

Topics:

- Jacobian for wheeled vehicles
- Kinematic constraints
- Mobility, steerability, and maneuverability
- Holonomic constraints

Notices:

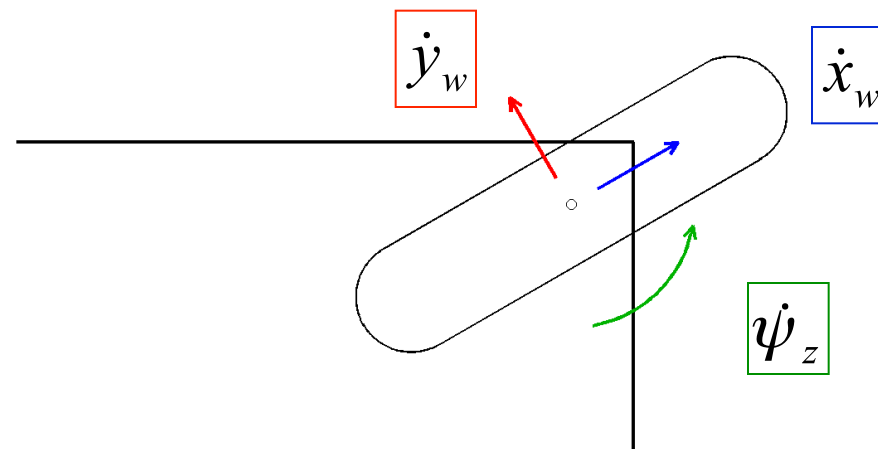
- HW 2 – due FRIDAY
- Prelab 3 and Lab 2 – due Monday, Oct. 29
- Wednesday Lab Group is now 6-9pm (not 5-8pm)

Kinematic Constraints in Rolling Machines

“It’s not the invention of the wheel that’s impressive, it’s the wheel and an axle!”

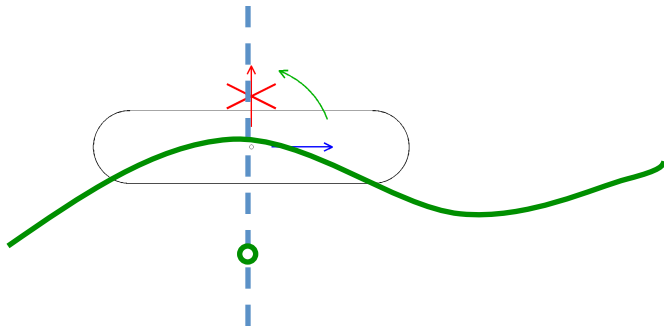
- Constraints for a standard wheel, rolling on a surface:

Direction for Instantaneous Velocity	Constraint: Allowed or Not?
$\dot{x}_w$: Orthogonal to Wheel Axis (“forward”)	ALLOWED (rolling direction)
$\dot{y}_w$: In Direction of Wheel Axis (“sideways”)	NOT ALLOWED (no lateral slip)
$\dot{\psi}_z$: Rotational, about contact point	ALLOWED (spin on contact point)



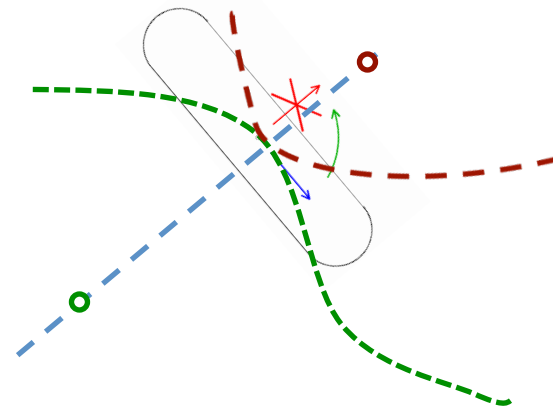
Instantaneous Center of Rotation (ICR)

- A standard wheel can both roll forward and turn:



At left:

Example instantaneous ICR
and long-term (curvy) path.



