

8.1 Performance Indices and Controller Tuning 565

5. *Estimate the sampling rate required for implementing a digital control system.*
6. *Develop and analyze a discrete-time control algorithm.*
7. *Apply MATLAB to analyze and design control systems using the concepts presented in this chapter.*

8.1 PERFORMANCE INDICES AND CONTROLLER TUNING

Once the form of the control law has been selected, the gains must be computed in light of the performance specifications. In the examples of the PID family of control laws in Sections 7.5, 7.6, and 7.7, the damping ratio, dominant time constant, and steady state error were taken as the primary indicators of system performance in the interests of simplicity. In practice, the criteria are usually more detailed. For example, the rise time and maximum overshoot as well as the other transient response specifications in Section 4.4 may be encountered. Requirements can also be stated in terms of frequency response characteristics, such as bandwidth, resonant frequency, and peak amplitude. Whatever specific form they take, a complete set of specifications for robust control system performance should generally include the following considerations, for given forms of the command and disturbance inputs:

1. Equilibrium specifications.
 - Stability margin, and
 - Steady state error.
2. Transient specifications.
 - Speed of response, and
 - Form of response (degree of damping).
3. Sensitivity specifications.
 - Sensitivity to parameter variations,
 - Sensitivity to model inaccuracies, and
 - Sensitivity to sensor error (noise rejection).
4. Nonlinear effects.
 - Stability margin, and
 - Final control element capabilities.

In addition to these performance stipulations, the usual engineering considerations of initial cost, weight, maintainability, and so forth must be taken into account. The considerations are highly specific to the chosen hardware, and it is difficult to deal with such issues in a general way. Here, we limit ourselves to the performance considerations listed. Two approaches exist for designing the controller. The proper one depends on the quality of the analytical description of the plant to be controlled. If an accurate model of the plant is easily developed, the approach is to design a specialized controller for the particular application. The range of adjustment of controller gains in this case can usually be made small because the accurate plant model allows the gains to be precomputed with confidence. This technique reduces the cost of the controller and can usually be applied to electromechanical systems.

The second approach is used when the plant is relatively difficult to model, which is often the case in process control. A standard controller with several control modes and wide ranges of gains is used, and the proper mode and gain settings are obtained first by testing the control design by computer simulation if a computer model of the process is available, and then by testing the controller on the process in the field.

Performance Indices

The performance criteria encountered thus far require a set of conditions to be specified; for example, one for steady state error, one for damping ratio, and one for the dominant time constant. If there are many such conditions and if the system is of high order with several gains to be selected, the design process can become quite complicated because transient and steady state criteria tend to drive the design in different directions. An alternative approach is specifying the system's desired performance by means of one analytical expression called a *performance index*. Powerful analytical and numerical methods are available that allow the gains to be systematically computed by minimizing (or maximizing) this index. This approach forms the basis for many advanced design methods. Here, we introduce the concept.

To be useful, a performance index must be selective. The index must have a sharply defined extremum in the vicinity of the gain values that give the desired performance. If the numerical value of the index does not change very much for large changes in the gains from their optimal values, the index will not be selective.

Any practical choice of a performance index must be easily computable, either analytically, numerically, or experimentally. Four common choices for an index are the following.

$$J = \int_0^{\infty} |e(t)| dt \quad (\text{IAE}) \quad (8.1-1)$$

$$J = \int_0^{\infty} t|e(t)| dt \quad (\text{ITAE}) \quad (8.1-2)$$

$$J = \int_0^{\infty} e^2(t) dt \quad (\text{ISE}) \quad (8.1-3)$$

$$J = \int_0^{\infty} te^2(t) dt \quad (\text{ITSE}) \quad (8.1-4)$$

where $e(t)$ is the system error. This error is usually the difference between the desired and the actual values of the output. However, if $e(t)$ does not approach zero as $t \rightarrow \infty$, the preceding indices will not have finite values. In this case, $e(t)$ can be defined as $e(t) = c(\infty) - c(t)$, where $c(t)$ is the output variable. If the index is to be computed numerically or experimentally, the infinite upper limit can be replaced by the limit t_f , where t_f is large enough so that $e(t)$ is negligible for $t > t_f$.

