

Topics include:

1. **Terminology:** manipulator, end effector, redundancy, singularity, ...
2. **Workspace:** Reachable vs Dexterous
3. **Kinematics:** Forward and Inverse problems
4. **The Jacobian matrix:** velocity kinematics
5. **Mechanical impedance:** Definition of; inertia; reflected inertia
6. **Work balance:** gear ratios; motor coefficients
7. **Motor dynamics model:** standard equations and block diagram
8. **Block diagrams:** understanding, creating and solving algebraically
9. **First- and second-order time response:** estimating parameters (Lab 2)
10. **Friction vs damping:** effect of time response; estimating parameters
11. **SISO control:** P, PD, PID; PID tuning methods; feedforward
12. **Control law partitioning:** “cancel out” all dynamics except mass (or inertia)
13. **Wheeled vehicles:** mobility, steerability, maneuver.; wheel types; ICR; ...
14. **Holonomic vs nonholonomic:** instantaneous DOFs vs long-term GC's
15. **Rotation matrices:** body frame vs inertial reference frame velocities

Notices:

- HW 2 due MONDAY (5pm).
- HW 3 (practice quiz) due: ?
- Midterm date: Thursday, May 10 (in class)

1. Terminology

- **Manipulator**: a multi-link arm
- **End Effector** (ee): The point(s) of the robot designed to interact w/ environment; e.g., the tool tip
- **Configuration**: specification of all points, geometrically, on a robot
- **Configuration space**: set of all possible configs
- **Kinematics**: geometric relationships, e.g., between joints and end effector
- **Degree of Freedom** (DOF): independent rotational or translational direction of motion that is presently possible
- **Revolute Joint**: rotation DOF
- **Prismatic Joint**: translational DOF
- **Pitch** (θ_y), **roll** (ϕ_x), **yaw** (ψ_z): rotation about y, x, and z axes, respectively
- **Joint variables, $\mathbf{q}$** : DOF for joints
- **Joint space**: set of all possible joint variable combinations
- **State space**: geometry AND dynamics; i.e., positions and velocities possible
- **Workspace**: Points reachable by ee, i.e., “reachable workspace” (next slide)
- **Kinematic redundancy**: when multiple joint solutions exist to put an end effector at a particular position and orientation
- **Accuracy**: How close to a desired target? (mean of the error wrt some goal)
- **Repeatability**: How close to other trials? (variance in error, trial after trial)

2. Workspace

- The **workspace** is the set of all points (“volume” or “area”) reachable by the end effector. When used alone, “workspace” generally means “reachable workspace”.
- **Reachable workspace**: see above
- **Dexterous workspace**: Reach at an arbitrary orientation. For a planar mechanism (in the x-y plane), this means at any angle ψ_z . In the more general 3D robot case, orientation includes all 3 rotational DOF, θ_y , ϕ_x , ψ_z .

Planar case:

- To achieve an arbitrary combination of (x, y, ψ_z) [**3 DOF** of end effector] requires at least how many joint DOFs?

3D case:

- To achieve an arbitrary combination of **6 DOF** (3 positions, 3 rotations) for an end effector requires at least how many joint DOFs?

2. Workspace

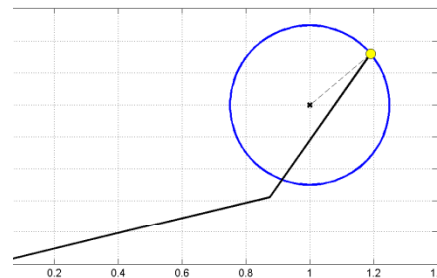
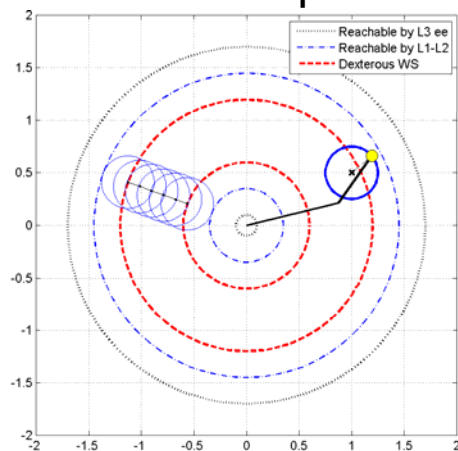
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Planar case:

