

1. Short review for midterm (Q&A)**2. Derivation of Lagrangian equations of motion (EOMs):**

- Review of holonomic / nonholonomic definitions

- The “Lagrangian”: $\longrightarrow \mathcal{L} = T^* - V$
 - Kinetic co-energy $\longrightarrow$
 - Potential energy $\longrightarrow$

- Lagrange’s equations

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) - \frac{\partial \mathcal{L}}{\partial q_i} = \Xi_i$$

Notices:

- Midterm date: Thursday, May 10 (in class)
- Office hours today in 3120A (lab), 3:30-4:30pm
- One-sided single sheet of notes (8.5 x 11 inches) for midterm
- HW 3 due tonight; solutions on web after 7pm

Topics for particular focus:

1. Terminology: manipulator, end effector, redundancy, singularity, ...
2. **Workspace: Reachable vs Dexterous**
3. Kinematics: Forward and backward
4. **The Jacobian matrix:** velocity kinematics
5. **Mechanical impedance:** Definition of; inertia; **reflected inertia**
6. Work balance: gear ratios; motor coefficients
7. Motor dynamics model: standard equations and block diagram
8. Block diagrams: understanding, creating and solving algebraically
9. **First- and second-order time response**
10. Friction vs damping: effects; estimating parameters
11. SISO control: P, PD, PID; PID tuning; feedforward
12. **Control law partitioning:** "cancel out" all dynamics except mass (or inertia)
13. **Wheeled vehicles:** mobility, steerability, maneuver.; wheel types; **ICR**; ...
14. Holonomic vs nonholonomic
15. Rotation matrices: body frame vs inertial reference frame velocities

$$\dot{\xi} = J(q)\dot{q}$$

Geometric calculation of J...

$$N^2$$

$$\frac{1}{N^2}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

Geometric calculation: of J..., and of ICR...

Notices:

- HW 2 due MONDAY (5pm).
- HW 3 (practice quiz) due: ?
- Midterm date: Thursday, May 10 (in class)

Holonomic / Nonholonomic

- **Nonholonomic** when:

- Accessible configuration space has higher dimension than the accessible velocity space.

i.e.,

- **More generalized variables** required to describe the long-term configurations that are achievable...
- **...than Degrees of Freedom** (DOFs) for local motion, instantaneously

$$\#GC = \xi_j > \delta\xi_k = \#DOF \iff \text{Nonholonomic}$$

$$\#GC = \xi_j = \delta\xi_k = \#DOF \iff \text{Holonomic}$$

Wheeled Robots

Common terminology differs from pure definition:

- “kinematic posture”: x, y, ϕ for body.
 - If we produce appropriate control laws, we can create an “apparent” holonomic system
 - The internal variables (wheel rotations) have not gone away
 - BUT, they are effectively “hidden” within a set of feedback laws...
 - 3 GC’s (longterm);
 - 3 “DDOF” (instantaneous DOF, which we will just call “DOF”)

If we care about actual wheel positions, then:

- “configuration posture”: $x, y, \phi, q_1, q_2, q_3$ (6 GC’s)
 - Omnibot from lab has an integrable constraint:
$$\phi_b = \frac{-r_w}{3L}(q_1 + q_2 + q_3)$$
 - 5 GC’s (longterm), since ϕ is a known function (above) of q_1, q_2 , and q_3
 - 3 DOF instantaneous ($dq_1/dt, dq_2/dt, dq_3/dt$)

Lagrangian Approach

1. Define generalized coordinates that are:

- Complete
- Independent

ξ_j ($j = 1, 2, 3, \dots$)

(For now, we will also assume system is holonomic...)

2. Force-dynamics relationship requires:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i} \right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \bar{E}_i$$

where:

$$\mathcal{L} = T^* - V$$

Lagrangian Approach

$$\xi_j \quad (j = 1, 2, 3, \dots)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i} \right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i$$

$$\mathcal{L} = T^* - V$$

Lagrangian Approach

Little “xi” is the set of DOF. Since we assume system is holonomic, there are the same number of GCs and DOFs.



