

1. Review Lagrangian Method (for EOM) :

$$\mathcal{L} = T^* - V$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i} \right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i$$

2. Examples

3. Use of Relative vs Absolute Coordinates

4. Non-conservative (damping) term “tricks”

Notices:

- Midterm was technically “Lecture 12”.
- **No labs until:**
FRIDAY, June 1 and MONDAY, June 4
Lab 6 (Segway-style car)

Lagrangian Approach: Recall

Little “xi” is the set of DOF. Since we assume system is holonomic, there are the same number of GCs and DOFs.



ξ_j ($j = 1, 2, 3, \dots$)

Big “Xi” represents all non-conservative forces, for each DOF. This includes both the ACTUATION and LOSSES (due to damping, most typically).

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i} \right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i$$



Fancy “L” is the “Lagrangian”



$$\mathcal{L} = T^* - V$$

V is potential energy



T* is kinetic CO-ENERGY



Lagrangian Approach: Procedure

0. Assume system is **holonomic**:

$$\#GC = \xi_j = \delta\xi_k = \#DOF \longleftrightarrow \text{Holonomic}$$

1. Pick **generalized coordinates** (DOFs) that are:

- **Complete**
- **Independent**
($j = 1, 2, 3, \dots$)

ξ_j

Little xi: GCs
(gen. coord's)

- Possible GC set(s) ***might not be unique!***
- **Any** independent and complete set is OK.
- Either **absolute or relative** coords is OK.

Lagrangian Approach: Procedure

2. Define kinetic (co-)energy, T . (*Technically*, T^*)

Kinetic energy includes the sum of “one-half mass time velocity-squared” for every particle:

$$T = \sum_i \frac{1}{2} m_i v_i^2$$

For a **rigid rotating body**, we can use moment of inertia, J :

$$T = \frac{1}{2} J \dot{\theta}^2$$

(Note: We are **ignoring** electrical kinetic or potential energy here, for now and focusing on **MECHANICAL energy**.)

Lagrangian Approach: Procedure

3. Define potential energy, V .

Potential energy can be stored in a **spring**:

$$V_{spring} = \frac{1}{2} k (x - x_o)^2$$

...and also includes any “mgh” of particles in a gravitational field:

$$V_{gravity} = mgh$$

Note: “h” is relative to any arbitrary (but fixed) inertial reference frame.

Lagrangian Approach: Procedure

4. Lagrangian is **kinetic minus potential** energy:

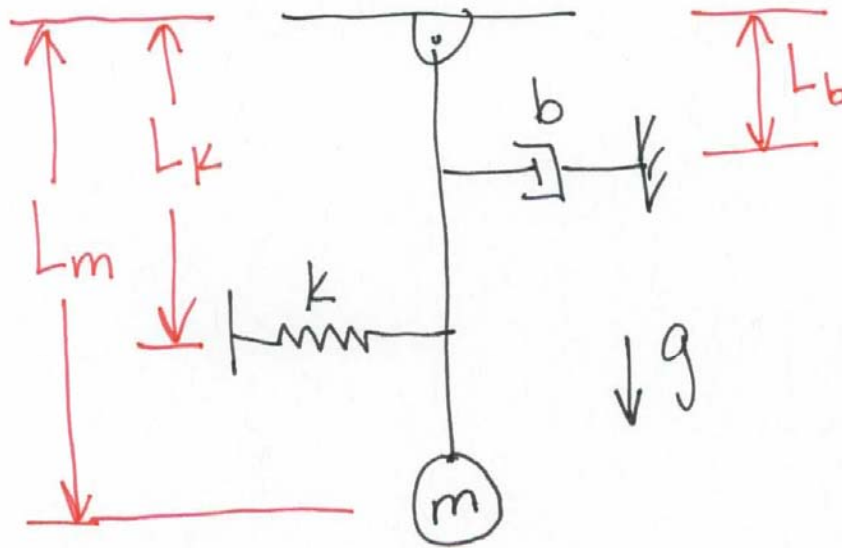
$$\mathcal{L} = T^* - V$$

5. For each GC (DOF), solve for an equation of motion:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_i} \right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i$$

Note: A system with **N generalized coordinates** (GCs) will have **N equations of motion**.

Spring-Damper Pendulum Example



One DOF.
One EOM.

Final answer is below:

$\approx_\theta$: non-conservative

$$mL_m^2 \ddot{\theta} + mgL_m \sin\theta + kL_k^2 \theta = -L_b^2 b \dot{\theta}$$

or, write as:

$$mL_m^2 \ddot{\theta} + bL_b^2 \dot{\theta} + kL_k^2 \theta + mgL_m \sin\theta = 0$$

Spring-Damper Pendulum: Derivation

- $GC(s)$?

- T ?

- V ?