The *integral-absolute-error (ITAE)* criterion (8.1-1) expresses mathematically that the designer is not concerned with the sign of the error, only its magnitude. In some applications, the IAE criterion describes the fuel consumption of the system. The index says nothing about the relative importance of an error occurring late in the response versus an error occurring early. Because of this, the index is not so selective as the *integral-of-time-multiplied-by-absolute-error criterion (ITAE)* (8.1-2). Because the multiplier t is small in the early stages of the response, this index weights early errors less heavily than later errors, which makes sense physically. No system can respond instantaneously, and the index is lenient accordingly, while punishing any system that allows a nonzero error to remain for a long time. Neither criterion allows highly underdamped or highly overdamped systems to be optimum. The ITAE criterion usually results in a system whose step response has a slight overshoot and well-damped oscillations. The *integral-squared-error (ISE)* and *integral-of-time-multiplied-by-squared-error (ITSE)* criteria are analogous to the IAE and ITAE criteria except that the square of the error is employed, for three reasons: (1) in

8.1 Performance Indices and Controller Tuning 567

some applications, the squared error represents the system's power consumption; (2) squaring the error weights large errors much more heavily than small errors; (3) the squared error is much easier to handle analytically. The derivative of a squared term is easier to compute than that of an absolute value and does not have a discontinuity at $e = 0$. These differences are important when the system is of high order with multiple error terms. This case is treated in Chapter 11.

The ISE and ITSE criteria are similar to the least squares technique for curve fitting. We can think of the process as one of finding the response curve that minimizes the total squared error. The ITSE index is analogous to weighted least squares, with the weighting factor being proportional to time. Systems designed with the ISE criterion tend to be fast and oscillatory. The ITSE index provides better selectivity than the ISE. The following example illustrates the procedure.

The general second-order system with zero steady state error for a step input has the form

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (8.1-5)$$

The step response can easily be found in terms of the parameters ζ and ω_n , and the various performance indices computed, but the algebra required is enough to make us think twice about attempting an analytical approach with a third-order system. The results for this model show that the ITAE index and the index

$$J = \int_0^\infty t(|e| + |\dot{e}|) dt \quad (8.1-6)$$

have the most sharply defined minima as functions of ζ . For these two indices, the optimum value of ζ occurs at $\zeta = 0.707$. In fact, the value $\zeta = 0.707$ is commonly used, even for transfer functions that are not identical to (8.1-5). The other indices might be more selective when applied to a different system and should not be discarded on the basis of this example.

EXAMPLE 8.1-1 Proportional Control of a Second-Order Plant

Consider proportional control of a second-order plant whose transfer function is

$$G_p(s) = \frac{1}{Is^2 + cs}$$

One application is position control. The closed-loop transfer function is

$$\frac{\Theta(s)}{\Theta_r(s)} = \frac{K_p}{Is^2 + cs + K_p} = \frac{K_p/I}{s^2 + (c/I)s + K_p/I} \quad (8.1-7)$$

where $\Theta(s)$ is the position and $\Theta_r(s)$ is the commanded position.

- (a) Use a performance index to select a value for the gain K_p for the case where $I = 10$ and $c = 1$.
- (b) Evaluate the performance of the controller.

SOLUTION

- (a) The transfer function (8.1-7) has the same form as (8.1-5), where $K_p/I = \omega_n^2$ and $c/I = 2\zeta\omega_n$. The ITAE index is minimized for $\zeta = 0.707$. Thus, for $I = 10$ and $c = 1$

568 Chapter 8 Control System Design: Tuning, Compensation, and Digital Control

we have $\omega_n = (1/10)/[2(0.707)] = 0.0707$. This gives $K_p/I = \omega_n^2 = (0.0707)^2$, or $K_p = 10(0.0707)^2 = 0.05$.

(b) The final value theorem shows that the steady state error for a step command will be zero. With $K_p = 0.05$, the closed-loop roots are given by $10s^2 + s + 0.05 = 0$, and are $s = (-1 \pm i)/20$. The time constant is $\tau = 20$ and the damped natural frequency is $\omega_d = 1/20$.