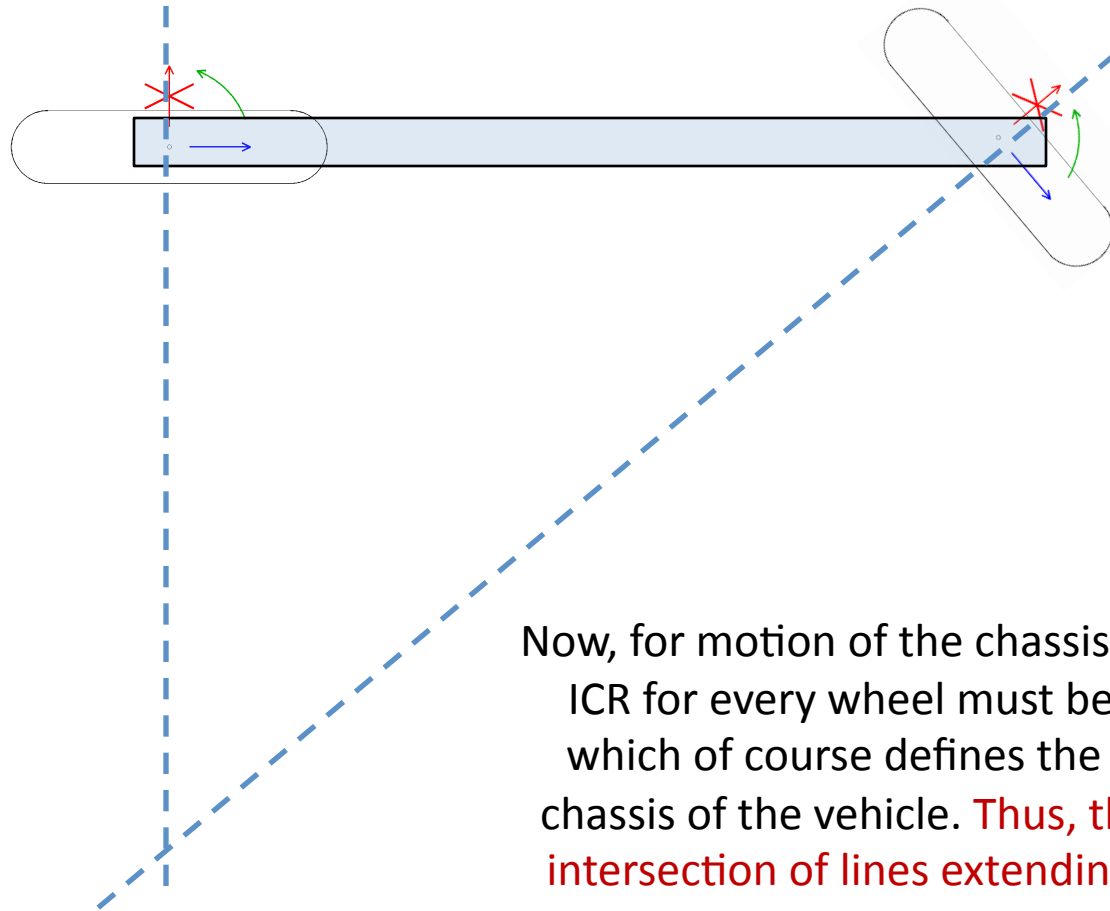
At right:

Note, the instantaneous
ICR could be to either side
of the wheel, or an infinity.

Thus, a single wheel can move to follow a path in which the **instantaneous center of rotation (ICR)** is anywhere along the line of its axis (i.e., along the “no-slip” direction).

Instantaneous Center of Rotation (ICR)

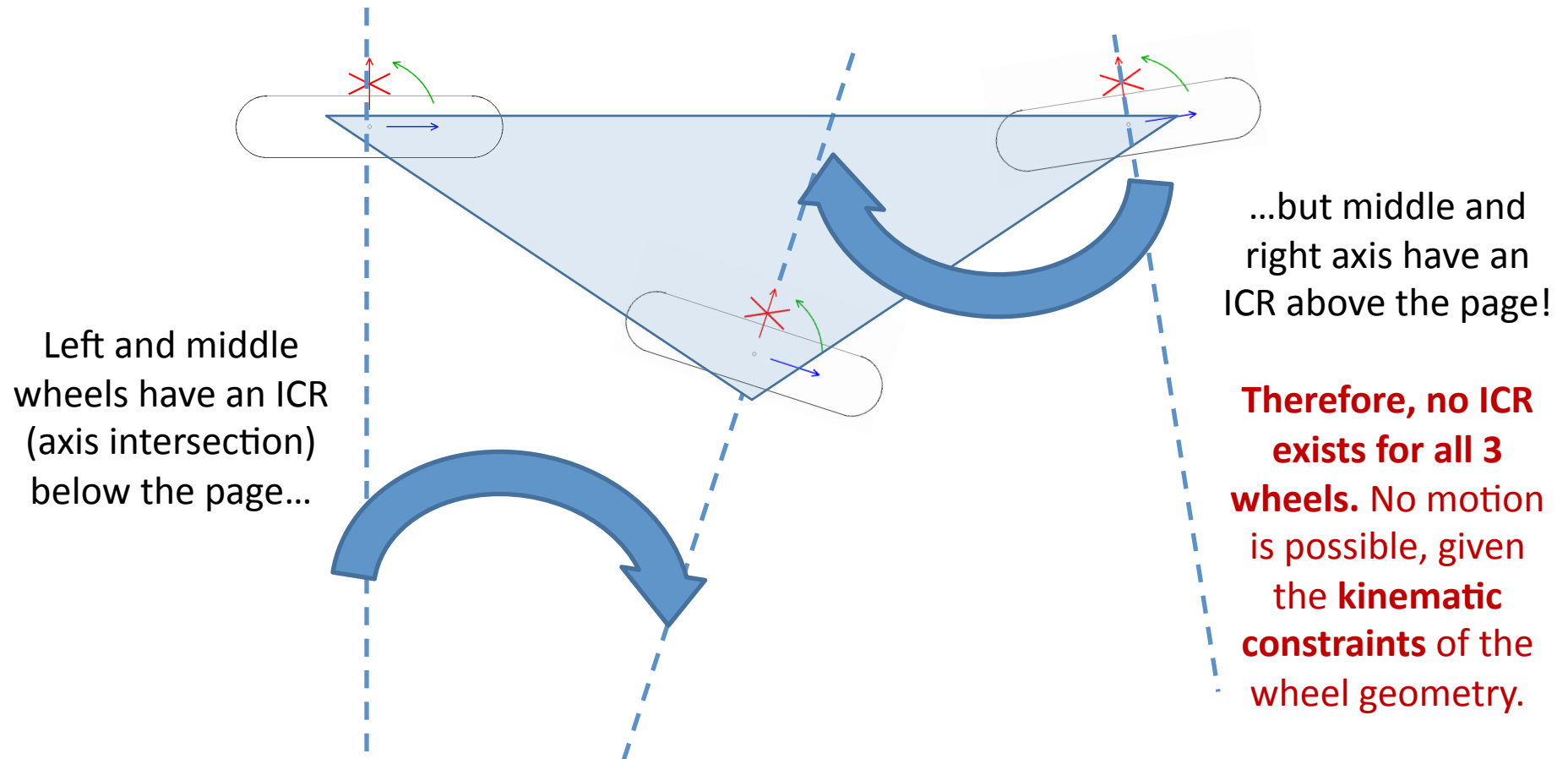
- Given two (or more) standard wheels on a single chassis:



