10–10 TUNING RULES FOR PID CONTROLLERS

It is interesting to note that more than half of the industrial controllers in use today utilize PID or modified PID control schemes. Analog PID controllers are mostly hydraulic, pneumatic, electric, and electronic types or their combinations. Currently, many of these are transformed into digital forms through the use of microprocessors.

Because most PID controllers are adjusted on-site, many different types of tuning rules have been proposed in the literature. With these tuning rules, PID controllers can be delicately and finely tuned. Also, automatic tuning methods have been developed, and some PID controllers may possess on-line automatic tuning capabilities. Many practical methods for bumpless switching (from manual operation to automatic operation) and gain scheduling are commercially available.

The usefulness of PID controls lies in their general applicability to most control systems. In the field of process control systems, it is a well-known fact that both basic and modified PID control schemes have proved their usefulness in providing satisfactory control, although they may not provide optimal control in many given situations.

PID control of plants. Figure 10–52 shows the PID control of a plant. If a mathematical model of the plant can be derived, then it is possible to apply various design techniques for determining the parameters of the controller that will meet the transient and steady-state specifications of the closed-loop system. However, if the plant is so complicated that its mathematical model cannot be easily obtained, then an analytical approach to the design of a PID controller is not possible. In that case, we must resort to experimental approaches to the tuning of PID controllers.

The process of selecting the controller parameters to meet given performance specifications is known as *controller tuning*. Ziegler and Nichols suggested rules for tuning PID controllers (to set values K_p, T_i, and T_d) based on experimental step responses or based on the value of K_p that results in marginal stability when only the proportional control action is used. Ziegler–Nichols rules, presented next, are convenient when mathematical models of plants are not known. (These rules can, of course, be applied to the design of systems *with* known mathematical models.)

Ziegler–Nichols rules for tuning PID controllers. Ziegler and Nichols proposed rules for determining values of the proportional gain K_p, integral time T_i, and derivative time T_d based on the transient-response characteristics of a given plant. Such determination of the parameters of PID controllers or tuning of PID controllers can be made by engineers on-site by experimenting on the plant. (Numerous tuning rules for PID controllers have been proposed since Ziegler and Nichols offered their rules. Here, however, we introduce only the Ziegler–Nichols tuning rules.)

The two methods called Ziegler–Nichols tuning rules are aimed at obtaining 25% maximum overshoot in the step response. (See Figure 10–53.)

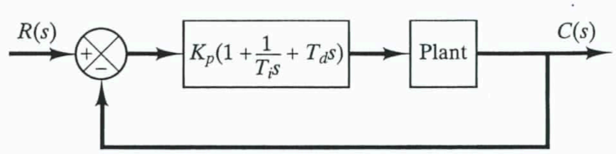

Figure 10–52 PID control of a plant.

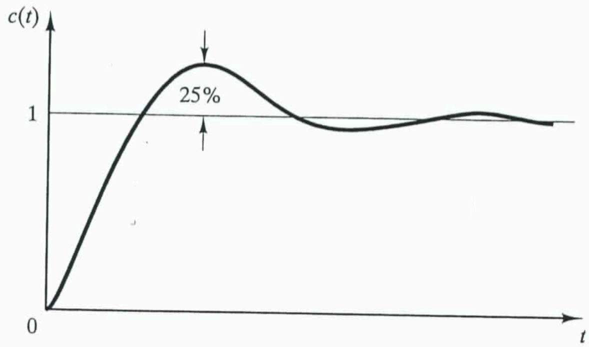

Figure 10–53 Unit-step response curve showing 25% maximum overshoot.

First method. In the first Ziegler–Nichols method, we obtain the response of the plant to a unit-step input experimentally, as shown in Figure 10–54. If the plant involves neither integrators nor dominant complex-conjugate poles, then such a unit-step response curve may look like an S-shaped curve, as shown in Figure 10–55. (If the response does not exhibit an S-shaped curve, this method does not apply.) Step-response curves of this nature may be generated experimentally or from a dynamic simulation of the plant.