Table 8.1-1 gives the transfer function forms, up to fourth-order, that minimize the ITAE index for a step input. Higher-order forms are also given in ([Graham, 1953]). Note that all the forms have zero steady state error for a step input and that none have numerator dynamics. Thus you can encounter transfer functions whose form is not in the table. For example, PI control of the plant $1/(Is + c)$ gives the closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{K_P s + K_I}{Is^2 + (c + K_P)s + K_I}$$

which has numerator dynamics. Thus a minimum ITAE index value for this system will not necessarily be achieved by choosing K_P and K_I to give $\zeta = 0.707$.

However, this form does give the minimum ITAE index value for a ramp input if the damping c is negligible. In this case, the transfer function becomes

$$\frac{C(s)}{R(s)} = \frac{K_P s + K_I}{Is^2 + K_P s + K_I}$$

Table 8.1-2 gives the transfer function forms, up to fourth-order, that minimize the ITAE index for a ramp input ([Graham, 1953]). Note that all the forms have zero steady state error for a ramp input and that all have numerator dynamics. The table

Table 8.1-1 Minimum ITAE Forms for Step Response

$\frac{\omega_n}{s + \omega_n}$
$\frac{\omega_n^2}{s^2 + 1.4\omega_n s + \omega_n^2}$
$\frac{\omega_n^3}{s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3}$
$\frac{\omega_n^4}{s^4 + 2.15\omega_n s^3 + 3.4\omega_n^2 s^2 + 2.7\omega_n^3 s + \omega_n^4}$

Table 8.1-2 Minimum ITAE Forms for Ramp Response

$\frac{3.2\omega_n s + \omega_n^2}{s^2 + 3.2\omega_n s + \omega_n^2}$
$\frac{3.25\omega_n^2 s + \omega_n^3}{s^3 + 1.75\omega_n s^2 + 3.25\omega_n^2 s + \omega_n^3}$
$\frac{5.14\omega_n^3 s + \omega_n^4}{s^4 + 2.41\omega_n s^3 + 4.93\omega_n^2 s^2 + 5.14\omega_n^3 s + \omega_n^4}$

8.1 Performance Indices and Controller Tuning 569

does not contain a first-order system because the steady state error for such a system cannot be made zero, and thus the ITAE index will be infinite.

EXAMPLE 8.1-2 PI Control and Ramp Inputs

Suppose that $I = 1$ and $c = 0$. The transfer function for the PI control system given previously becomes

$$\frac{C(s)}{R(s)} = \frac{K_P s + K_I}{s^2 + K_P s + K_I}$$

Compute the gain values required to obtain zero steady state error for a ramp input after approximately $t = 1$.

SOLUTION

Comparing the transfer function with the first entry in Table 8.1-2, we see that

$$\begin{aligned} K_P &= 3.2\omega_n \\ K_I &= \omega_n^2 \end{aligned}$$

We are free to choose ω_n to obtain some desired performance measure, such as a desired time constant, but not the damping ratio, because the damping ratio of the optimal form is fixed at $\zeta = 1.6$.

$$\zeta = \frac{3.2\omega_n}{2\sqrt{\omega_n^2}} = 1.6$$

The two real roots are $s = -2.849\omega_n$ and $s = -0.351\omega_n$. To obtain zero steady state error for a ramp input after approximately $t = 1$, we select the dominant time constant to be $\tau = 0.25$. This requires that $0.351\omega_n = 4$, or $\omega_n = 11.396$. Thus the two roots are $s = -4$ and $s = -32.47$. The value of $\omega_n = 11.396$ gives $K_P = 36.5$ and $K_I = 130$.

The choice of the control action for trapezoidal command inputs where the plant has the form $1/(Is+c)$ present a challenge to the designer because such inputs consist of ramp functions and a step function. PI control action gives the optimum form for a ramp input, but not for a step input. In Section 8.7 we will see a control action called PDF that gives the optimal form for a step input, but not for a ramp.

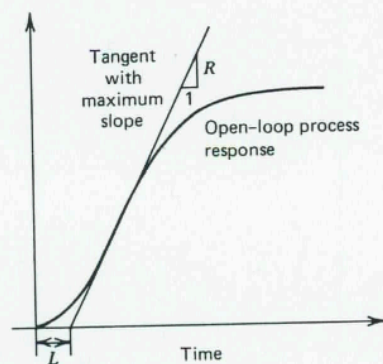
As we have seen, the choice of a performance index is somewhat subjective. Thus the performance of different designs should be compared using the same performance index for each. Although the ITAE index is perhaps the most widely used, there are others in use. Table 8.1-3 gives the optimal forms that give the so-called "minimum time solution" for a step input ([Graham, 1953]). Systems designed with this index tend to approach the steady-state response more quickly than with the ITAE index, but are more oscillatory. Note that the first and second order forms are identical to the ITAE forms.