Now, for motion of the chassis to occur, the ICR for every wheel must be the same, which of course defines the ICR for the chassis of the vehicle. **Thus, the ICR is the intersection of lines extending the wheel axes. Recall, this assumes NO WHEEL SLIP.**

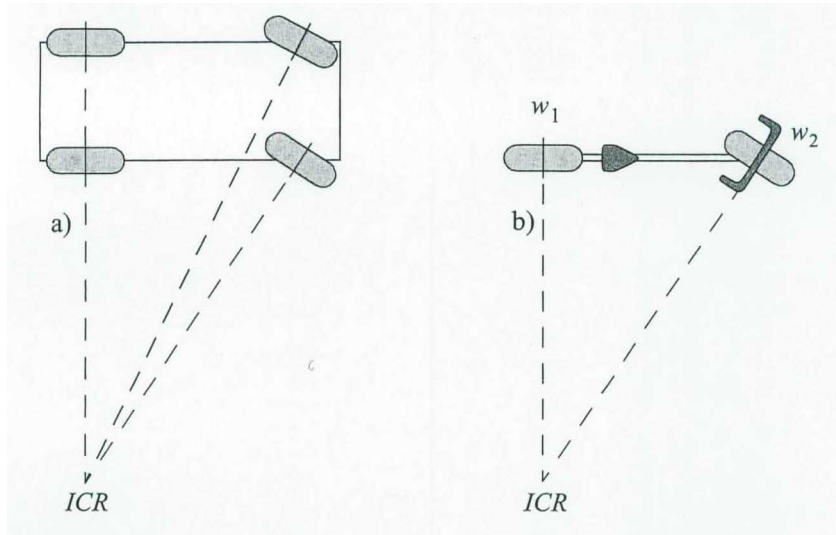
Instantaneous Center of Rotation (ICR)

- If all wheel axes do not coincide at a single point, no ICR exists, and no motion can occur without wheel slip!



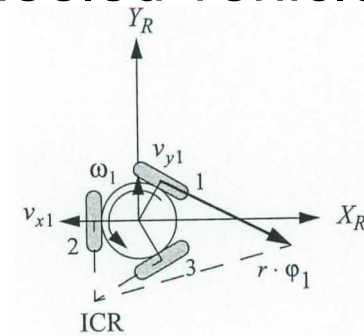
Instantaneous Center of Rotation (ICR)

- Examples for Siegwart of ICR for wheeled vehicles:



a) Four-wheel car, with Ackerman steering

b) bicycle



c) omnibot

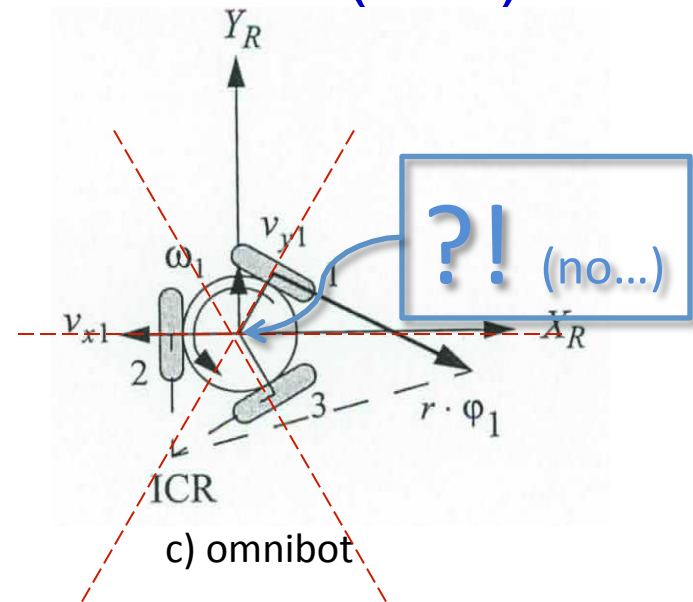


Labs 3 and 4:
Omnibot control.

(From Siegwart and Nourbakhsh, pp. 68 and 65, respectively.)

Instantaneous Center of Rotation (ICR)

- Omnibot example:



