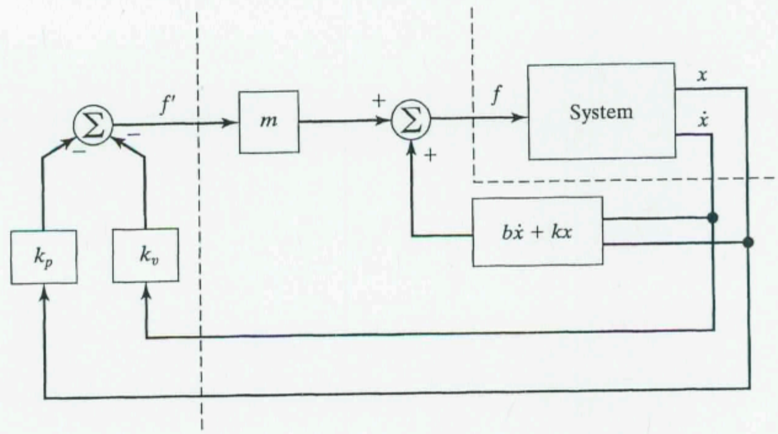

FIGURE 9.8: A closed-loop control system employing the partitioned control method.

This is the equation of motion for a unit mass. We now proceed as if (9.44) were the open-loop dynamics of a system to be controlled. We design a control law to compute f', just as we did before:

$$f' = -k_v\dot{x} - k_p x. \tag{9.45}$$

Combining this control law with (9.44) yields

$$\ddot{x} + k_v\dot{x} + k_p x = 0. \tag{9.46}$$

Under this methodology, the setting of the control gains is simple and is independent of the system parameters; that is,

$$k_v = 2\sqrt{k_p} \tag{9.47}$$

must hold for critical damping. Figure 9.8 shows a block diagram of the partitioned controller used to control the system of Fig. 9.6.

EXAMPLE 9.5

If the parameters of the system in Fig. 9.6 are $m = 1$, $b = 1$, and $k = 1$, find α, β, and the gains k_p and k_v for a position-regulation control law that results in the system's being critically damped with a closed-loop stiffness of 16.0.

We choose

$$\begin{aligned} \alpha &= 1, \\ \beta &= \dot{x} + x, \end{aligned} \tag{9.48}$$

so that the system appears as a unit mass from the fictitious f' input. We then set gain k_p to the desired closed-loop stiffness and set $k_v = 2\sqrt{k_p}$ for critical damping.

This gives

$$\begin{aligned} k_p &= 16.0, \\ k_v &= 8.0. \end{aligned} \tag{9.49}$$