Note that the closed-form solution for the response is not required to evaluate a performance index. For a given set of parameter values, the response and resulting index value can be computed numerically using widely available numerical methods for solving differential equations, such as those available in MATLAB.

570 Chapter 8 Control System Design: Tuning, Compensation, and Digital Control

Table 8.1-3 Minimum Time Forms for Step Response

$\frac{\omega_n}{s + \omega_n}$
$\frac{\omega_n^2}{s^2 + 1.4\omega_n s + \omega_n^2}$
$\frac{\omega_n^3}{s^3 + 1.55\omega_n s^2 + 2.1\omega_n^2 s + \omega_n^3}$
$\frac{\omega_n^4}{s^4 + 1.6\omega_n s^3 + 3.15\omega_n^2 s^2 + 2.45\omega_n^3 s + \omega_n^4}$

**Figure 8.1-1** Process signature for a unitstep input. The parameters R and L are determined for use with the process-reaction method of Ziegler and Nichols.**The Ziegler-Nichols Rules**

The difficulty of obtaining accurate transfer function models for some processes has led to the development of empirically based rules of thumb for computing the optimum gain values for a controller. Perhaps the most commonly used guidelines are the Ziegler-Nichols rules, which have proved so helpful that they are still in use 60 years after their development ([Ziegler, 1942], [Ziegler, 1943]). Other guidelines have been developed; see for example [Seborg, 1989]. The Ziegler-Nichols rules actually consist of two separate methods. The first method requires the open-loop step response of the plant, while the second uses the results of experiments performed with the controller already installed. While primarily intended for use with systems for which no analytical model is available, the rules are also helpful sometimes even when a model can be developed.

Ziegler and Nichols developed their rules from experiments and by analyzing various industrial processes. Using the IAE criterion with a unit step response, they found that controllers adjusted according to the following rules usually had a step response that was oscillatory but with enough damping so that the second overshoot was less than 25% of the first (peak) overshoot. This is the *quarter-decay criterion*, and it is sometimes used as a specification. The first method, the *process reaction method*, relies on the fact that many processes have an open-loop step response like that shown in Figure 8.1-1. This process signature is characterized by two parameters, R and L . The constant R is the slope of a line tangent to the steepest part of the response curve, and the constant L is the time at which this line intersects the time axis. First- and second-order linear systems do not yield positive values for L , and so the method cannot be applied to such systems. However, third-order and higher linear systems with sufficient damping do yield such a response. If so, the

8.1 Performance Indices and Controller Tuning 571

Table 8.1-4 The Ziegler-Nichols Rules

Controller Transfer Function: $G(s) = K_p + \frac{K_I}{s} + K_D s = K_p \left(1 + \frac{1}{T_I s} + T_D s \right)$		
T_I = Reset Time; T_D = Derivative Time		
Control Mode	Process Reaction Method	Ultimate Cycle Method
P control	$K_p = 1/RL$	$K_p = 0.5K_{pu}$
PI control	$K_p = 0.9/RL$	$K_p = 0.45K_{pu}$
	$T_I = 3.3L$	$T_I = 0.83P_u$
PID control	$K_p = 1.2/RL$	$K_p = 0.6K_{pu}$
	$T_I = 2L$	$T_I = 0.5P_u$
	$T_D = 0.5L$	$T_D = 0.125P_u$

Ziegler-Nichols rules recommend the controller settings given in Table 8.1-1. Note that the rules express the gains in terms of the *reset time* T_I and the *derivative time* T_D . The relation between the two forms of the PID control law is

$$G(s) = K_p + \frac{K_I}{s} + K_D s = K_p \left(1 + \frac{1}{T_I s} + T_D s \right) \quad (8.1-8)$$

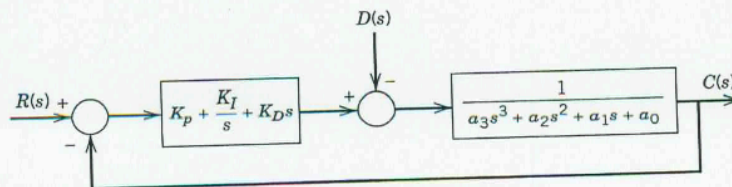
Thus $K_I = K_p/T_I$ and $K_D = K_p T_D$. Many industrial controllers require you to enter values for K_p , T_I , and T_D , rather than values for K_I and K_D . Some controllers require the value of $1/T_I$, which is called the *reset rate*.