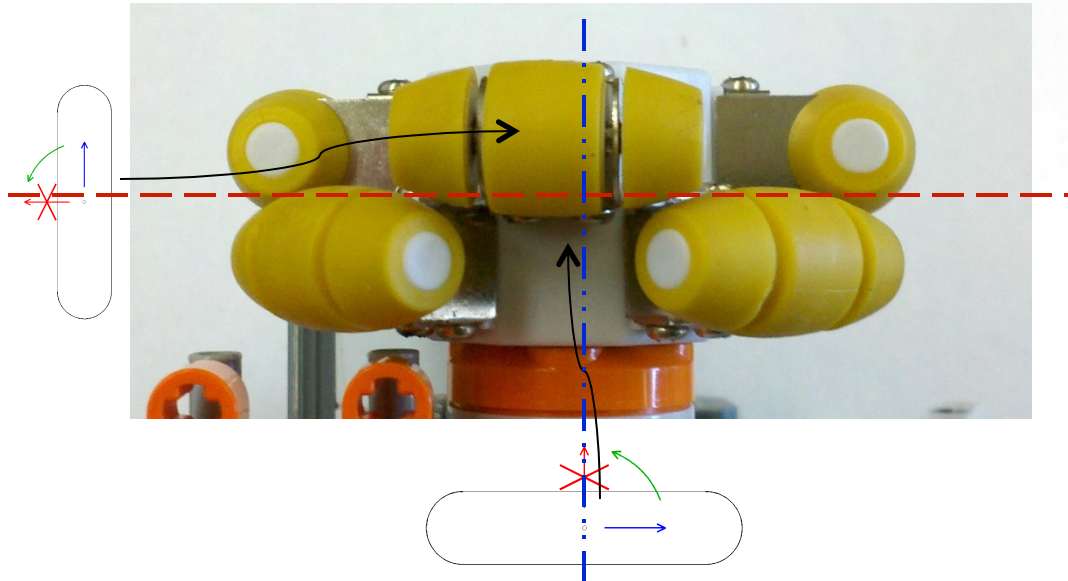
Wait a minute! Isn't the ICR always in the CENTER here?

No, because wheels 2 and 3 use their “free rollers” here, while only wheel 1 is powered.

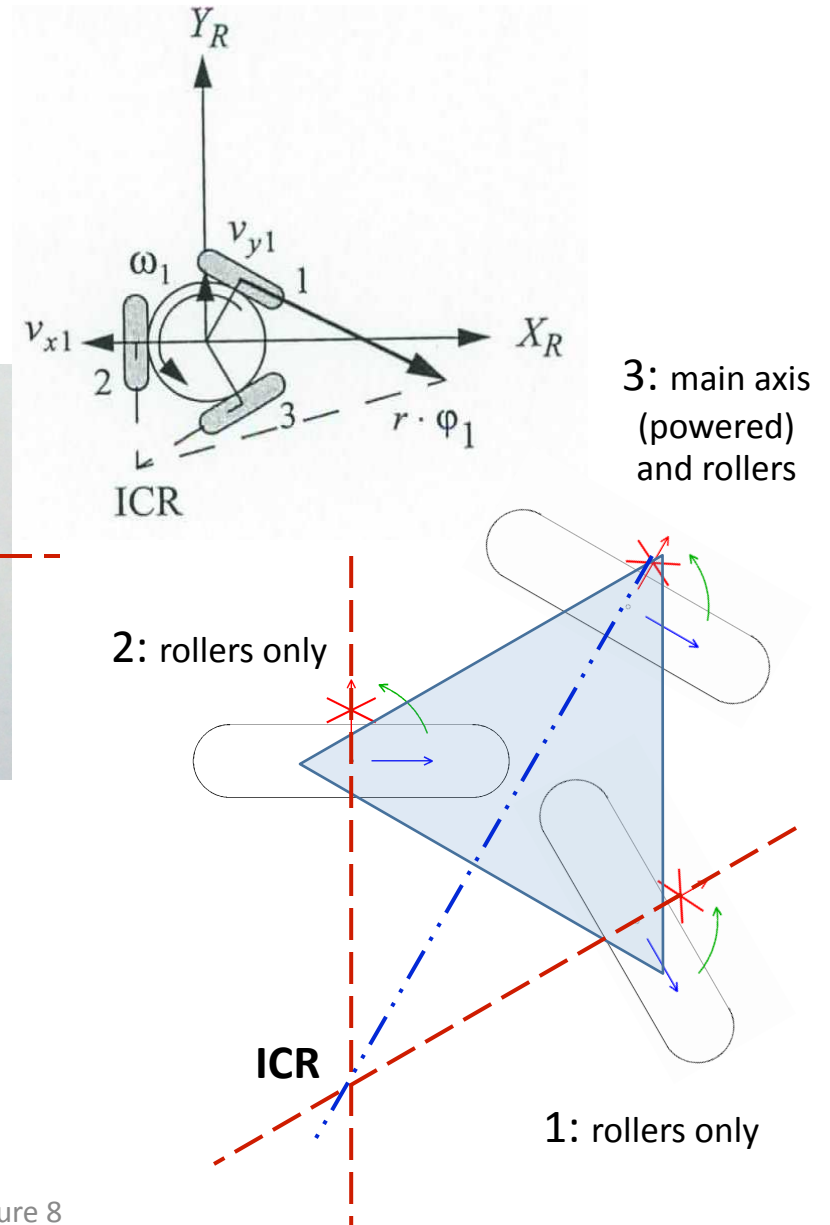
Instantaneous Center of Rotation (ICR)

- Omnibot example :

Each **ROLLER** has an axis of rotation, perpendicular to the power wheel axis



The **POWERED** axis must be controlled actively by motors (not “freely” rotating).



Wheel Types

Fives types. Two categorizations (below).

Case 1: Kinematic constraint is imposed: no lateral slip allowed.