The S-shaped curve may be characterized by two constants—the delay time L and a time constant T—determined by drawing a line tangent to the S-shaped curve at the inflection point. These constants are determined by the intersections of the tangent line with the time axis and the line $c(t) = K$, as shown in Figure 10–55. The transfer function $C(s)/U(s)$ may then be approximated by a first-order system with a transport lag as follows:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts + 1}$$

Ziegler and Nichols suggested setting the values of K_p, T_i, and T_d according to the formula shown in Table 10–4.

Notice that the PID controller tuned by the first Ziegler–Nichols method gives

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right) \\ &= 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s} \end{aligned}$$

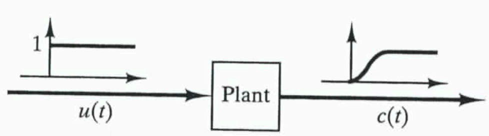

Figure 10–54 Unit-step response of a plant.

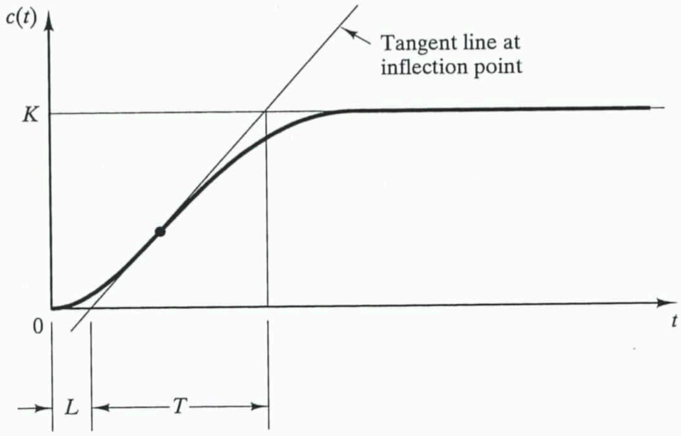

Figure 10–55 S-shaped response curve.

Thus, the PID controller has a pole at the origin and double zeros at $s = -1/L$.

Second method. In the second Ziegler–Nichols method, we first set $T_i = \infty$ and $T_d = 0$. Using the proportional control action only (see Figure 10–56), we increase K_p from 0 to a critical value K_{cr} at which the output first exhibits sustained oscillations. (If the output does not exhibit sustained oscillations for whatever value K_p may take, then this method does not apply.) Thus the critical gain K_{cr} and the corresponding period P_{cr} are determined experimentally. (See Figure 10–57.) Ziegler and Nichols suggested that we set the values of the parameters K_p, T_i, and T_d according to the formula shown in Table 10–5.

Notice that the PID controller tuned by the second Ziegler–Nichols method gives

$$
\begin{aligned}
G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\
&= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right) \\
&= 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s}
\end{aligned}
$$

TABLE 10–4 Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

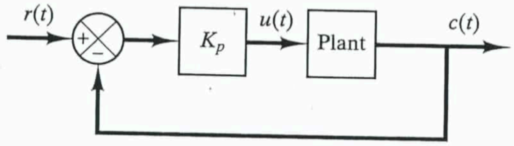

Figure 10–56 Closed-loop system with a proportional controller.

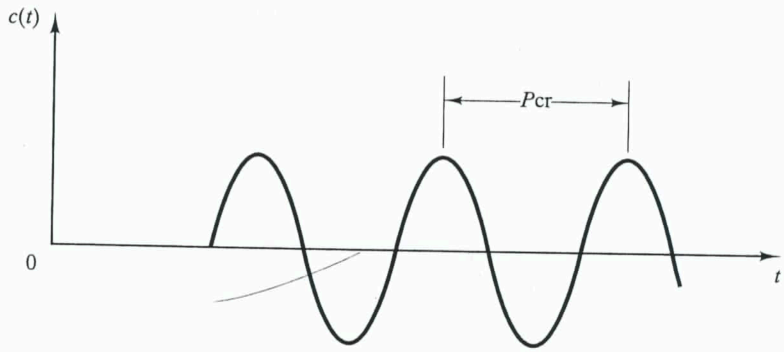

Figure 10–57 Sustained oscillation with period P_{cr}.