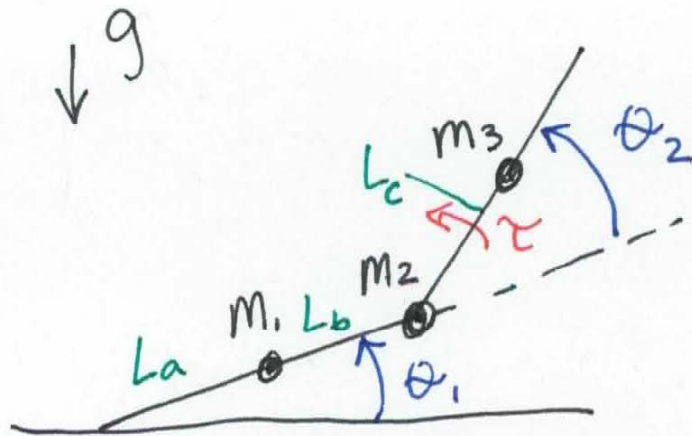
- L ?

Spring-Damper Pendulum: Derivation

- **Now solve for EOM(s):**

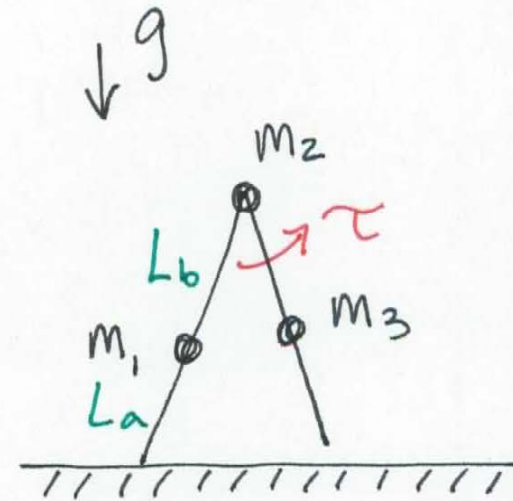
Two-Link Pendulum (“Walker”) Example

acrobot



OR

walker



$$L = L_a + L_b$$
$$L_c = L_b$$

Two-Link Pendulum (“Walker”) Example

- **GC(s) ?** Let's use *relative* theta 2 here...
- **T ?**
- **V ?**
- **L ?**

Two-Link Pendulum (“Walker”) Example

- **Set-up form to solve EOM:**
- **Be careful for non-conservative term(s):
[One torque input, at “elbow”]**

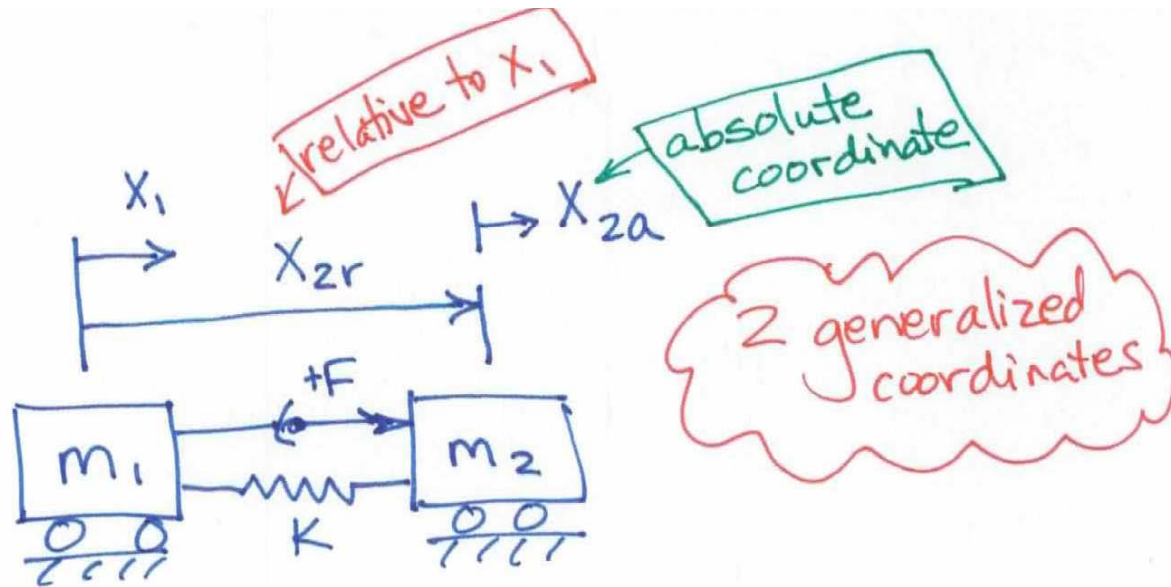
Relative vs Absolute Coordinates

- As mentioned earlier, **valid GCs can be either absolute** (wrt an inertial reference frame) **or relative**.
- The **non-conservative terms** must be written to be **compatible** with the choice of absolute vs relative coordinates!
- **Example:** For the acrobot (relative coords), **tau appears ONLY in the “theta 2” EOM!**