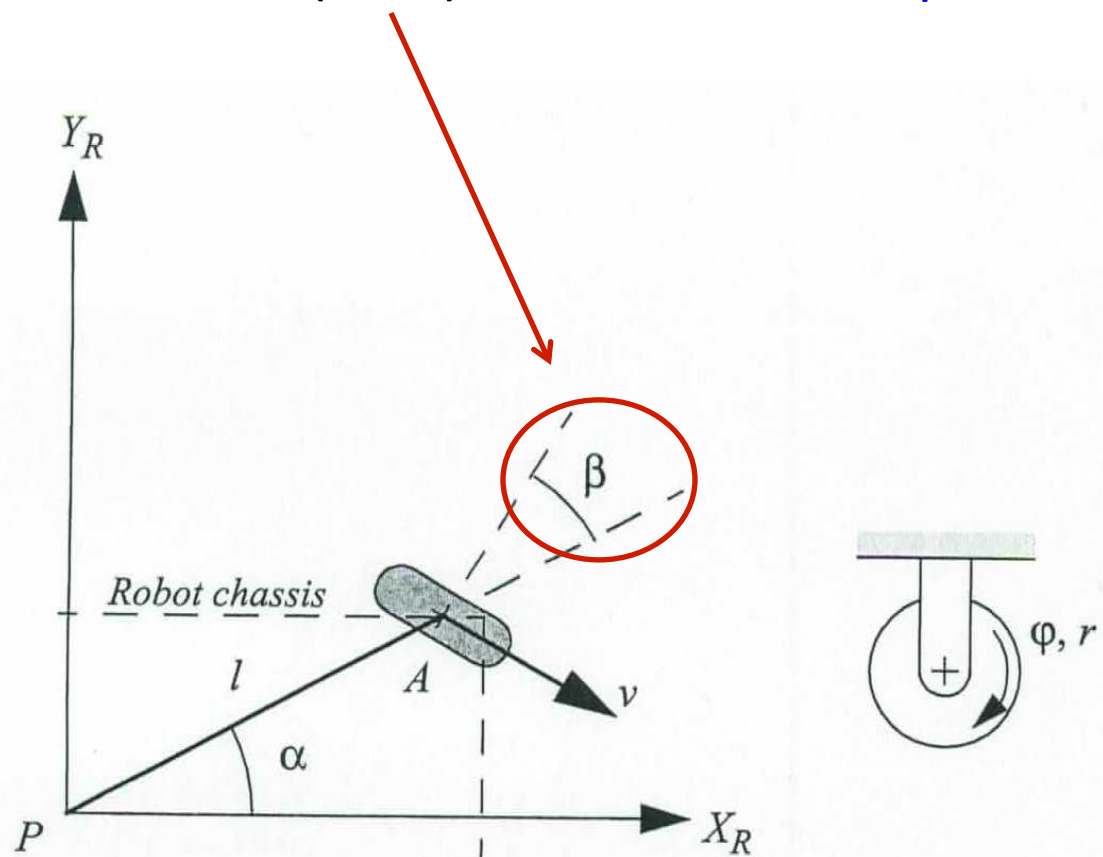
- Fixed standard wheel
- Steerable standard wheel

Case 2: No kinematic constraint, because there are additional rolling degrees of freedom in the wheel, to prevent kinematic constraints.

- Castor wheel
- Omnidirection, or “Swedish”, wheel
- Spherical wheel

Fixed standard wheel

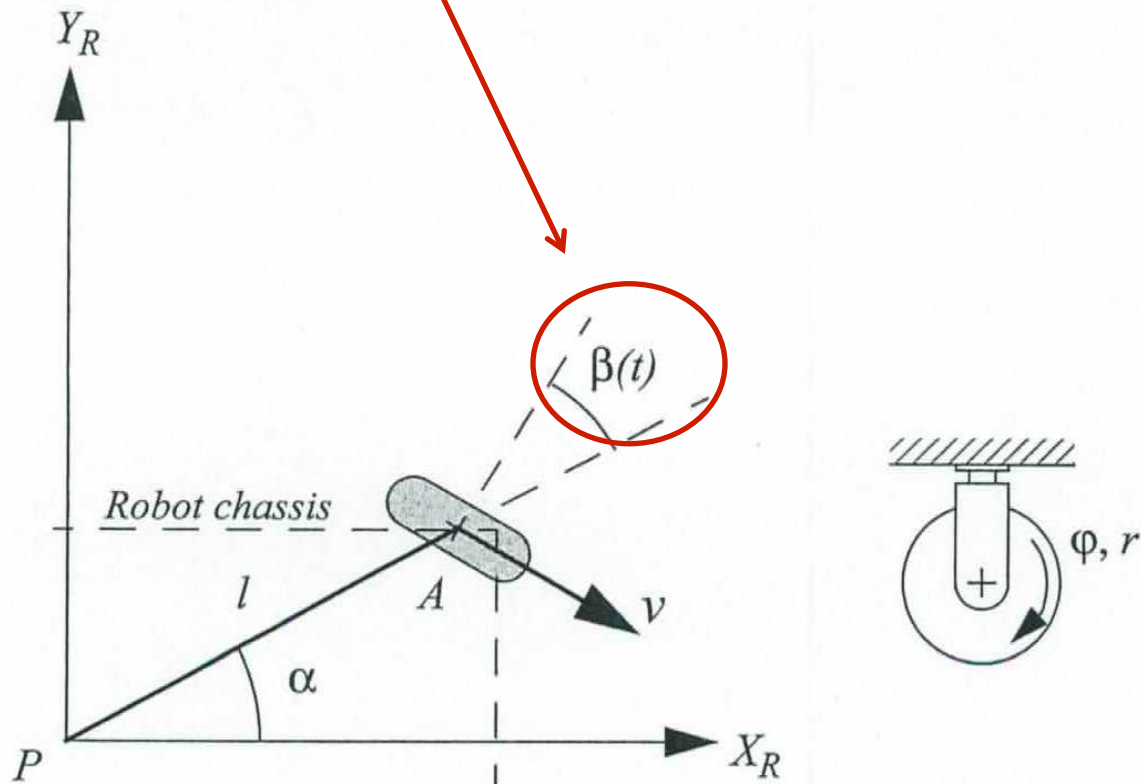
Orientation of wheel (beta) is **FIXED**, with respect to chassis.