Thus, the PID controller has a pole at the origin and double zeros at $s = -4/P_{cr}$.

Comments. Ziegler–Nichols tuning rules (and other tuning rules presented in the literature) have been widely used to tune PID controllers in process control systems for which the plant dynamics are not precisely known. Over many years, such tuning rules have proved to be highly useful. Ziegler–Nichols tuning rules can, of course, be applied to plants whose dynamics are known. (If plant dynamics are known, many analytical and graphical approaches to the design of PID controllers are available in addition to Ziegler–Nichols tuning rules.)

If the transfer function of the plant is known, a unit-step response may be calculated or the critical gain K_{cr} and critical period P_{cr} may be calculated. Then, with those calculated values, it is possible to determine the parameters K_p, T_i, and T_d from Table 10–4 or Table 10–5. However, the real usefulness of Ziegler–Nichols tuning rules (and other tuning rules) becomes apparent when the plant dynamics are *not* known, so that no analytical or graphical approaches to the design of controllers are available.

TABLE 10–5 Ziegler–Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Generally, for plants with complicated dynamics, but no integrators, Ziegler–Nichols tuning rules can be applied. If, however, the plant has an integrator, the rules may not be applicable in some cases, either because the plant does not exhibit the S-shaped response or because the plant does not exhibit sustained oscillations no matter what gain K is chosen.

If the plant is such that Ziegler–Nichols rules can be applied, then the plant with a PID controller tuned by such rules will exhibit approximately 10% to 60% maximum overshoot in step response. On the average (obtained by experimenting on many different plants), the maximum overshoot is approximately 25%. (This is quite understandable, because the values suggested in Tables 10–4 and 10–5 are based on the average.) In a given case, if the maximum overshoot is excessive, it is always possible (experimentally or otherwise) to fine-tune the closed-loop system so that it will exhibit satisfactory transient responses. In fact, Ziegler–Nichols tuning rules give an educated guess for the parameter values and provide a starting point for fine-tuning.

Example 10–11

Consider the control system shown in Figure 10–58, in which a PID controller is used to control the system. The PID controller has the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Although many analytical methods are available for the design of a PID controller for this system, let us apply a Ziegler–Nichols tuning rule for the determination of the values of parameters K_p, T_i, and T_d. Then we will obtain a unit-step response curve and check whether the designed system exhibits approximately 25% maximum overshoot. If the maximum overshoot is excessive (40% or more), fine-tune the system and reduce the amount of the maximum overshoot to approximately 25%.

Since the plant has an integrator, we use the second Ziegler–Nichols method. Setting $T_i = \infty$ and $T_d = 0$, we obtain the closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{K_p}{s(s+1)(s+5) + K_p}$$

The value of K_p that makes the system marginally stable so that sustained oscillations occur can be obtained by the use of Routh's stability criterion. Since the characteristic equation for the closed-loop system is

$$s^3 + 6s^2 + 5s + K_p = 0$$

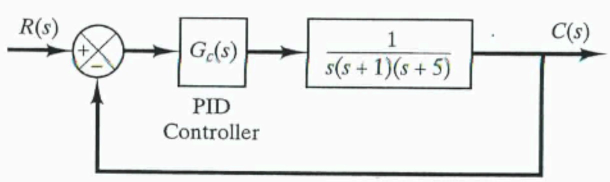

Figure 10–58 PID-controlled system.

the Routh array becomes

$$\begin{array}{lll} s^3 & 1 & 5 \\ s^2 & 6 & K_p \\ s^1 & \dfrac{30 - K_p}{6} & \\ s^0 & K_p & \end{array}$$