ξ_j ($j = 1, 2, 3, \dots$)

Big “Xi” represents all non-conservative forces, for each DOF. This includes both the ACTUATION and LOSSES (due to damping, most typically).

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i} \right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i$$



Fancy “L” is the “Lagrangian”



$$\mathcal{L} = T^* - V$$

V is potential energy



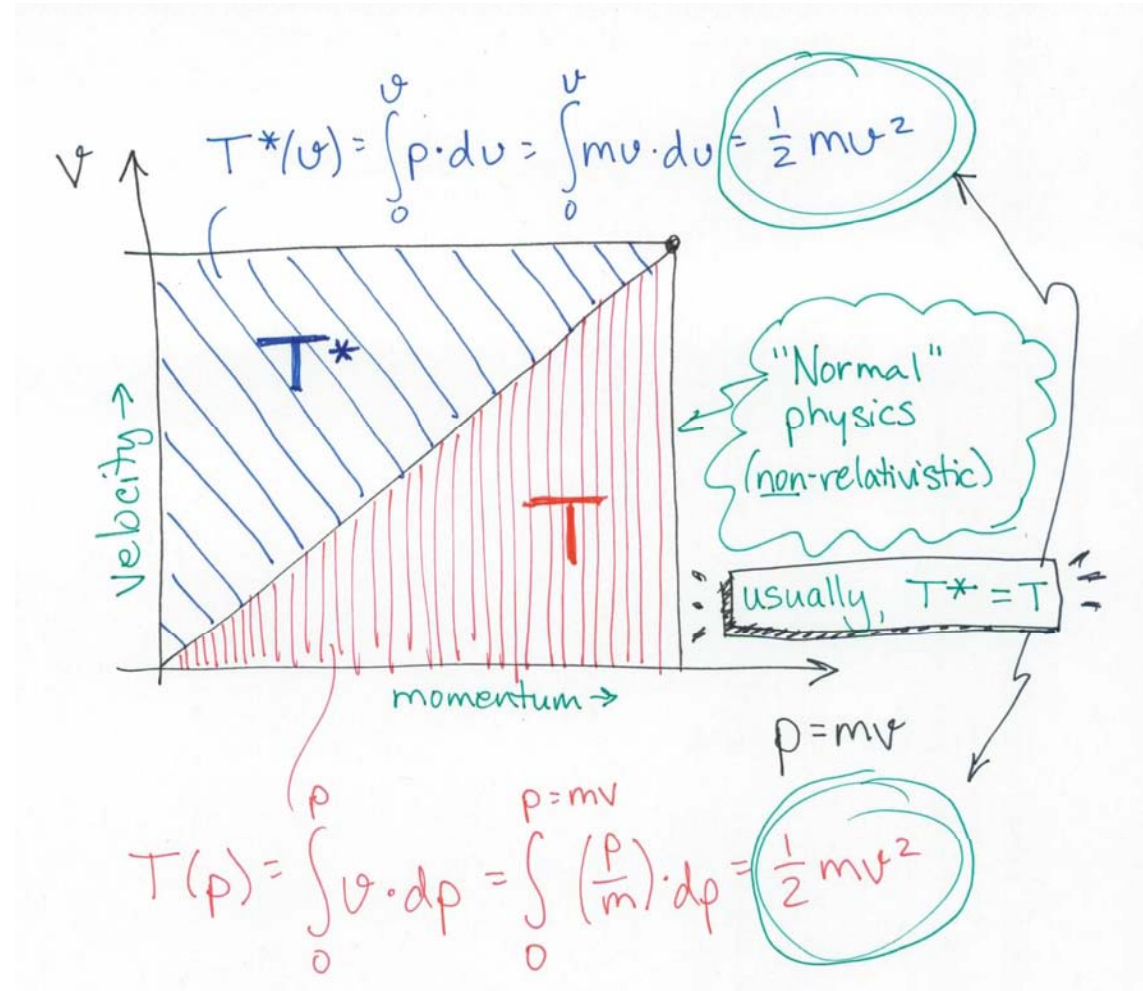
T* is kinetic CO-ENERGY

Kinetic Co-energy

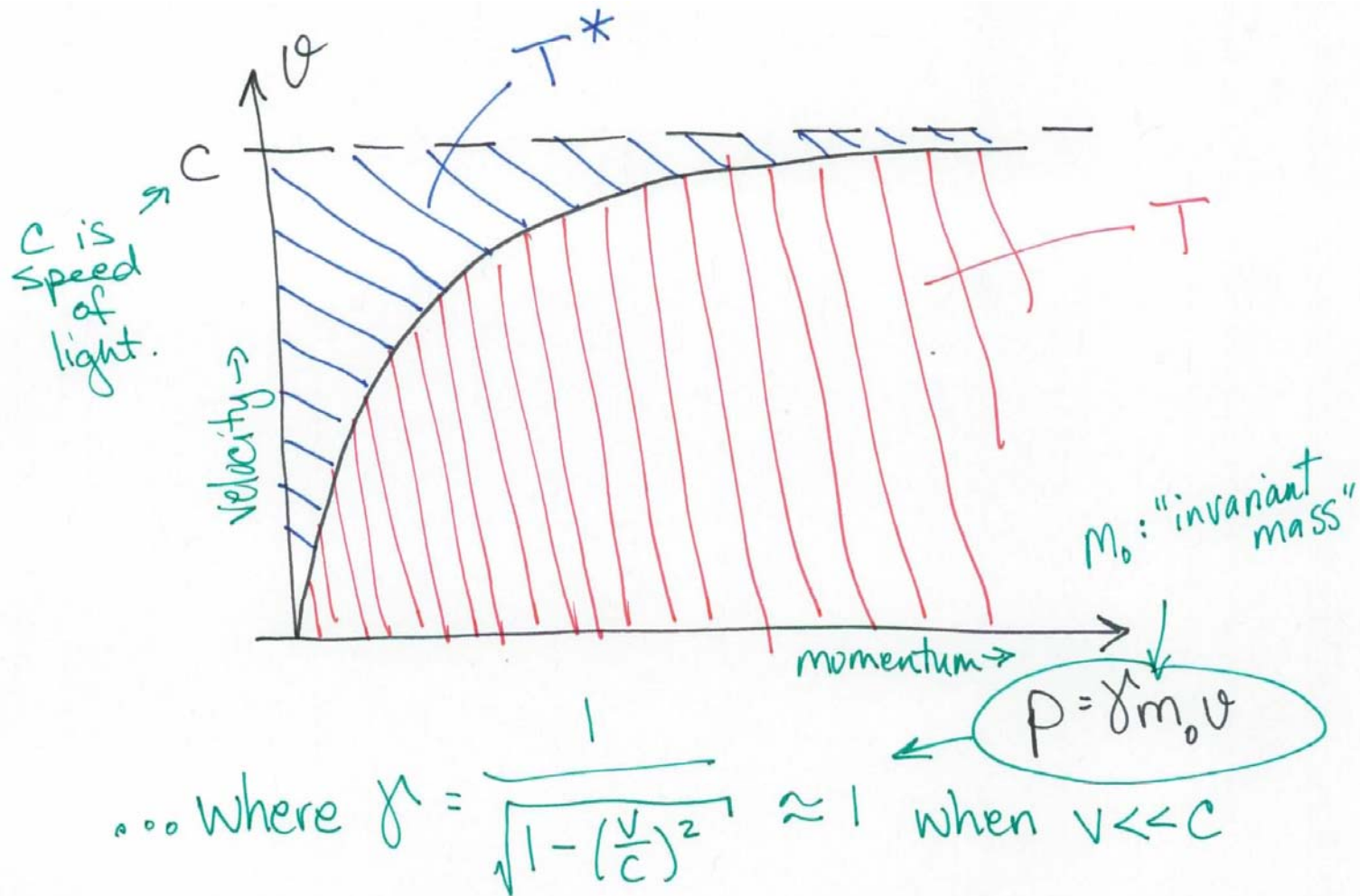
Usually, kinetic energy is identical to kinetic “co-energy”.