(From Siegwart and Nourbakhsh, p. 54.)

Steerable standard wheel

Orientation of wheel (beta) can be steered over time, actively.

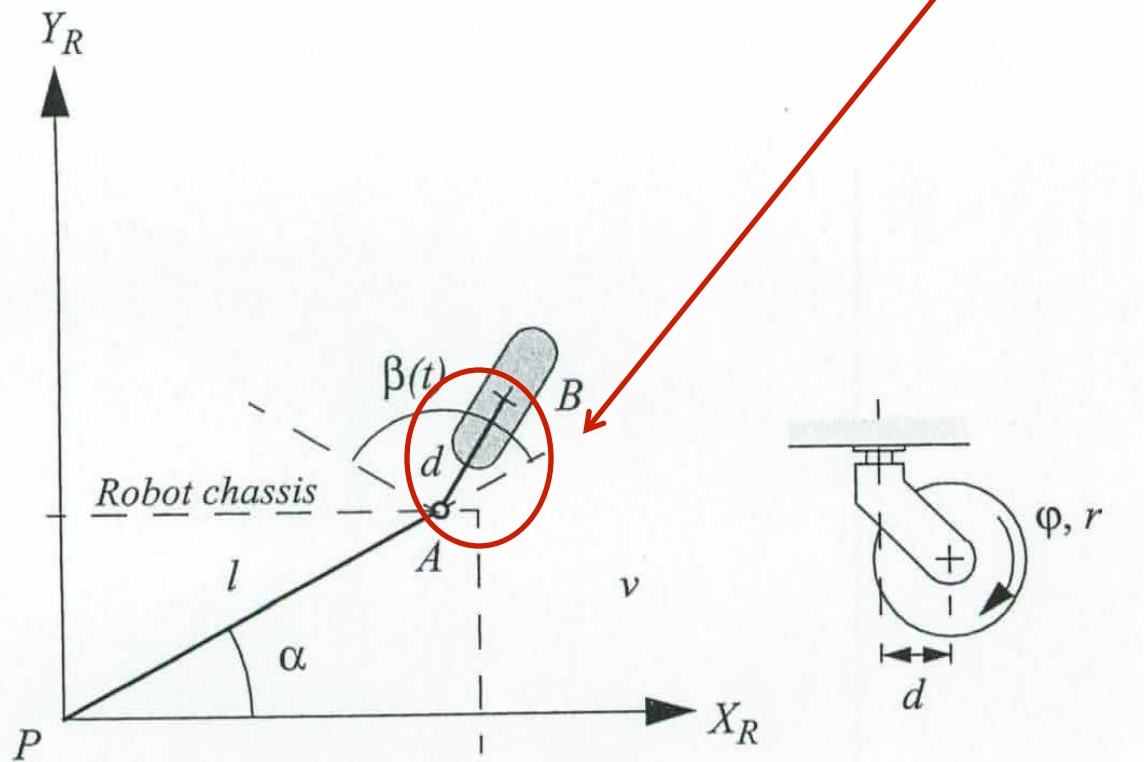


(From Siegwart and Nourbakhsh, p. 56.)

Castor wheel

Wheel can again steer wrt chassis, but **axis of rotation** does NOT pass through the ground contact point. It is **offset by distance d** .

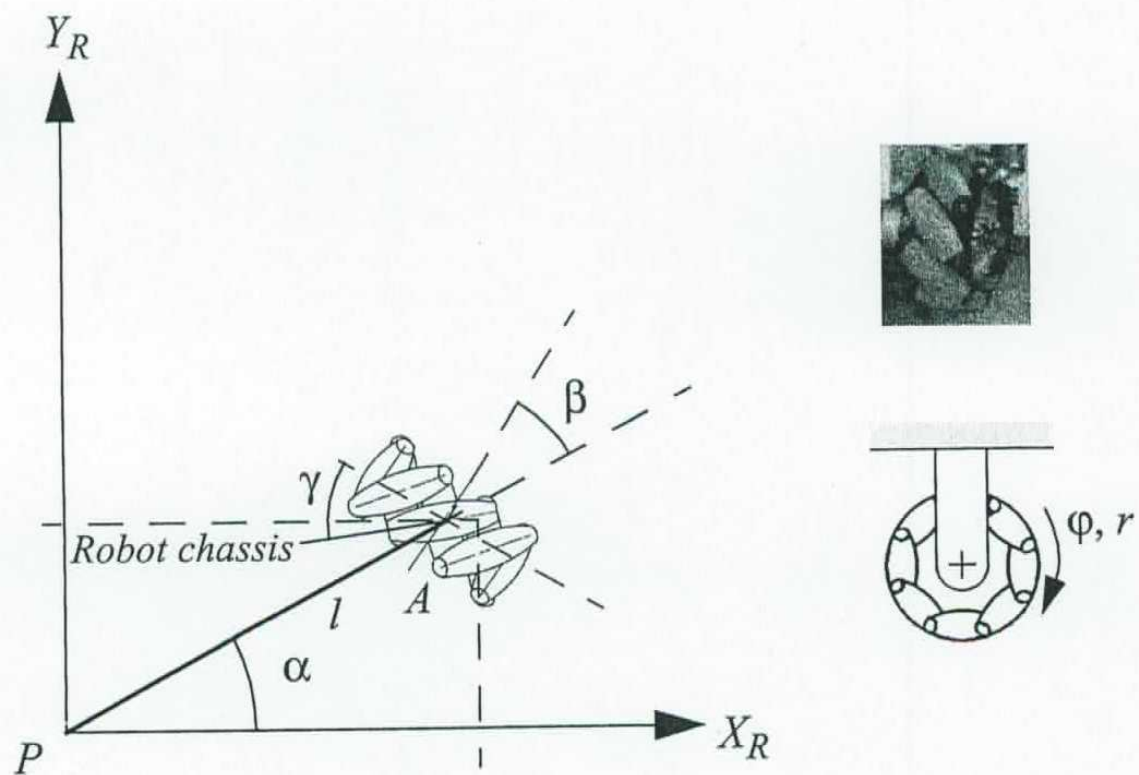
Given any $\dot{\xi}_I$, there exists some $\dot{\varphi}$ and $\dot{\beta}$ such that constraints are met.