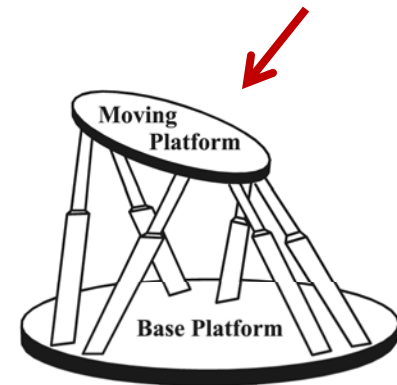
- To achieve an arbitrary combination of (x, y, ψ_z) [**3 DOF** of end effector] requires at least how many joint DOFs? **3 (e.g., 3-link arm)**

3D case:

- To achieve an arbitrary combination of **6 DOF** (3 positions, 3 rotations) for an end effector requires at least how many joint DOFs? **6 (e.g., Stewart platform)**



ECE/ME 179D - Lecture 10



3. Kinematics

- Forward kinematics: Given a set of joint positions, q , there exists one solution for the end effector
- Inverse kinematics (IK): Given a desired end effector position, there may be multiple solutions (redundancy) or there may be no solution (infeasible).

4. The Jacobian matrix

- Velocity kinematics
- Matrix equations that relate velocities of the joints, dq/dt , to velocities at the end effector: $\xi = J(q)\dot{q}$ via Spong convention, or as: $\dot{\xi} = J(q)\dot{q}$
...because some books define “xi” as position, rather than as velocity.
- Can solve for numerically, differentiating relationship for position (kinematics)
- But can ALSO sometimes solve for a given geometry through inspection...

5. Mechanical Impedance

- Effort: Force (analogous to voltage, usually the “source”)
- Flow: Velocity, dx/dt (analogous to current)
- Capacitance (Spring): $Z_e = 1/(Cs)$, $Z_m = k/s$ (or K/s , rotational system)
- Resistance (Damping): $Z_e=R$, $Z_m=b$ (or B)
- Inductance (Mass): $Z_e=Js$, $Z_m=ms$ (or J_s)
- Understand “reflected inertia”

6. Work Balance

- Rate of work is POWER:
 - Force times velocity
 - Voltage times current
- Gear ratios and electro-mechanical coefficients describe a work balance

7. Motor Dynamics Model

- Motor constant
- Resistance (and inductance)
- Effect of back emf (adds damping)

Block diagram:

8. Block Diagrams

- Be able to write equations for block diagrams
- Be able to write block diagrams from equations
- Be able to REDUCE a block diagram, to derive a desired input-output relationship.
- Know that gear ratios and transformation coefficients (such as the motor torque and back emf coefficient) typically appear TWICE in a loop. i.e., same N (not $1/N$), and $K_{\text{tau}} = K_{\text{back-emf}}$
- Know whether an impedance or “inverse impedance” goes in an empty block.