The *ultimate-cycle method* uses experiments with the controller in place. All control modes except proportional are turned off, and the process is started with the proportional gain K_p set at a low value. The gain is slowly increased until the process begins to exhibit sustained oscillations. Denote the period of these oscillations by P_u , and the corresponding ultimate gain by K_{pu} . The Ziegler-Nichols recommendations are given in Table 8.1-4 in terms of these parameters. The proportional gain is lower for PI control than for P control and higher for PID control, because I action increases the order of the system and thus tends to destabilize it; thus, a lower gain is needed. On the other hand, D action tends to stabilize the system; hence, the proportional gain can be increased without degrading the stability characteristics. Because rules were developed for a typical case out of many types of processes, final tuning of the gains by simulation or in the field is usually necessary.

Sometimes the Ziegler-Nichols rules can be applied to an analytical model. The following example shows how this is done with the ultimate-cycle method.

EXAMPLE 8.1-3 Ultimate-Cycle Method for a Third-Order Plant

Figure 8.1-2(a) shows a PID control system for a third-order plant. Because of the I action, its closed-loop transfer function has a fourth-order denominator. Without

**Figure 8.1-2** PID control of a third-order plant.

572 Chapter 8 Control System Design: Tuning, Compensation, and Digital Control

I action, the closed loop system is third-order. In either case, none of our methods thus far can be used to find the PID gains because these methods are limited to first and second-order systems. Use the ultimate-cycle method to find the PID gains.

SOLUTION

The ultimate-cycle method starts by using proportional action. The characteristic equation of the closed-loop system shown in Figure 8.1-2, using only P action, can be found by setting $K_I = K_D = 0$. It is

$$a_3s^3 + a_2s^2 + a_1s + a_0 + K_p = 0 \quad (8.1-9)$$

where K_p is the proportional gain. To apply the ultimate-cycle method, we must first find the ultimate period P_u and the associated gain K_{pu} . To do this analytically, we note that sustained oscillations occur when a pair of roots is purely imaginary and the rest of the roots have negative real parts. To find when this occurs, set $s = i\omega_u$, where ω_u is the as-yet-unknown frequency of oscillation. Then $s^2 = -\omega_u^2$ and $s^3 = -i\omega_u^3$. Substituting these into (8.1-9) and collecting real and imaginary parts, we obtain

$$(a_0 + K_{pu} - a_2\omega_u^2) + i(a_1\omega_u - a_3\omega_u^3) = 0$$

Both the real and imaginary terms must be zero, so we obtain two equations to solve for the two unknowns ω_u and K_{pu} . These are

$$a_0 + K_{pu} - a_2\omega_u^2 = 0 \quad (8.1-10)$$

$$\omega_u(a_1 - a_3\omega_u^2) = 0 \quad (8.1-11)$$

The latter equation yields the nonoscillatory solution $\omega_u = 0$ as well as the one of interest—namely,

$$\omega_u = \pm \sqrt{\frac{a_1}{a_3}} \quad (8.1-12)$$

Substituting this into (8.1-10), we see that the ultimate gain is given by

$$K_{pu} = a_2\omega_u^2 - a_0 = \frac{a_2a_1}{a_3} - a_0 \quad (8.1-13)$$

The ultimate period is

$$P_u = \frac{2\pi}{\omega_u} \quad (8.1-14)$$

In order for sustained oscillations to occur, the third root must be negative. The first two roots are $s_1 = i\omega_u$ and $s_2 = -i\omega_u$. The third root can be determined by multiplying the three factors and comparing with the polynomial (8.1-9) after the cubic coefficient has been normalized to unity. This gives

$$(s - i\omega_u)(s + i\omega_u)(s - s_3) = s^3 - s_3s^2 + \omega_u^2s - s_3\omega_u^2 = 0$$