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- **Example:** For the acrobot (relative coords), **tau appears ONLY in the “theta 2” EOM!**

Example: Two-cart spring system



Absolute:

(A) $\xi_1 = x_1$
 $\xi_2 = x_{2a}$

$x_1 = x_1$
 $x_2 = x_{2a}$

Relative:

(R) $\xi_1 = x_1$
 $\xi_2 = x_{2r}$

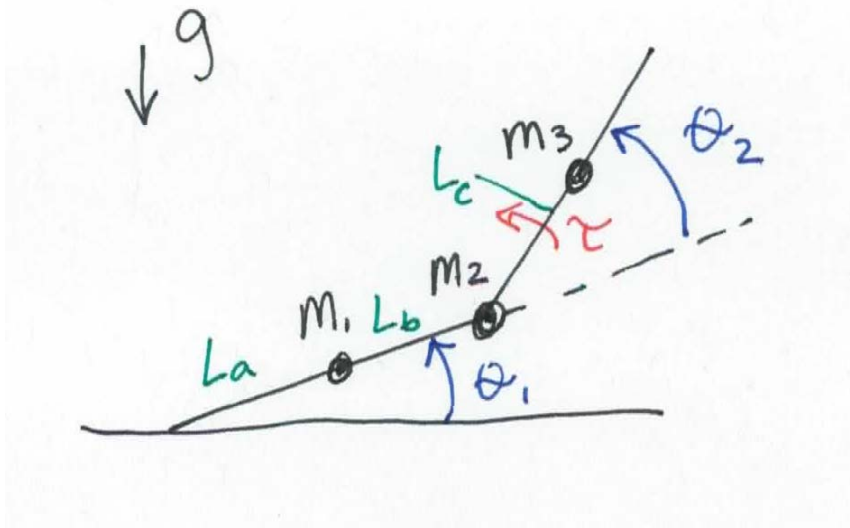
$x_1 = x_1$
 $x_2 = x_1 + x_{2r}$

**Choice of
GCs is NOT
UNIQUE**

Example: Two-cart spring system

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Example: Acrobot (Rel vs Abs)

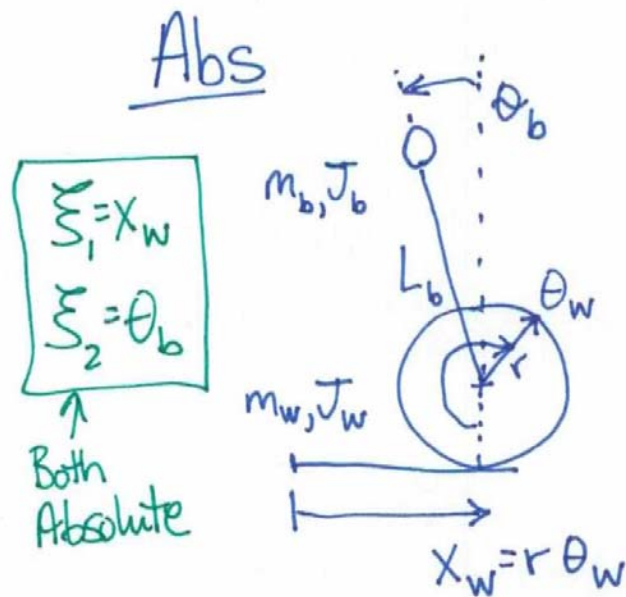


Theta 2 can be
relative (as shown)
OR absolute.

Example: Acrobot (Rel vs Abs)

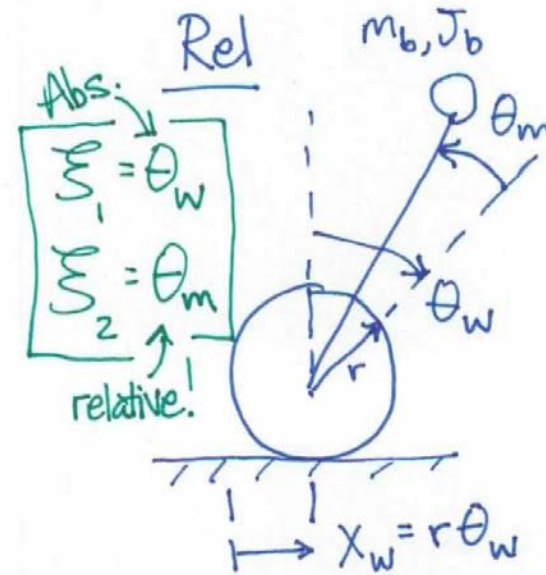
Example: Acrobot (Rel vs Abs)

Example: “Segway” style inverted pendulum



$$\mathbb{E}_{1a} = -\tau / r$$

$$\mathbb{E}_{2a} = \tau$$



$$\mathbb{E}_{1r} = 0$$

$$\mathbb{E}_{2r} = \tau$$

Example: “Segway” style inverted pendulum



Example: “Segway” style inverted pendulum