Examining the coefficients of the first column of the Routh array, we find that sustained oscillation will occur if $K_p = 30$. Thus, the critical gain K_{cr} is

$$K_{cr} = 30$$

With the ~~gain K_p set~~ equal to $K_{cr}(= 30)$, the characteristic equation becomes

$$s^3 + 6s^2 + 5s + 30 = 0$$

To find the frequency of the sustained oscillations, we substitute $s = j\omega$ into this characteristic equation and obtain

$$(j\omega)^3 + 6(j\omega)^2 + 5(j\omega) + 30 = 0$$

or

$$6(5 - \omega^2) + j\omega(5 - \omega^2) = 0$$

from which we find the frequency of the sustained oscillation to be $\omega^2 = 5$ or $\omega = \sqrt{5}$. Hence, the period of sustained oscillation is

$$P_{cr} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{5}} = 2.8099$$

From Table 10–5, we determine

$$K_p = 0.6K_{cr} = 18$$
$$T_i = 0.5P_{cr} = 1.405$$
$$T_d = 0.125P_{cr} = 0.35124$$

The transfer function of the PID controller is thus

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 18\left(1 + \frac{1}{1.405s} + 0.35124s\right) \\ &= \frac{6.3223(s + 1.4235)^2}{s} \end{aligned}$$

The PID controller has a pole at the origin and double zero at $s = -1.4235$. A block diagram of the control system with the designed PID controller is shown in Figure 10–59.

Next, let us examine the unit-step response of the system. The closed-loop transfer function $C(s)/R(s)$ is given by

$$\frac{C(s)}{R(s)} = \frac{6.3223s^2 + 18s + 12.811}{s^4 + 6s^3 + 11.3223s^2 + 18s + 12.811}$$

The unit-step response of this system can be obtained easily with MATLAB. (See MATLAB Program 10–7.) The resulting unit-step response curve is shown in Figure 10–60. The maximum overshoot in the unit-step response is approximately 62%, is

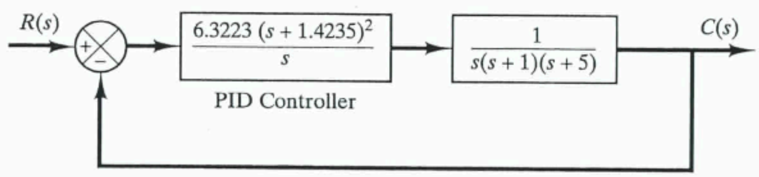

Figure 10–59 Block diagram of the system with PID controller designed by the use of the second Ziegler–Nichols tuning method.

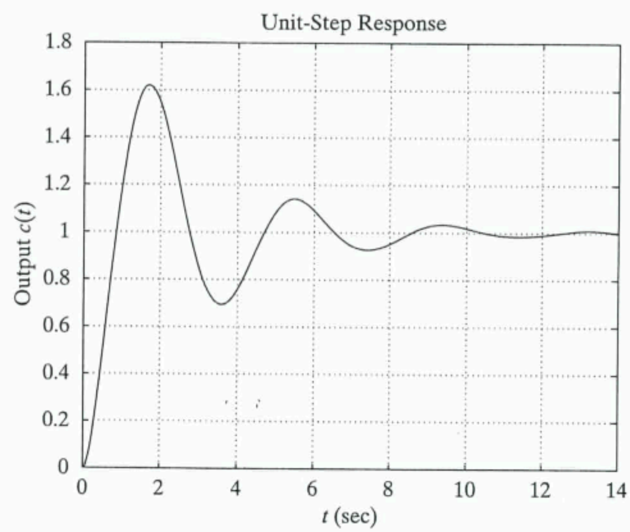

Figure 10–60 Unit-step response curve of PID-controlled system designed by the use of the second Ziegler–Nichols tuning method.