$$T = \sum_i \frac{1}{2} m_i v_i^2$$

For a rigid rotating body:

$$T = \frac{1}{2} J \dot{\theta}^2$$


Relativistic (and other unusual) Physics

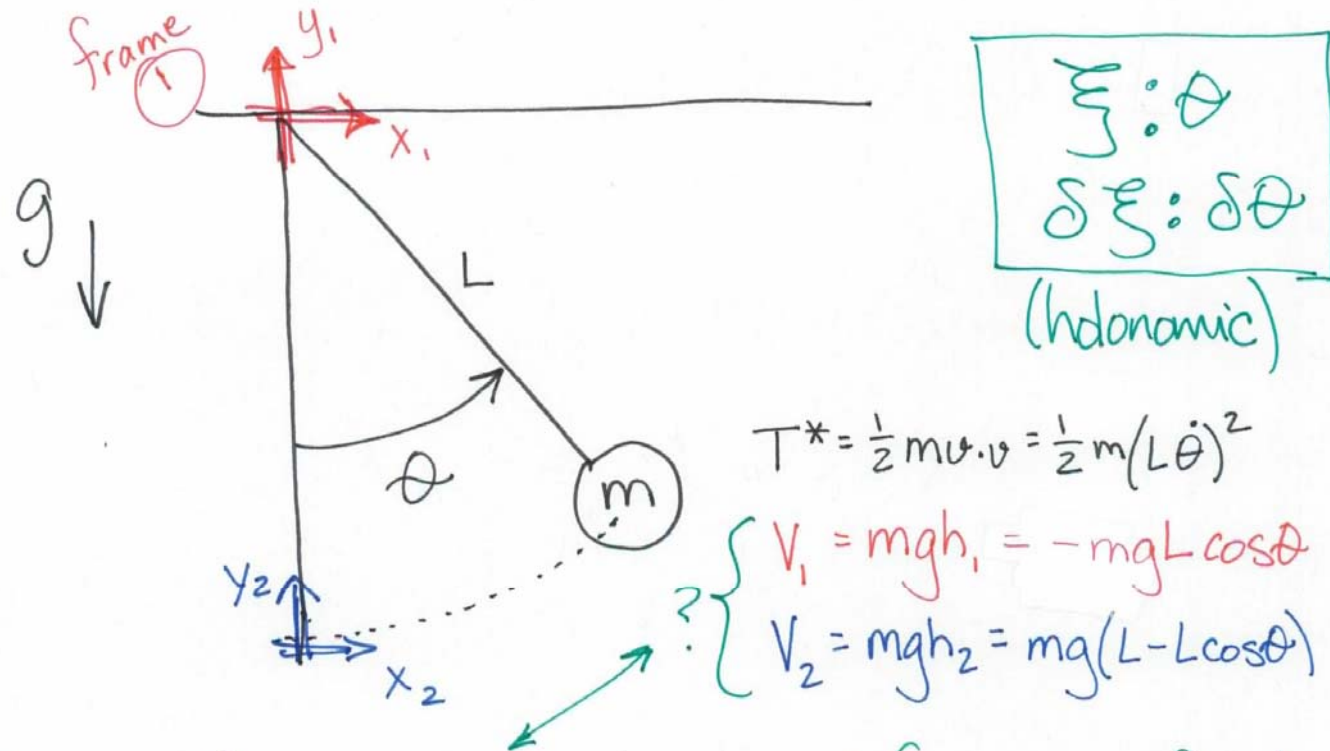


Note: That said, just assume $T^*=T$ in ECE/ME 179d...

Relativistic (and other unusual) Physics

Note: That said, just assume $T^*=T$ in ECE/ME 179d...