Dividing (8.1-9) by a_3 gives

$$s^3 + \frac{a_2}{a_3}s^2 + \frac{a_1}{a_3}s + \frac{a_0}{a_3} + \frac{K_p}{a_3} = 0$$

Comparing the two polynomials, we immediately see that $s_3 = -a_2/a_3$. If $s_3 < 0$, the values given by (8.1-13) and (8.1-14) can be used with Table 8.1-4 to compute the optimum PID gains. Note that this analysis uses only proportional control, but the gains

8.1 Performance Indices and Controller Tuning 573

for other control laws can be computed with these results. The power of the method is shown by the fact that we did not have to solve for the roots of the cubic (8.1-9). Note also that I action applied to this system would result in a fourth-order characteristic equation to deal with were it not for the Ziegler-Nichols rules.

The method used in the previous example to find K_{pu} can be used for any system to find the value of K_p that results in instability. Another convenient result of this example is the result for the third root s_3 . In general, for the polynomial

$$s^n + a_{n-1}s^{n-1} + \cdots + a_1s + a_0 = 0 \quad (8.1-15)$$

the sum of the roots equals $-a_{n-1}$. This fact is frequently very useful for locating the remaining roots once the dominant roots have been selected.

EXAMPLE 8.1-4 Control of a Third-Order Plant

Use the ultimate-cycle method to design P, PI, and PID controllers for the following plant. Compare the step responses for each design.

$$G_p(s) = \frac{1}{(s+1)(s+2)(s+3)}$$

SOLUTION

Proportional control applied to this plant gives a characteristic equation of the form in (8.1-9), where $a_3 = 1$, $a_2 = 6$, $a_1 = 11$, and $a_0 = 6$. From the results in Example 8.1-3, we have

$$\begin{aligned} K_{pu} &= 60 \\ \omega_u &= \sqrt{11} = 3.317 \\ P_u &= \frac{2\pi}{3.317} = 1.89 \end{aligned}$$

For P control, Table 8.1-4 gives $K_p = 0.5(60) = 30$. For PI control

$$\begin{aligned} K_p &= 0.45(60) = 27 \\ T_I &= 0.83(1.89) = 1.57 \\ K_I &= \frac{K_p}{T_I} = 17.18 \end{aligned}$$

Similarly, for PID control

$$\begin{aligned} K_p &= 0.6(60) = 36 \\ T_I &= 0.5(1.89) = 0.947 \\ T_D &= 0.125(1.89) = 0.236 \\ K_I &= 36/0.947 = 38.1 \\ K_D &= K_p T_D = 36(0.236) = 8.52 \end{aligned}$$

The closed-loop transfer function for P control is

$$T(s) = \frac{30}{s^3 + 6s^2 + 11s + 36}$$

574 Chapter 8 Control System Design: Tuning, Compensation, and Digital Control

For PI control,

$$T(s) = \frac{27s + 17.18}{s^4 + 6s^3 + 11s^2 + 33s + 17.18}$$

For PID control,

$$T(s) = \frac{8.52s^2 + 36s + 38.1}{s^4 + 6s^3 + 19.52s^2 + 42s + 38.1}$$

The unit step responses are shown in Figure 8.1-3. The steady state error is 0.1667 for P control and zero for PI and PID. Note the improvement as the control law is changed from P to PI to PID. First, the steady state error is eliminated, and then the oscillations and settling time are reduced.

The Ziegler-Nichols rules are used to obtain gain values that are “in the ballpark.” The engineer can use these values as a starting point for tuning the gains to obtain improved performance.

EXAMPLE 8.1-5 Tuning a PID Controller

Consider the PID controller designed in Example 8.1-4. Apply tuning to see if you can reduce the overshoot.

SOLUTION

When the controller transfer function is expressed in the form

$$G(s) = K_p \left(1 + \frac{1}{T_I s} + T_D s \right)$$

we can see that changing the proportional gain K_p changes the proportional, integral, and derivative gains by the same factor. Because high overshoots are associated with high gains, let us reduce all three gains by decreasing K_p . The value computed in Example 8.1-4 was $K_p = 36$. Arbitrarily trying $K_p = 5$, we obtain the step response shown in Figure 8.1-4. The overshoot has been decreased quite a bit without affecting the settling time very much. You might be able to obtain a better response by trying different values of K_p , T_D , and T_I .

