ECE/ME 179D

Lagrangian EOM – Part 1

Lecture 11

1. Short review for midterm (Q&A)
2. Derivation of Lagrangian equations of motion (EOMs):

- **Review of holonomic / nonholonomic definitions**
- **The "Lagrangian":** $\mathcal{L} = T^* - V$
  - **Kinetic co-energy**
  - **Potential energy**
- **Lagrange's equations**

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = \Xi_i$$

Notices:

- Midterm date: Thursday, May 10 (in class)
- Office hours today in 3120A (lab), 3:30-4:30pm
- One-sided single sheet of notes (8.5 x 11 inches) for midterm
- HW 3 due tonight; solutions on web after 7pm

ECE/ME 179D

Review for Midterm

Lecture 11

Topics for particular focus:

1. **Terminology**: manipulator, end effector, redundancy, singularity, …
2. **Workspace**: Reachable vs Dexterous
3. **Kinematics**: Forward and
4. **The Jacobian matrix**: velocity kinematics
5. **Mechanical impedance**: Definition of; inertia; reflected inertia
6. **Work balance**: gear ratios; motor coefficients
7. **Motor dynamics model**: standard equations and block diagram
8. **Block diagrams**: understanding, creating and solving algebraically
9. **First- and second-order time response**
10. **Friction vs damping**: effect … timating parameters
11. **SISO control**: P, PD, PID; PID … edforward
12. **Control law partitioning**: "cancel out" all dynamics except mass (or inertia)
13. **Wheeled vehicles**: mobility, steerability, maneuv.; wheel types; ICR; …
14. **Holonomic vs nonholo** … C's
15. **Rotation matrices**: body frame vs inertial reference frame velocities

$\boxed{\dot{\xi} = J(q)\dot{q}}$

Geometric calculation of J…

$\boxed{N^2}$ $\boxed{\dfrac{1}{N^2}}$

$\boxed{s^2 + 2\zeta\omega_n s + \omega_n^2}$

Geometric calculation: of J…, and of ICR…

Notices:

- HW 2 due MONDAY (5pm).
- HW 3 (practice quiz) due: ?
- Midterm date: Thursday, May 10 (in class)

Holonomic / Nonholomic

- **Nonholonomic** when:
  - Accessible configuration space has higher dimension than the accessible velocity space.

i.e.,
  - **More generalized variables** required to describe the long-term configurations that are achievable…
  - **…than Degrees of Freedom** (DOFs) for local motion, instantaneously

$$\#GC = \xi_j > \delta\xi_k = \#DOF \longleftrightarrow \text{Nonholonomic}$$

$$\#GC = \xi_j = \delta\xi_k = \#DOF \longleftrightarrow \text{Holonomic}$$

Wheeled Robots

Common terminology differs from pure definition:
- "kinematic posture": x, y, φ for body.
  - If we produce appropriate control laws, we can create an "apparent" holonomic system
    - The internal variables (wheel rotations) have not gone away
    - BUT, they are effectively "hidden" within a set of feedback laws…
  - 3 GC's (longterm);
  - 3 "DDOF" (instantaneous DOF, which we will just call "DOF")

If we care about actual wheel positions, then:
- "configuration posture": x, y, φ, q1, q2, q3 (6 GC's)
  - Omnibot from lab has an integrable constraint:

$$\phi_b = \frac{-r_w}{3L}\left(q_1 + q_2 + q_3\right)$$

  - 5 GC's (longterm), since φ is a known function (above) of q1, q2, and q3
  - 3 DOF instantaneous (dq1/dt, dq2/dt, dq3/dt)

Lagrangian Approach

1. Define generalized coordinates that are:
   – Complete
   – Independent

   ξ_j (j = 1, 2, 3, …)

   (*For now, we will also assume system is holonomic*…)

2. Force-dynamics relationship requires:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i}\right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i$$

   where:

$$\mathcal{L} = T^* - V$$

Lagrangian Approach

ξ_j (j = 1, 2, 3, …)

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i}\right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i$$

$$\mathcal{L} = T^* - V$$

Lagrangian Approach

Little "xi" is the set of DOF. Since we assume system is holonomic, there are the same number of GCs and DOFs.

ξ_j (j = 1, 2, 3, …)

Big "Xi" represents all non-conservative forces, for each DOF. This includes both the ACTUATION and LOSSES (due to damping, most typically).

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i}\right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i$$

Fancy "L" is the "Lagrangian"

