ECE/ME 179D

Wheeled Vehicles

Lecture 8

Topics:

- Kinematic constraints
- Mobility, steerability, and maneuverability
- Holonomic constraints

Notices:

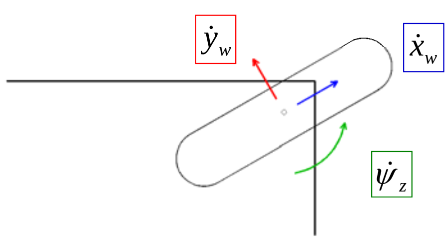
- Prelab 4 due TODAY (dropbox)
- HW 2 due next week (Friday), to be posted soon

Kinematic Constraints in Rolling Machines

"It's not the invention of the wheel that's impressive, it's the wheel and an axle!"

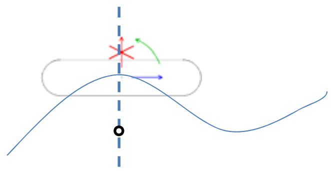
- Constraints for a standard wheel, rolling on a surface:

Direction for Instantaneous Velocity	Constraint: Allowed or Not?
$\boxed{\dot{x}_w}$: Orthogonal to Wheel Axis ("forward")	**ALLOWED (rolling direction)**
$\boxed{\dot{y}_w}$: In Direction of Wheel Axis ("sideways")	**NOT ALLOWED (no lateral slip)**
$\boxed{\dot{\psi}_z}$: Rotational, about contact point	**ALLOWED (spin on contact point)**

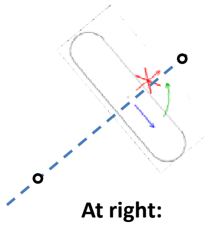


Instantaneous Center of Rotation (ICR)

- A standard wheel can both roll forward and turn:



At left:
Example instantaneous ICR and long-term (curvy) path.

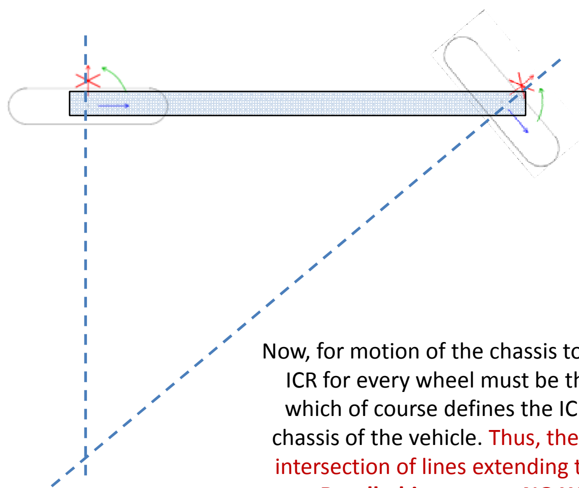


At right:
Note, the instantaneous ICR could be to either side of the wheel, or an infinity.

Thus, a single wheel can move to follow a path in which the **instantaneous center of rotation (ICR)** is anywhere along the line of its axis (i.e., along the "no-slip" direction).

Instantaneous Center of Rotation (ICR)

- Given two (or more) standard wheels on a single chassis:

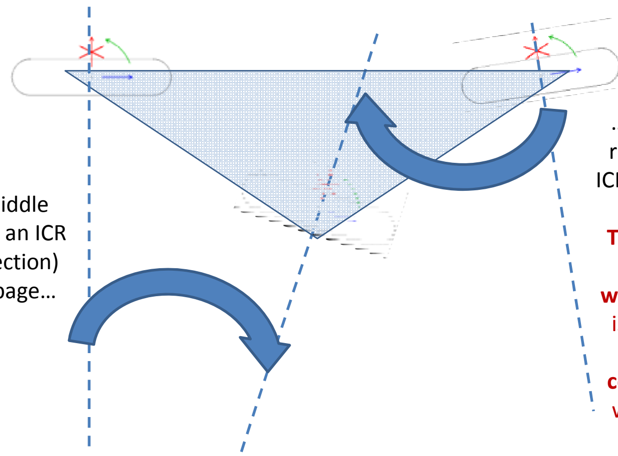


Now, for motion of the chassis to occur, the ICR for every wheel must be the same, which of course defines the ICR for the chassis of the vehicle. Thus, the ICR is the intersection of lines extending the wheel axes. **Recall, this assumes NO WHEEL SLIP.**

Instantaneous Center of Rotation (ICR)

- If all wheel axes do not coincide at a single point, no ICR exists, and no motion can occur without wheel slip!