excessive. It can be reduced by fine-tuning the controller parameters, possibly on the computer. We find that, by keeping $K_p = 18$ and by moving the double zero of the PID controller to $s = -0.65$, that is, using the PID controller

$$G_c(s) = 18\left(1 + \frac{1}{3.077s} + 0.7692s\right) = 13.846\frac{(s + 0.65)^2}{s} \qquad (10\text{–}51)$$

MATLAB Program 10–7

```
>> t = 0:0.01:14;
>> num = [0   0   6.3223   18   12.811];
>> den = [1   6   11.3223   18   12.811];
>> step(num,den,t)
>> grid
>> title('Unit-Step Response')
>> xlabel('t'); ylabel('Output c(t)')
```

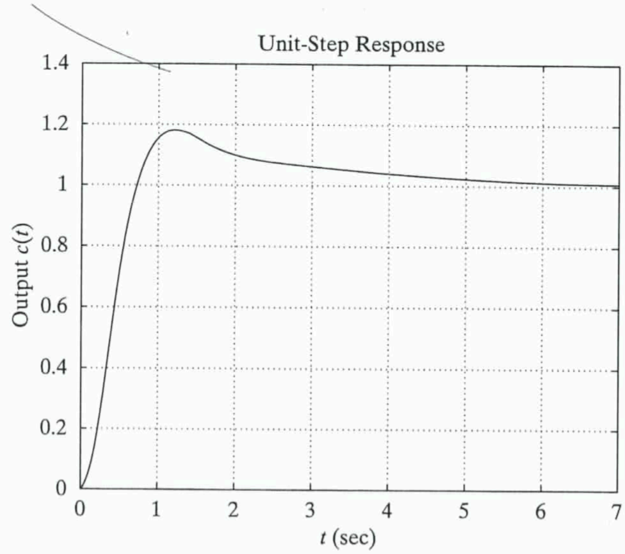

Figure 10–61 Unit-step response of the system shown in Figure 10–58 with PID controller with parameters $K_p = 18$, $T_i = 3.077$, and $T_d = 0.7692$, or $G_c(s) = 13.846(s + 0.65)^2/s$.

the maximum overshoot in the unit-step response can be reduced to approximately 18%. (See Figure 10–61.) If the proportional gain K_p is increased to 39.42, without changing the location of the double zero ($s = -0.65$), that is, if the PID controller

$$G_c(s) = 39.42\left(1 + \frac{1}{3.077s} + 0.7692s\right) = 30.322\,\frac{(s + 0.65)^2}{s} \tag{10–52}$$

is used, then the speed of response is increased, but the maximum overshoot is also increased, to approximately 28%, as shown in Figure 10–62. Since the maximum overshoot in this case is fairly close to 25% and the response is faster than the system with $G_c(s)$ given by Equation (10–51), we may consider $G_c(s)$, as given by Equation (10–52), to be acceptable. Then the tuned values of K_p, T_i, and T_d become

$$K_p = 39.42, \qquad T_i = 3.077, \qquad T_d = 0.7692$$

It is interesting to observe that these values are approximately twice the values suggested by the second Ziegler–Nichols tuning method. The important thing to note here is that the Ziegler–Nichols rule has provided a starting point for fine-tuning.

It is instructive to note that, in the case where the double zero is located at $s = -1.4235$, increasing the value of K_p increases the speed of response, but as far as the percentage maximum overshoot is concerned, varying gain K_p has very little effect. The reason for this may be seen from the root-locus analysis. Figure 10–63 shows the root-locus diagram for the system designed with the second Ziegler–Nichols tuning method. Since the dominant branches of root loci are along the $\zeta = 0.3$ lines for a considerable range of K, varying the value of K (from 6 to 30) will not change the damping ratio of the dominant closed-loop poles very much. However, varying the location of the double zero

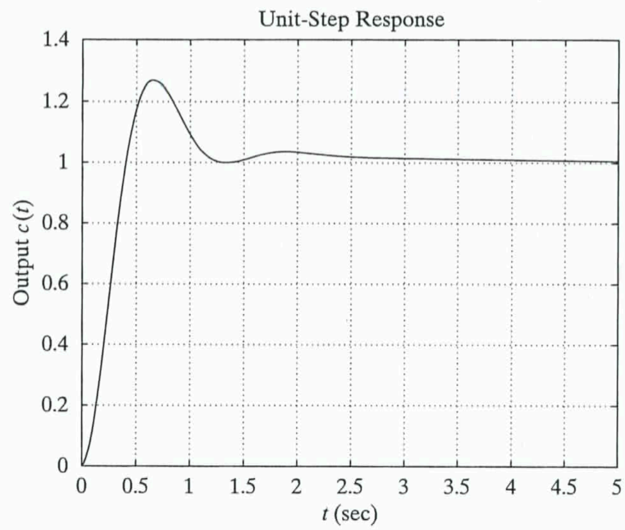

Figure 10–62 Unit-step response of the system shown in Figure 10–58 with PID controller with parameters $K_p = 39.42$, $T_i = 3.077$, and $T_d = 0.7692$, or $G_c(s) = 30.322(s + 0.65)^2/s$.

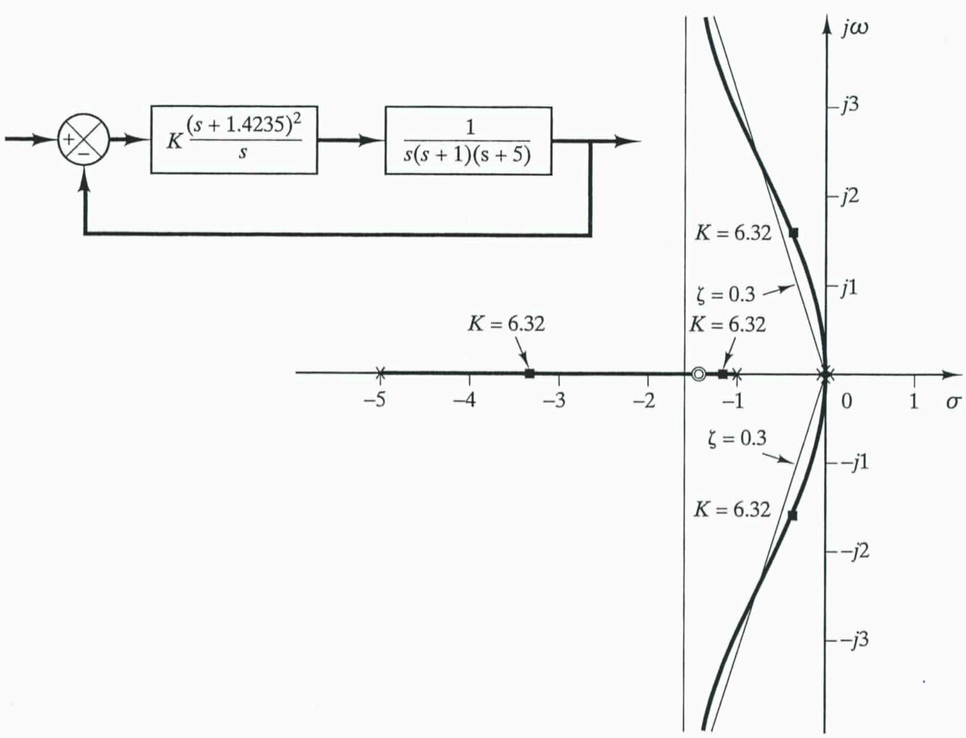

Figure 10–63 Root-locus diagram of system when PID controller has double zero at $s = -1.4235$.