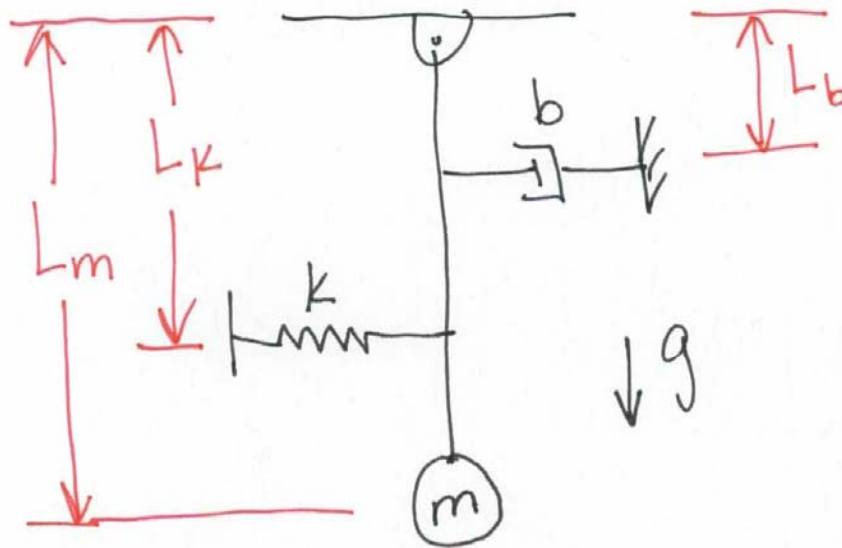
Simple, Passive Pendulum Example



? Does it matter where we define our fixed (inertial) reference frame?

Simple, Passive Pendulum Example

Spring-Damper Pendulum Example



$\approx_\theta$: non-conservative

$$mL_m^2 \ddot{\theta} + mgL_m \sin\theta + kL_k^2 \theta = -L_b^2 b \dot{\theta}$$

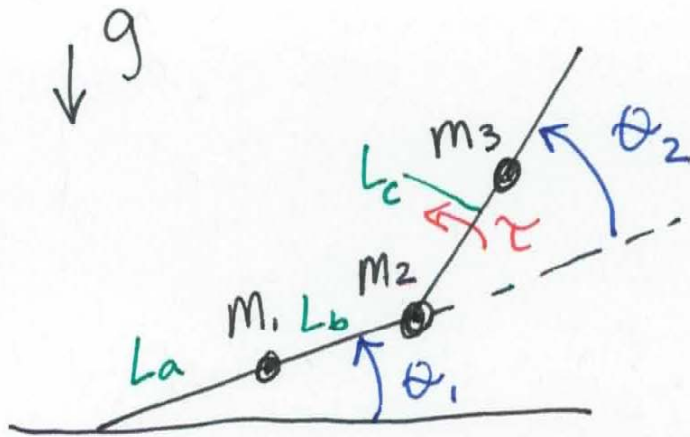
or, write as:

$$mL_m^2 \ddot{\theta} + bL_b^2 \dot{\theta} + kL_k^2 \theta + mgL_m \sin\theta = 0$$

Spring-Damper Pendulum Example

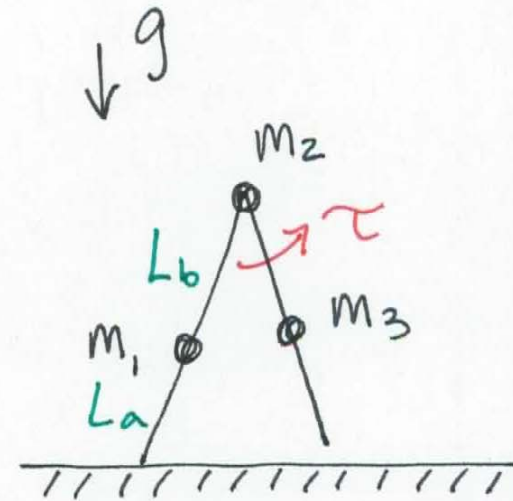
Two-Link Pendulum (“Walker”) Example

acrobot



OR

walker



$$L = L_a + L_b$$
$$L_c = L_b$$

Two-Link Pendulum (“Walker”) Example