Left and middle wheels have an ICR (axis intersection) below the page…

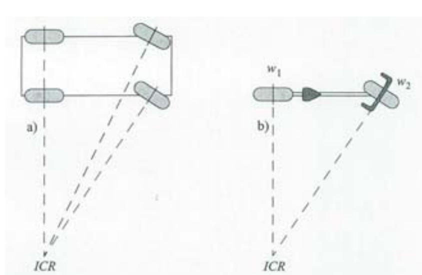


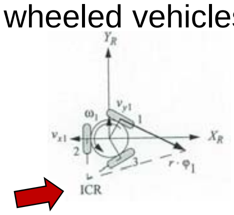
…but middle and right axis have an ICR above the page!

Therefore, no ICR exists for all 3 wheels. No motion is possible, given the **kinematic constraints** of the wheel geometry.

Instantaneous Center of Rotation (ICR)

- Examples for Siegwart of ICR for wheeled vehicles:





c) omnibot

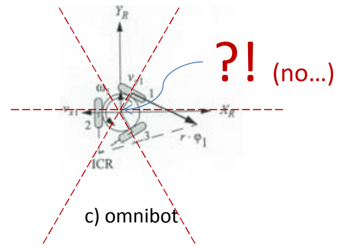
a) Four-wheel car, with Ackerman steering

b) bicycle

(*From Siegwart and Nourbakhsh, pp. 68 and 65, respectively*.)

Instantaneous Center of Rotation (ICR)

- Omnibot example:



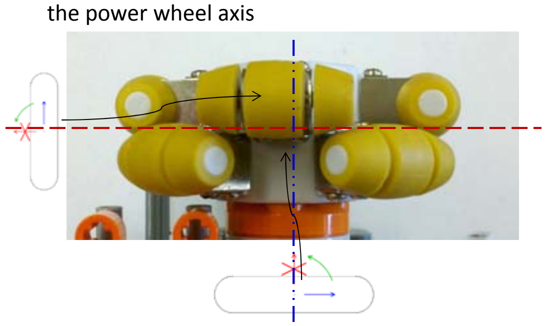
Wait a minute! Isn't the ICR in the CENTER here?

No, because wheels 2 and 3 use their "free rollers" here, while only wheel 1 is powered.

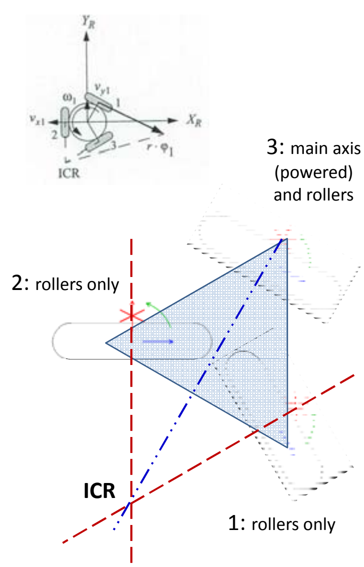
Instantaneous Center of Rotation (ICR)

- Omnibot example :

Each **ROLLER** has an axis of rotation, perpendicular to the power wheel axis



The **POWERED** axis must be controlled actively by motors (not "freely" rotating).



Wheel Types

Fives types. Two categorizations (below).

Case 1: Kinematic constraint is imposed: no lateral slip allowed.

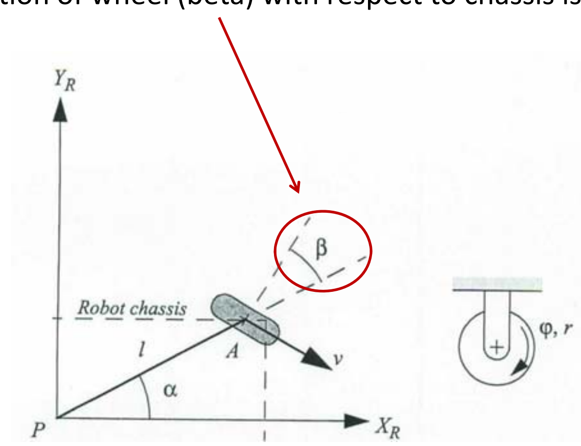
- Fixed standard wheel
- Steerable standard wheel

Case 2: No kinematic constraint, because there are additional rolling degrees of freedom in the wheel, to prevent kinematic constraints.

- Castor wheel
- Omnidirection, or "Swedish", wheel
- Spherical wheel

Fixed standard wheel

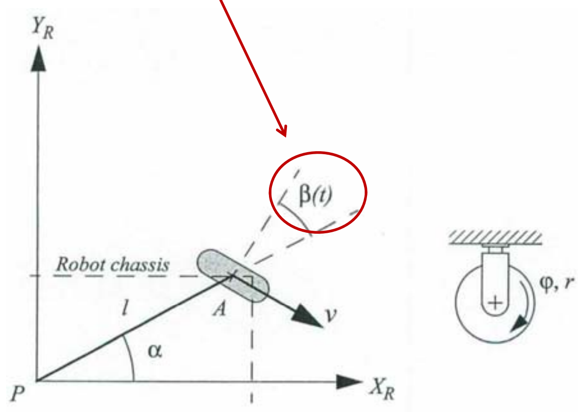
Orientation of wheel (beta) with respect to chassis is FIXED.



