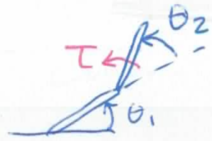
ECE/ME 179d

Lecture 18

Partial Feedback Linearization, for two-link acrobot



$$\boxed{M\ddot{\theta} + C\dot{\theta} + G = u}$$

$$\underbrace{\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}}_{(n \times n)} \underbrace{\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}}_{(n \times 1)} = \underbrace{-C\dot{\theta} + G + u}_{(n \times 1)}$$

Write as:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\boxed{M\ddot{\theta} = F + u}$$

Now, solve directly for $\ddot{\theta}$:

$$\ddot{\theta} = M^{-1}(F + u)$$

inverse of M

or

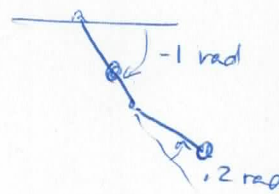
$$\ddot{\theta} = \bar{F} + \bar{u} * \tau$$

single torque @ elbow

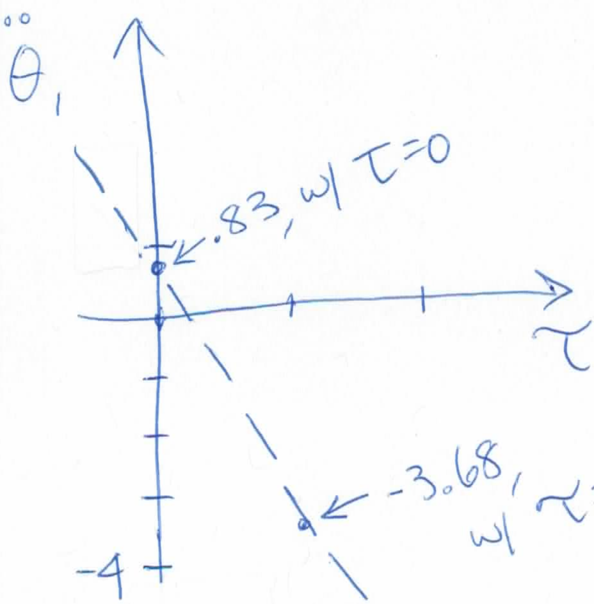
For acrobot,

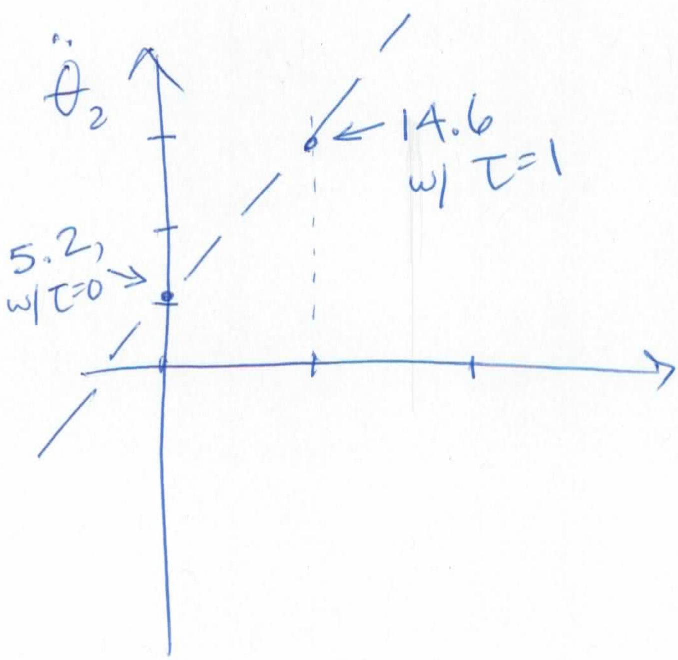
Example,

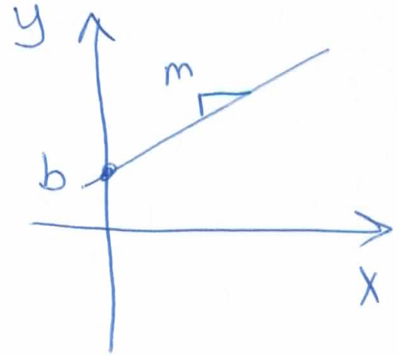
$$X = \begin{array}{cl} \left[\begin{array}{c} -1 \\ .2 \\ .1 \\ .1 \end{array}\right] & \begin{array}{l} \leftarrow \theta_1 \\ \leftarrow \theta_2 \\ \leftarrow \dot{\theta}_1 \\ \leftarrow \dot{\theta}_2 \end{array} \end{array}$$



- Let $\begin{cases} \tau_0 = 0 \\ \tau_1 = 1 \end{cases}$
- Solve for $\ddot{\theta}_1$ & $\ddot{\theta}_2$, given each test value for τ.