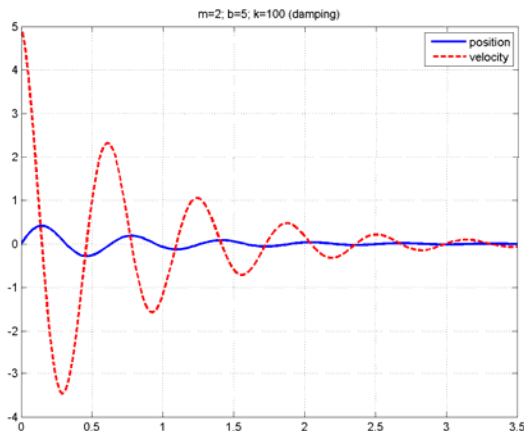
9. 1st- and 2nd-order responses

$$\frac{K}{Ts + 1}$$

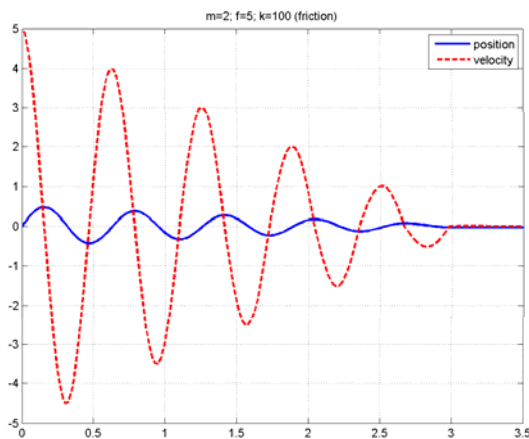
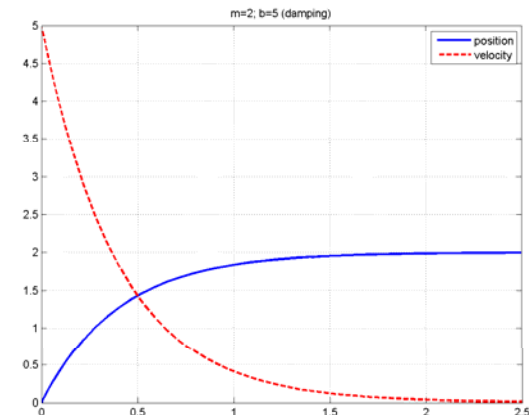
$$\frac{A\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

10. Friction vs Damping

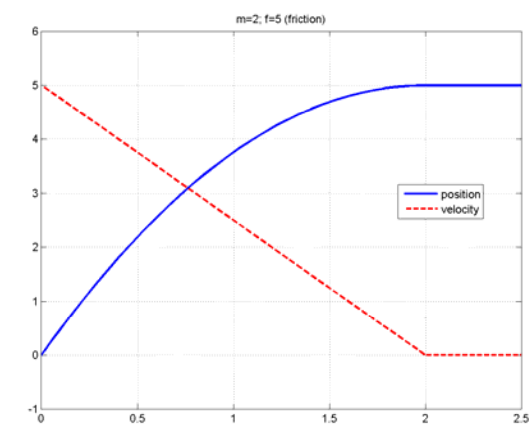
- Nonlinear Coulomb friction: $F = -f \cdot \text{sign}(dx/dt)$
- Linear viscous damping: $F = -b \cdot dx/dt$
- Friction non-linearity can result in steady state errors and make it difficult to predict the final value of a time response.



Damping:
Exponential decay



Friction:
Linear decay



11. Single-input single-output (SISO) control

- Proportional (P): A gain times difference in position. Acts like “stiffness”.
- Proportional-derivative (PD): K_p times error in position (“stiffness”), and K_d times negative velocity (or times error in velocity) (“damping”). Whether K_d is used in the forward or feedback path affects numerator (zeros) of transfer function, but not denominator (poles). Similar in function to a LEAD controller, adding phase to the open-loop TF at crossover frequency.
- Proportional-integral-derivative (PID): Also includes a gain times the integral of error. Integrator increases gain of the open-loop TF at low frequencies, thereby eliminating steady-state error.