V is potential energy

$$\mathcal{L} = T^* - V$$

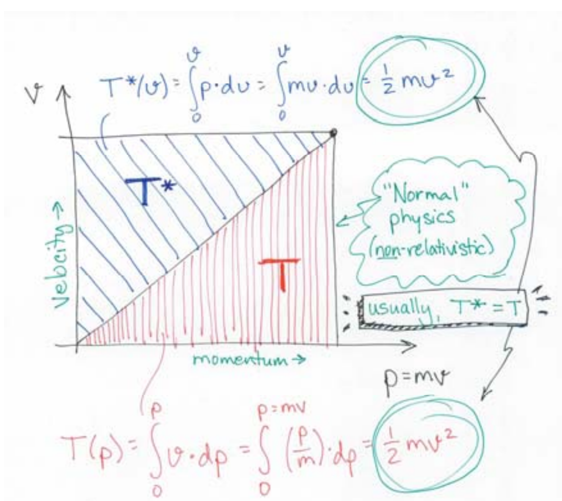
T* is kinetic CO-ENERGY

Kinetic Co-energy

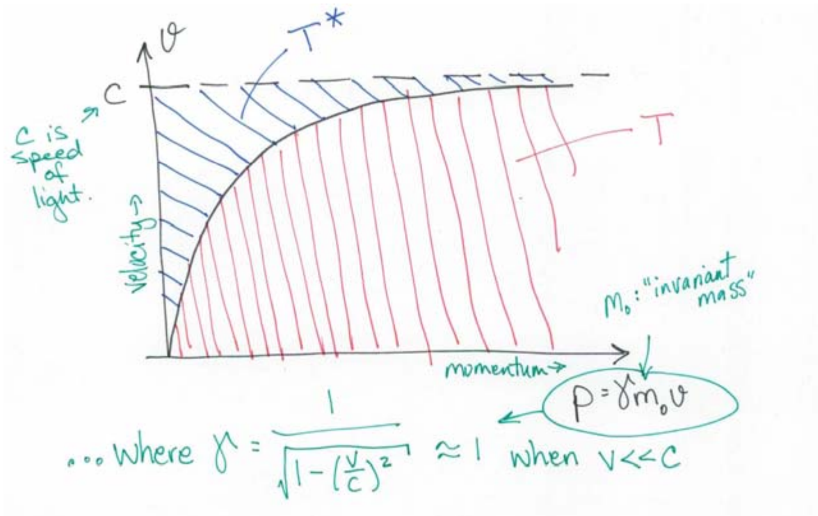
Usually, kinetic energy is identical to kinetic "co-energy".

$$T = \sum_i \tfrac{1}{2} m_i v_i^2$$

For a rigid rotating body: $T = \tfrac{1}{2} J \dot{\theta}^2$



Relativistic (and other unusual) Physics



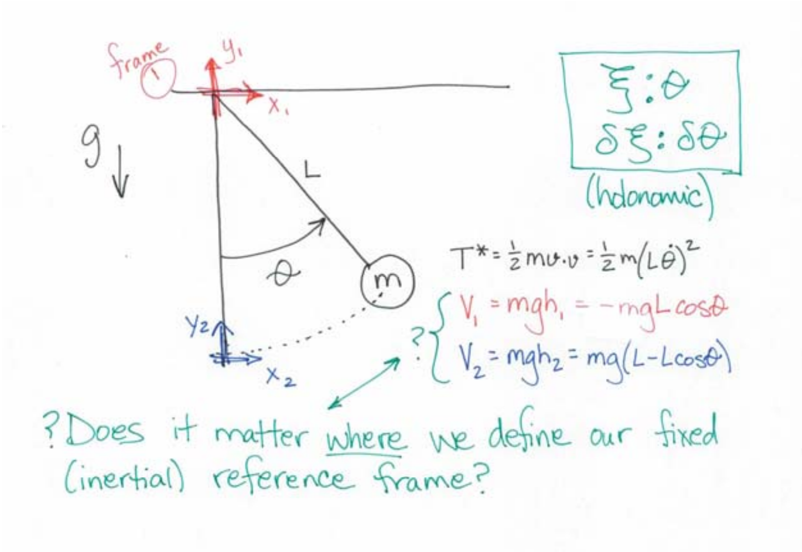
Note: That said, just assume T*=T in ECE/ME 179d…

Relativistic (and other unusual) Physics

Note: That said, just assume T*=T in ECE/ME 179d…

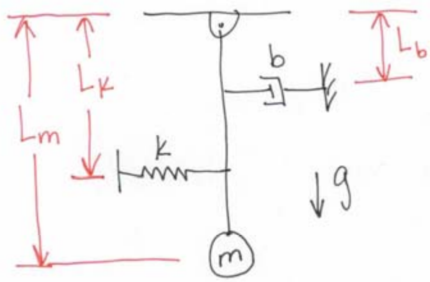
ECE/ME 179D - Lecture 11

Simple, Passive Pendulum Example



Simple, Passive Pendulum Example

Spring-Damper Pendulum Example



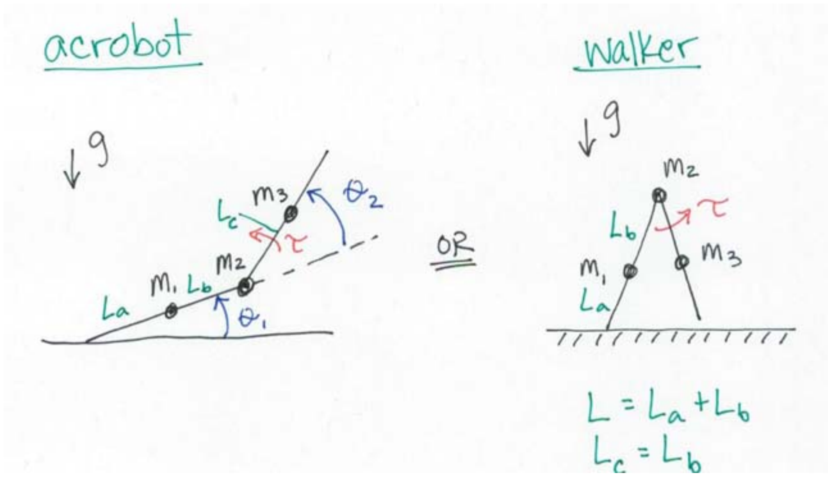
$$mL_m^2\ddot{\theta} + mgL_m\sin\theta + kL_k^2\theta = \overbrace{-L_b^2 b\dot{\theta}}^{\Xi_\theta \text{ : non-conservative}}$$

or, write as:

$$mL_m^2\ddot{\theta} + bL_b^2\dot{\theta} + kL_k^2\theta + mgL_m\sin\theta = 0$$

Spring-Damper Pendulum Example

Two-Link Pendulum ("Walker") Example



Two-Link Pendulum ("Walker") Example