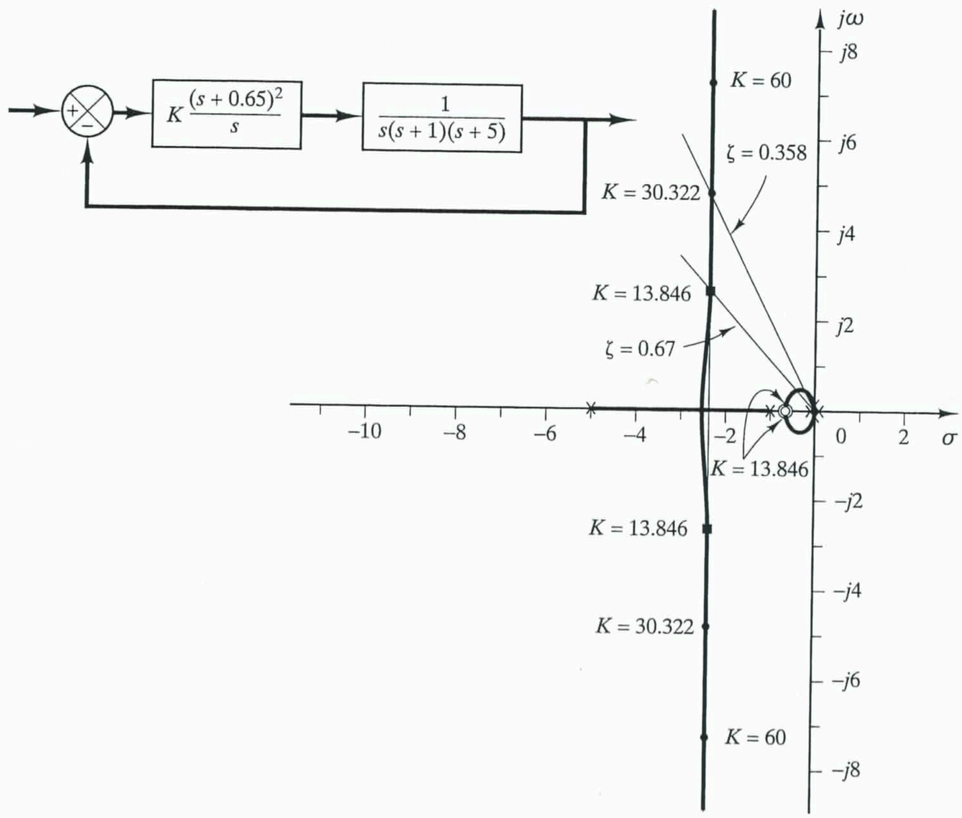

Figure 10–64 Root-locus diagram of system when PID controller has double zero at $s = -0.65$. $K = 13.846$ corresponds to $G_c(s)$ given by Equation (10–51) and $K = 30.322$ corresponds to $G_c(s)$ given by Equation (10–52).

has a considerable effect on the maximum overshoot, because the damping ratio of the dominant closed-loop poles can be changed significantly. This can also be seen from the root-locus analysis. Figure 10–64 shows the root-locus diagram for the system with PID controller with a double zero at $s = -0.65$. Notice the change of the root-locus configuration, making it possible to change the damping ratio of the dominant closed-loop poles.

In the figure, notice that, in the case where the system has gain $K = 30.322$, the closed-loop poles at $s = -2.35 \pm j4.82$ act as dominant poles. Two additional closed-loop poles are very near the double zero at $s = -0.65$, with the result that these closed-loop poles and the double zero almost cancel each other. The dominant pair of closed-loop poles indeed determines the nature of the response. By contrast, when the system has $K = 13.846$, the closed-loop poles at $s = -2.35 \pm j2.62$ are not quite dominant, because the two other closed-loop poles near the double zero at $s = -0.65$ have a considerable effect on the response. The maximum overshoot in the step response in this case (18%) is much larger than in the case of a second-order system having only dominant closed-loop poles. (In the latter case, the maximum overshoot in the step response would be approximately 6%.)