(From Siegwart and Nourbakhsh, p. 57.)

Omnidirection wheel (Swedish wheel)

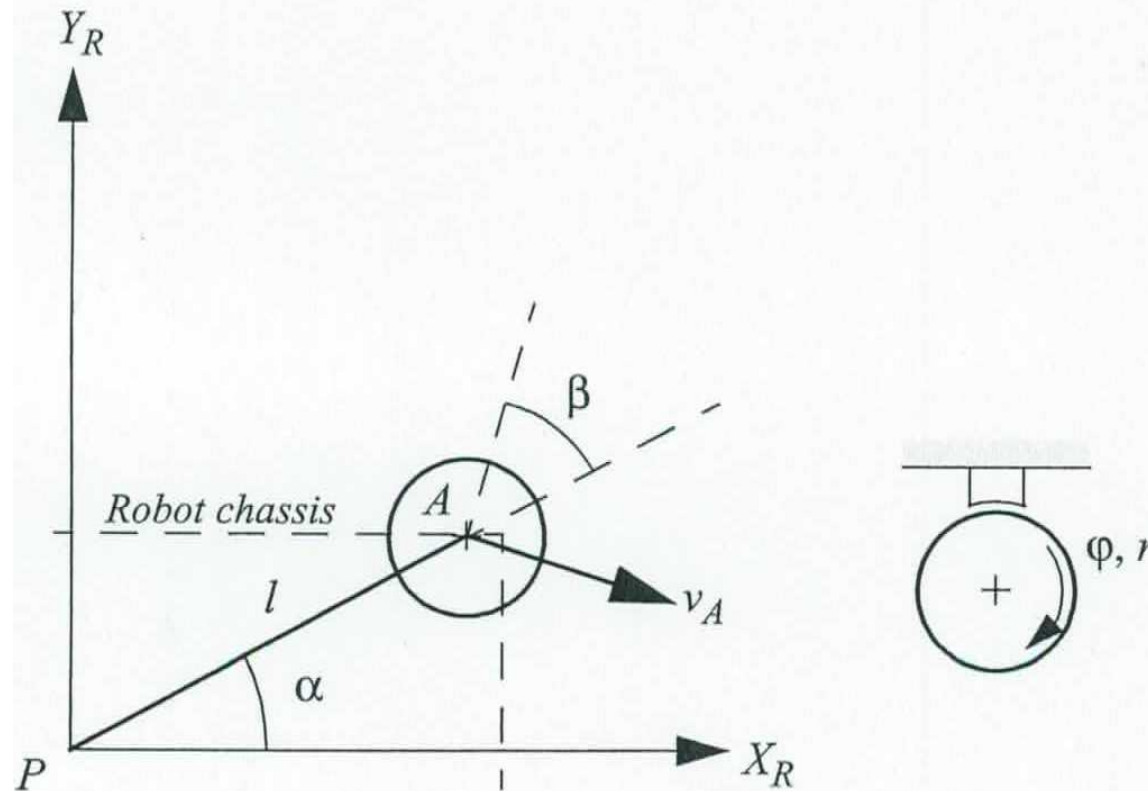
Wheel has both a powered axis and a set of rollers that can spin freely.



(From Siegwart and Nourbakhsh, p. 59.)

Spherical wheel

No constraints on motion. Allows free rotation in any direction.