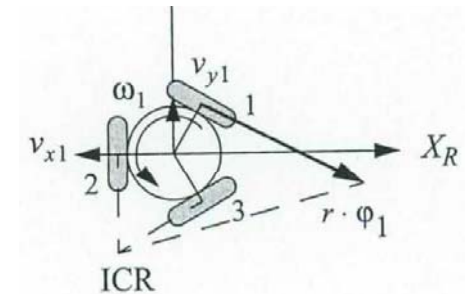
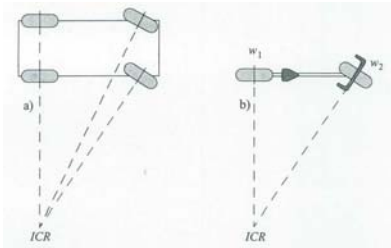
12. Control law partitioning (SISO)

- Divide control law into two parts:
 1. First, feedback a force (or torque) that exactly cancels all terms EXCEPT mass time acceration:
 2. Then, use simple PD control to set damping ratio (ζ) and natural frequency (ω_n) of the desired, 2nd-order closed loop response
- Note you can “cancel out” both linear terms AND nonlinear terms (such as gravity or Coulomb friction)

Example:

13. Wheeled Vehicles

- At most 3 DOF for the CHASSIS: x, y, and phi
- **Mobility**: DDOF via wheel velocities, without steering (no reorienting wheels)
- **Steerability**: DOF for ICR, by steering, only
- **Maneuverability**: at most 3. Sum of the other two. (See eqn at bottom.)
- **Instantaneous center of rotation** (ICR): What is allowed, given any no-slip constraints
 - Note for the **OMNIBOT**, calculating ICR can be tricky. Remember the direction of the free rollers is at an angle (90 deg?) to the main (driven) axis!



Omnidirectional	Differential	Omni-Steer	Tricycle	Two-Steer
$\delta_M = 3$	$\delta_M = 2$	$\delta_M = 3$	$\delta_M = 2$	$\delta_M = 3$
$\delta_m = 3$	$\delta_m = 2$	$\delta_m = 2$	$\delta_m = 1$	$\delta_m = 1$
$\delta_s = 0$	$\delta_s = 0$	$\delta_s = 1$	$\delta_s = 1$	$\delta_s = 2$

$$\delta_M = \delta_m + \delta_s$$

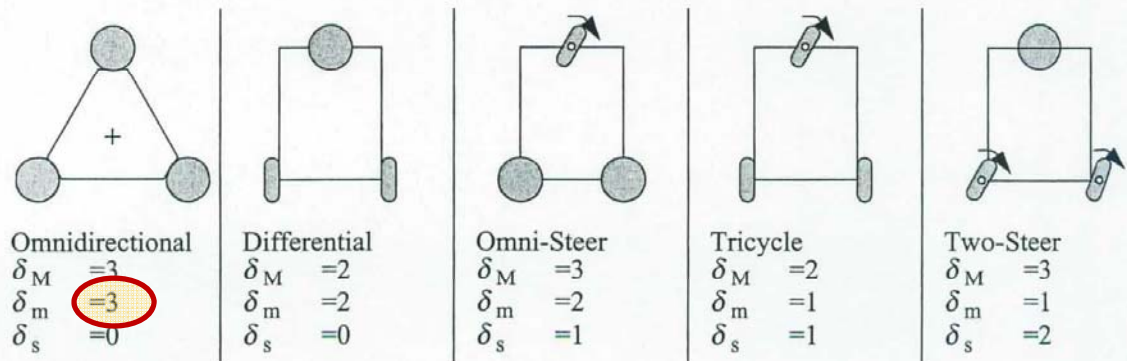
14. Holonomic vs Nonholonomic

- Generalized variables
 - Long-term coordinates needed
- Admissible variations
 - Immediate DDOF allowed
- Independent and complete: know definitions

$$DDOF = \delta_m \leq \delta_M \leq DOF$$

$$GC = \xi_j > \delta \xi_k = DOF \iff \text{Nonholonomic}$$

$$GC = \xi_j = \delta \xi_k = DOF \iff \text{Holonomic}$$



$$\delta_M = \delta_m + \delta_s$$

15. Rotation matrices

- Rotation matrices are quite important in kinematics.
- However, we touch on only the basics in this class (since our focus is in dynamics and control).
- You should understand how to transform velocity in one coordinate frame (e.g., wrt omnibot body frame) to velocity in the GLOBAL frame