8.2 SENSITIVITY AND ROBUST CONTROLLER DESIGN

Consider the function $y = f(x)$. If the parameter x changes by an amount Δx , then y changes by the amount Δy . If Δx is small, Δy can be estimated from the slope dy/dx as follows.

$$\Delta y = \frac{dy}{dx} \Delta x$$

The *relative* or percent change in y is $\Delta y/y$. It is related to the relative change in x as follows.

$$\frac{\Delta y}{y} = \frac{dy}{dx} \frac{\Delta x}{y}$$

8.2 Sensitivity and Robust Controller Design 575

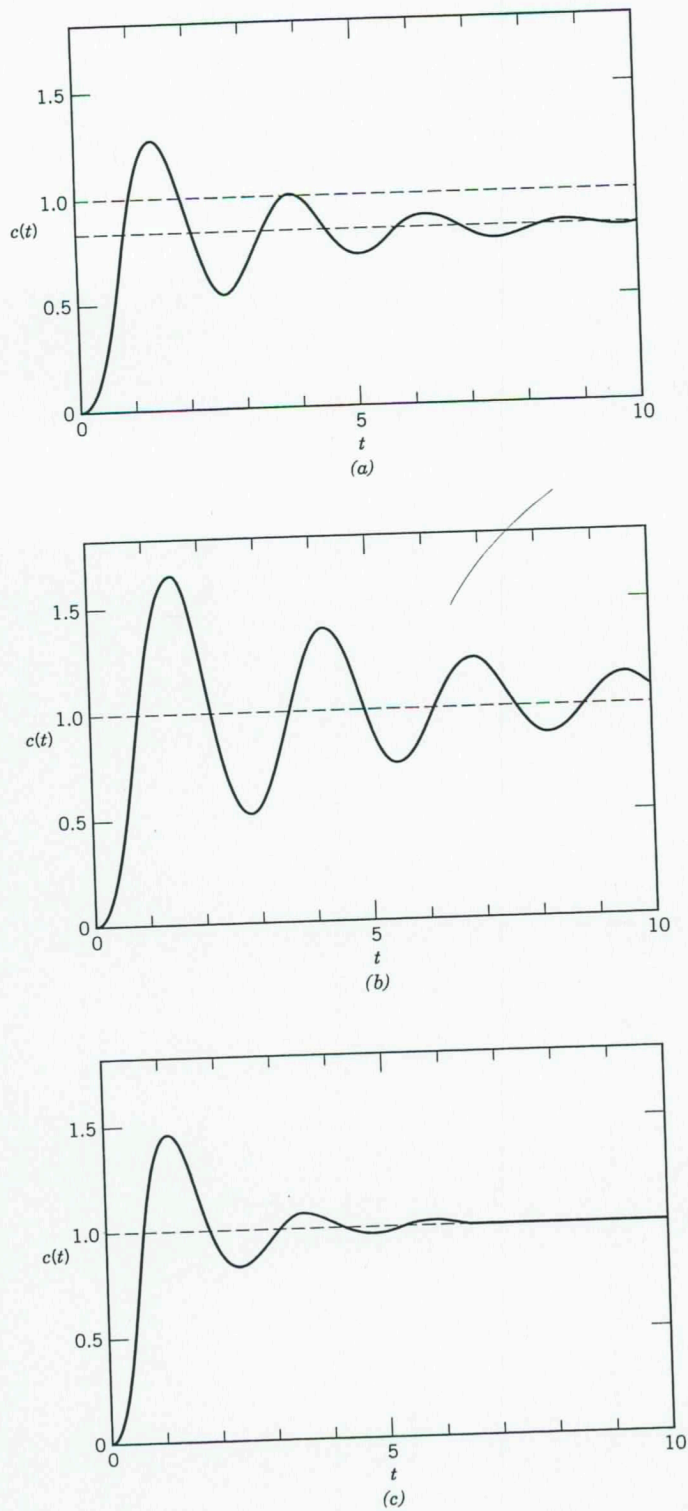


Figure 8.1-3 Unit step responses of the control systems designed in Example 8.1-3. (a) P-control. (b) PI-control (c) PID-control.