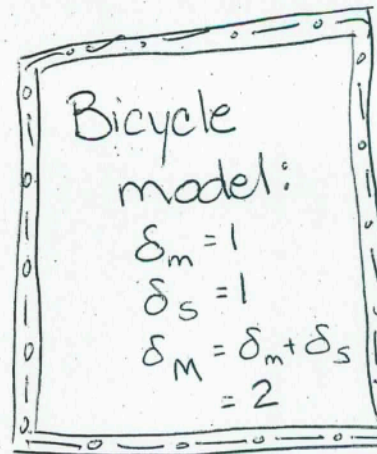
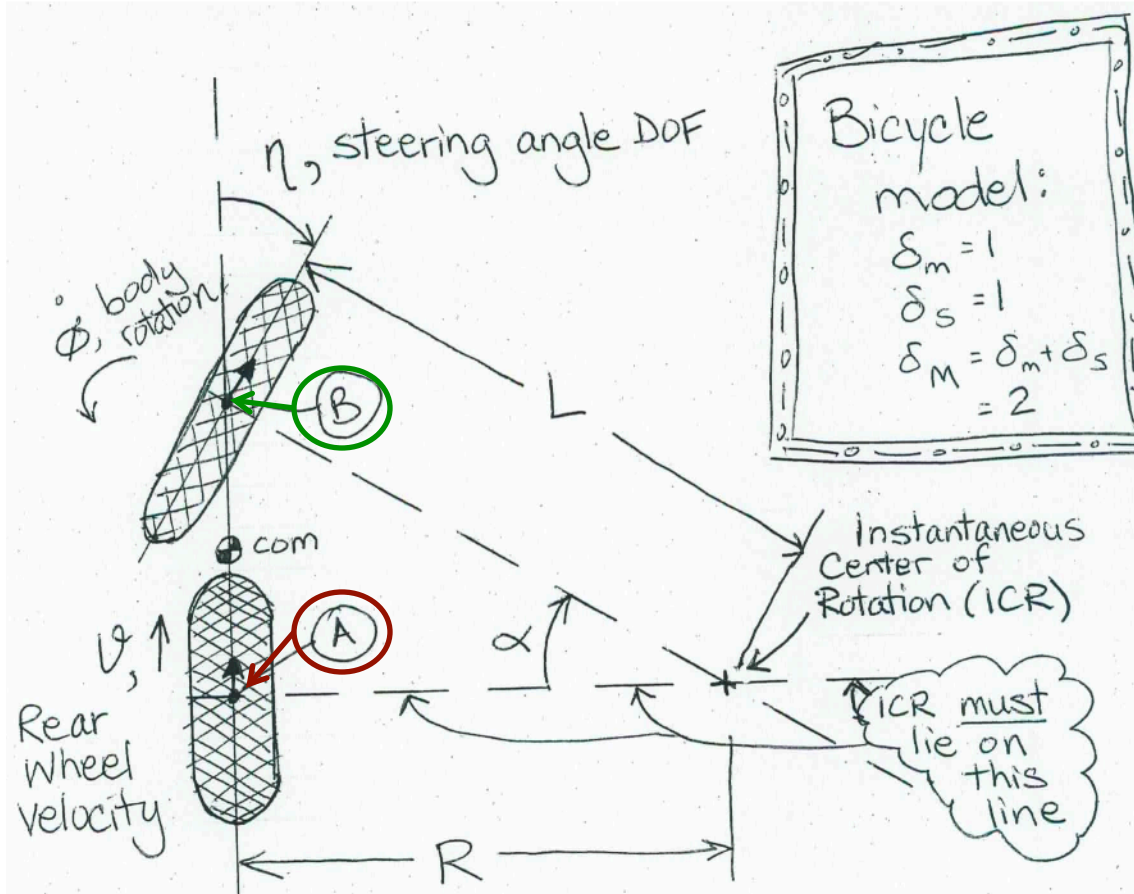
(From Siegwart and Nourbakhsh, p. 60.)

Mobility, Steerability, and Maneuverability

- Degree of **mobility**, δ_m : **Instantaneous** DOF of robot chassis due to commanding different **wheel velocities** (without turning wheels). (aka “Differential DOF”: DDOF)
- Degree of **steerability**, δ_s : **Instantaneous** DOF of robot chassis due to **reorienting the wheel**, which changes the ICR location on the plane, without actually changing the points of contact of the wheels.
- Degree of **maneuverability**, δ_M : The sum of differential and steering DOFs. Out of $\dot{x}$, $\dot{y}$, and $\dot{\phi}$, how many of these instantaneous velocity components can be set independently? Ans: δ_M

$$\delta_M = \delta_m + \delta_s$$

Maneuverability Example: A bicycle



- ① Pt "A" has 3 velocity components:
 $\dot{x}_A, \dot{y}_A, \dot{\phi}_A$ (body rotation)

$$\dot{x}_A = 0$$

$$\dot{y}_A = \vec{V}$$

$$\dot{\phi}_A = \frac{-1}{R} \vec{V}$$

$\dot{x}_A$ always zero.
 Only two independent, instantaneous velocity components can be set!

$\therefore$ maneuverability is: $\delta_M = 2$

- ② Consider Pt "B" location, instead:

$$\dot{x}_B, \dot{y}_B, \dot{\phi}_B = \dot{\phi}_A = \dot{\phi}_{\text{body}}$$

$$L = \frac{R}{\cos \alpha}$$

$$\dot{x}_B = \left(\frac{L}{R}\right) \vec{V} \sin \alpha$$

$$\dot{y}_B = \left(\frac{L}{R}\right) \vec{V} \cos \alpha$$

$$\dot{\phi}_B = \dot{\phi}_A = \frac{-1}{R} \vec{V}$$

Still, only 2 velocities can be set independently

$$\delta_M = \delta_m + \delta_s = 1 + 1 = 2$$

Definitions for Mobile Robotics

- **Workspace:** The set of possible configurations, i.e., of allowable combinations of DOFs (degrees of freedom).
- **Degrees of Freedom (DOF):** Independent variables specifying allowable configurations in the workspace, for all achievable (long-term) motions of the vehicle.
- **Differential Degrees of Freedom (DDOF):** Mobility, or instantaneous DOF of motion, δ_m .
- **Holonomic robot:** A robot with zero nonholonomic constraints. (That is, the long-term DOF can be determined from a one-to-one mapping from internal positions and angles and are NOT “path dependent”.)

Holonomic and Nonholonomic Constraints

- A **holonomic** constraint can be expressed as an explicit function of **position variables, only**. (i.e., not requiring velocity variables.)
- A **nonholonomic** constraint requires a differential relationship. (i.e., velocity terms like $\dot{x}_w$ or $\dot{\xi}_w$ *cannot be avoided* when describing how motion is constrained.)

Examples:

- 2-link robot arm (e.g., Labs 1 and 2): holonomic.
End configuration is NOT “path dependent”.
- Unicycle (or standard car): nonholonomic.
End configuration is “path dependent”.

(See examples at board...)