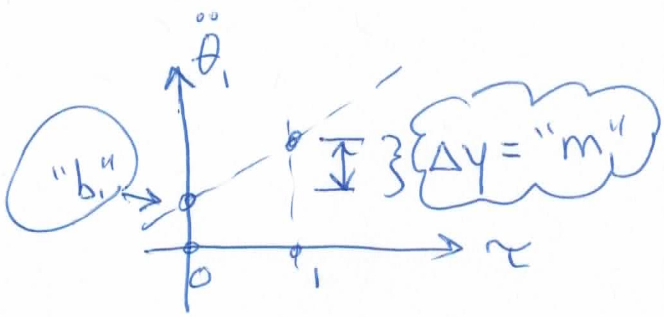


$y = mx + b$

$b = \ddot{\theta}_1(\tau = 0)$

$$\ddot{\theta}_1 = m_1 \tau + b_1$$

$m_1 = (\ddot{\theta}_1(\tau=1) - \ddot{\theta}_1(\tau=0))$



- To get a desired $\ddot{\theta}_1 = \ddot{\theta}_1^{des}$,

$$\boxed{\tau = \left(\frac{1}{m_1}\right)\left(\ddot{\theta}_1^{des} - b_1\right)}$$

- Likewise to get a desired $\ddot{\theta}_2 = \ddot{\theta}_2^{des}$,

$$\boxed{\tau = \left(\frac{1}{m_2}\right)\left(\ddot{\theta}_2^{des} - b_2\right)}$$

Can do <u>one</u> or the other; NOT BOTH!
(one input, ∴ one controlled $\ddot{\theta}$, instantaneously...)

Control Idea

Still can stabilize! (Controllable...)

- Drive θ_2 to bend to some desired angle, as a fn of velocity of first link, $\dot{\theta}_1$.

$$\boxed{\dot{\theta}_1 < 0 \rightarrow \theta_2^{des} = -40^\circ}$$

$$\boxed{\dot{\theta}_1 \geq 0 \rightarrow \theta_2^{des} = +40^\circ}$$

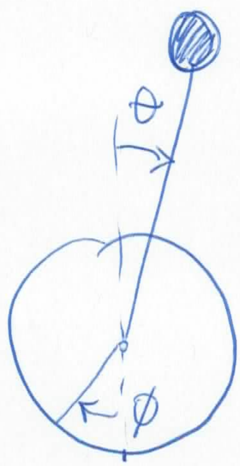
- Linearize dynamics of θ_2, as it is driven to θ_2^{des}.

$$\boxed{\ddot{\theta}_2 + \underbrace{K_d}_{2\zeta\omega_n}\dot{\theta}_2 + \underbrace{K_p}_{\omega_n^2}(\theta_2 - \theta_2^{des}) = 0}$$

Segway

$$M\begin{bmatrix}\ddot{\phi}\\ \ddot{\theta}\end{bmatrix} + C\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\end{bmatrix} + G = u$$

$$X \equiv \begin{bmatrix}\phi\\ \theta\\ \dot{\phi}\\ \dot{\theta}\end{bmatrix}$$



State space:

2 GC's (position vars) means 4 states (pos + vel.)

First, write as:

$$\boxed{\tilde{M}\dot{X} = \tilde{A}X + \tilde{B}u}$$

Then,

$$\dot{X} = (M^{-1}\tilde{A})\dot{X} + (M^{-1}\tilde{B})u$$

$$\boxed{\dot{X} = AX + Bu}$$

where $\boxed{\begin{array}{l} A \equiv M^{-1}\tilde{A} \\ B \equiv M^{-1}\tilde{B}\end{array}}$

in MATLAB, use $\boxed{A = M \backslash Abar}$ instead of inverse(M)*Ab for speed