(*From Siegwart and Nourbakhsh, p. 54.*)

Steerable standard wheel

Orientation of wheel (beta) can be steered over time, actively.

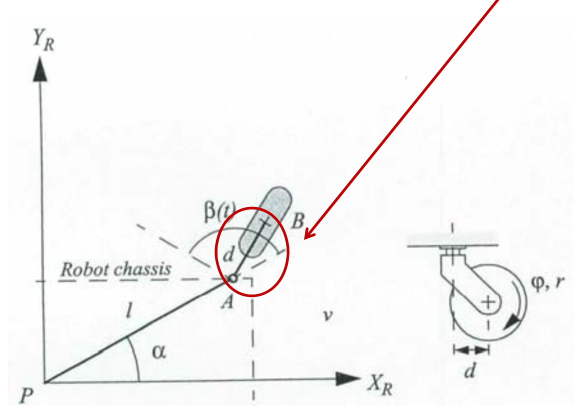


(*From Siegwart and Nourbakhsh, p. 56.*)

Castor wheel

Wheel can again steer wrt chassis, but axis of rotation does NOT pass through the ground contact point. It is offset by distance d.

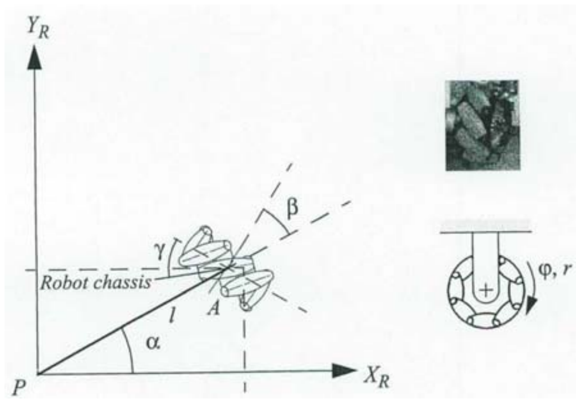
Given any $\dot{\xi}_I$, there exists some $\dot{\varphi}$ and $\dot{\beta}$ such that constraints are met.



(*From Siegwart and Nourbakhsh, p. 57.*)

Omnidirection wheel (Swedish wheel)

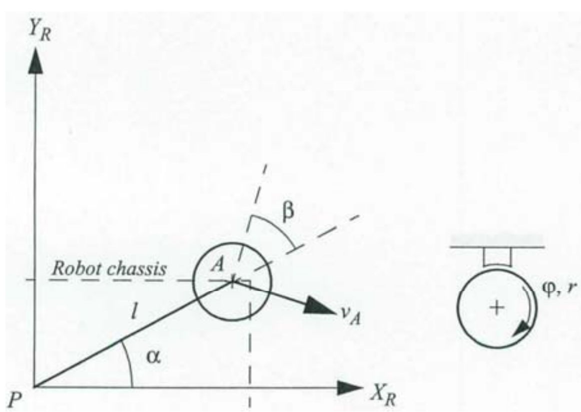
Wheel has both a powered axis and a set of rollers that can spin freely.



(*From Siegwart and Nourbakhsh, p. 59.*)

Spherical wheel

No constraints on motion. Mechanism allow free rotation in any direction.



(*From Siegwart and Nourbakhsh, p. 60.*)

Mobility, Steerability, and Maneuverability

- Degree of **mobility**, δ_m : **Instantaneous** DOF of robot chassis due to commanding different **wheel velocities** (without turning wheels).
- Degree of **steerability**, δ_s : **Instantaneous** DOF of robot chassis due to **reorienting the wheel**. i.e., the DOF of changing the ICR location on the plane, without actually changing the points of contact of the wheels.
- Degree of **maneuverability**, δ_M : The **long-term** DOF of robot.

$$\delta_M = \delta_m + \delta_s$$

Definitions for Mobile Robotics

- Workspace: The set of possible configurations, i.e., of allowable combinations of DOFs (degrees of freedom).
- Degrees of Freedom (DOF): Maneuverability over the long term.
- Differential Degrees of Freedom (DDOF): Mobility, or instantaneous DOF of motion.
- Holonomic robot: One with zero nonholonomic constraints.

Holonomic and Nonholonomic Constraints

- A **holonomic** constraint can be expressed as an explicit function of **position variables, only**. (i.e., not requiring velocity variables.)
- A **nonholonomic** constraint requires a differential relationship. (i.e., velocity terms like $\dot{x}_w$ or $\dot{\xi}_w$ *cannot be avoided* when describing how motion is constrained.)

Examples: