```
% Example of AUTOMATED GENERATION OF LAGRANGIAN EOM
%
% Note: This file requires the shareware function "fulldiff.m"
% to do chain rule for time derivatives.
%
% Example system:
%              ____----|                            |  g (gravity)
%      ____----       |-| b (damper)              \|/
% |----                :                            v
% |                   [m] (mass)  -  -  -  -
% |                    <                    :
% |  (constant         >  k (spring)       dm (rel displace of mass)
% | arm length)        <                    :
% |       La   ____----        -  -  -  -  -
% |    ____----  alpha, tau (acting wrt inertial frame on alpha)
% |----  -  -  -  -  -  -

% Process
% =======

% 0. Clear workspace (desirable):
clear all
format compact % more readable when single-spaced

% 1. Declare constant and variable symbolically.
syms m k b alpha dm g La tau

% 2. Define INDEPENDENT and COMPLETE set of Generalized Coordinates.
GC = {alpha, dm}  % note CURLY BRACKET notation here and elsewhere!!

% 3. Define any DEPENDENT variables that are convenient. (For example,
%    we may wish to define potential energy in terms of absolute height.)
xm = La*cos(alpha)       % absolute x position of center of mass (COM)
ym = La*sin(alpha) + dm  % absolute height of COM

% 4. Define TIME DERIVATIVES of the dependent variables:
dxm = fulldiff(xm,GC)  % input arg GC are the independent vars of time
dym = fulldiff(ym,GC)
ddm = fulldiff(dm,GC)
for n=1:length(GC)
    dGC{n} = fulldiff(GC{n},GC)
end

% 5. Write expressions for...
%    a. T* (kinetic energy):
Tstar = (1/2)*m*(dxm^2 + dym^2)
%    b. V (potential energy):
V = m*g*ym + (1/2)*k*dm^2
%    c. L = Tstar - V, except "L" might already be defined previously
Lagran = Tstar - V
```

```
% 6. For each GC, calculate LHS of EOM:
%   d/dt(partial L wrt dxi_n) - (partial L wrt xi_n)
for n=1:length(GC)
    LHS{n} = fulldiff(diff(Lagran,dGC{n}),GC) - diff(Lagran,GC{n})
end

% 7. Define NON-CONSERVATIVE forces (or torque) for RHS of each EOM:
RHS{1} = tau;     % absolute torque on alpha DOF
RHS{2} = -b*ddm;  % damping force; dm is a relative coordinate

% 8. Print equations nicely on screen!
for n=1:length(GC)
    fprintf('\n__Equation %d:__\n',n)
    pretty(simplify(LHS{n})) % simplify, then print in "pretty" way...
    fprintf('  equals\n')
    pretty(simplify(RHS{n}))
    latex_output{n} = [latex(simplify(LHS{n})), ' = ',...
        latex(simplify(RHS{n}))]; % to print in LATEX
end

% Now, the contents of latex_output{1} and latex_output{2} can be
% copied into your favorite LaTeX editor.  To display nicely will
% still require some "hand editing" without latex, however!
```

Auto-Generation of Equations of Motion Example, in LaTeX

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Abstract

This example equation comes from the output to m-file eom_4bar_example.

1 MATLAB auto-formatting for EOMs

By default, MATLAB's `latex` function tends to display many variables in a non-*italic* Roman font which is not pleasing to the eye, nor easy to read. Below are our two equations of motion for the 4-bar spring-mass-damper system, for example.

The first EOM MATLAB generates is:

```
\mathrm{La}\, m\, \left(\mathrm{La}\, \mathrm{d2alpha} + \mathrm{d2dm}\,
\cos\!\left(\mathrm{alpha}\right) + g\,
\cos\!\left(\mathrm{alpha}\right)\right) = \mathrm{tau}
```

In latex formatting, this becomes:

$$\mathrm{La}\, m\, \left(\mathrm{La}\, \mathrm{d2alpha} + \mathrm{d2dm}\, \cos\!\left(\mathrm{alpha}\right) + g\, \cos\!\left(\mathrm{alpha}\right)\right) = \mathrm{tau} \tag{1}$$

Similarly, the second equation output from MATLAB is:

```
- \mathrm{La}\, m\, \sin\!\left(\mathrm{alpha}\right)\,
{\mathrm{dalpha}}^2 + \mathrm{dm}\, k + \mathrm{d2dm}\,
m + g\, m + \mathrm{La}\, \mathrm{d2alpha}\, m\,
\cos\!\left(\mathrm{alpha}\right) = - b\, \mathrm{ddm}
```

which becomes:

$$- \mathrm{La}\, m\, \sin\!\left(\mathrm{alpha}\right)\, {\mathrm{dalpha}}^2 + \mathrm{dm}\, k + \mathrm{d2dm}\, m + g\, m + \mathrm{La}\, \mathrm{d2alpha}\, m\, \cos\!\left(\mathrm{alpha}\right) = - b\, \mathrm{ddm} \tag{2}$$

2 Hand-editing within LaTeX

Making sense of the equations requires some hand editing. The equations are more easily understood when re-formatted *by hand* as shown below.
An improved latex expression for the first EOM is given below:

```
L_a m \left(L_a \ddot{\alpha} + ( \ddot{d}_m
+ g ) \cos (\alpha) \right) = \tau
```

This becomes:

$$L_a m \left(L_a \ddot{\alpha} + (\ddot{d}_m + g) \cos (\alpha) \right) = \tau \tag{3}$$

Similarly, the second equation can be written nicely as:

```
-L_a m \sin (\alpha) {\dot{\alpha}}^2 + d_m k +
(\ddot{d}_m + g) m + L_a \ddot{\alpha} m \
cos (\alpha) = - b \dot{d}_m
```

And this becomes:

$$-L_a m \sin (\alpha) {\dot{\alpha}}^2 + d_m k + (\ddot{d}_m + g) m + L_a \ddot{\alpha} m \cos (\alpha) = - b \dot{d}_m \tag{4}$$

Appendix: Example System

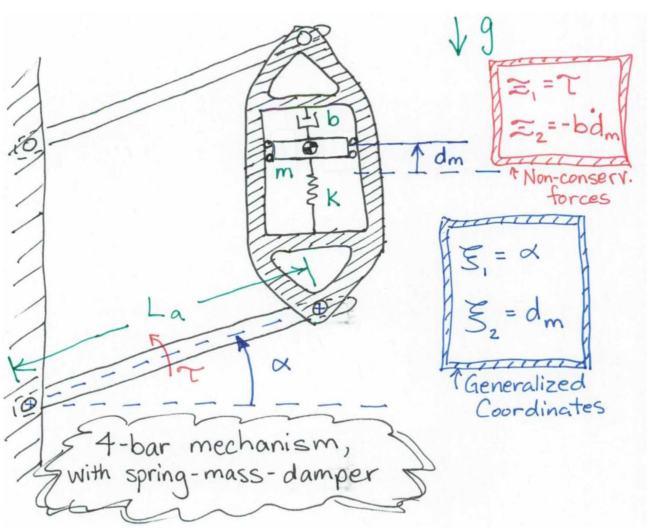

Figure 1: 4-bar mechanism with spring-mass-damper