

ECE/ME 179D Holonomic vs Nonholonomic Systems Lecture 9

Topics:

- Wheeled vehicle types: $\delta_M = \delta_m + \delta_s$
- Holonomic constraints
- Examples

Notices:

- HW 2 due date:
- Midterm date:
- Lab 4 due at your Lab 5 session.

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Recall: Mobility, Steerability, and Maneuverability

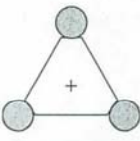
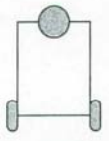
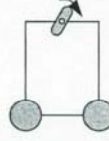
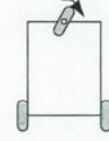
- Degree of **mobility**, δ_m : **Instantaneous** DOF of robot due to **wheel velocities** (without turning wheels). Also called the differential degrees of freedom (DDOF).
- Degree of **steerability**, δ_s : **Instantaneous** DOF of robot due to **reorienting the wheel**. (DOF of ICR on the plane.)
- Degree of **maneuverability**, δ_M : All **instantaneous** DOF.

$$\delta_M = \delta_m + \delta_s$$
- DOF: The **long-term** DOF. For a wheeled vehicle on a plane, this is at most 3. (x, y, and theta)

$$DDOF = \delta_m \leq \delta_M \leq DOF$$

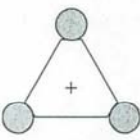
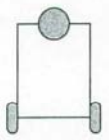
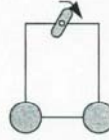
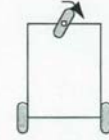
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Wheeled Vehicle Types: $\delta_M = \delta_m + \delta_s$

					
3. δ_M					
1. δ_m					
2. δ_s					

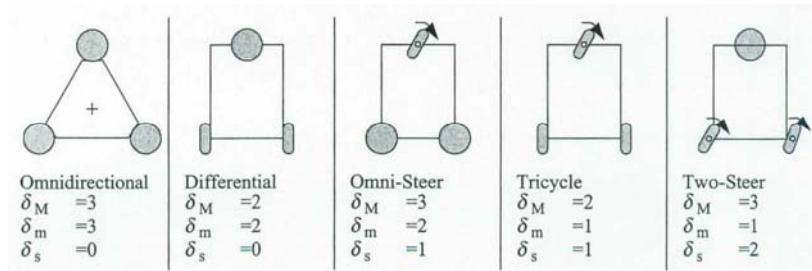
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Wheeled Vehicle Types: $\delta_M = \delta_m + \delta_s$

				
Omnidirectional $\delta_M = 3$ $\delta_m = 3$ $\delta_s = 0$	Differential $\delta_M = 2$ $\delta_m = 2$ $\delta_s = 0$	Omn-steer $\delta_M = 3$ $\delta_m = 2$ $\delta_s = 1$	Tricycle $\delta_M = 2$ $\delta_m = 1$ $\delta_s = 1$	Two-Steer $\delta_M = 3$ $\delta_m = 1$ $\delta_s = 2$

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Wheeled Vehicle Types: $\delta_M = \delta_m + \delta_s$



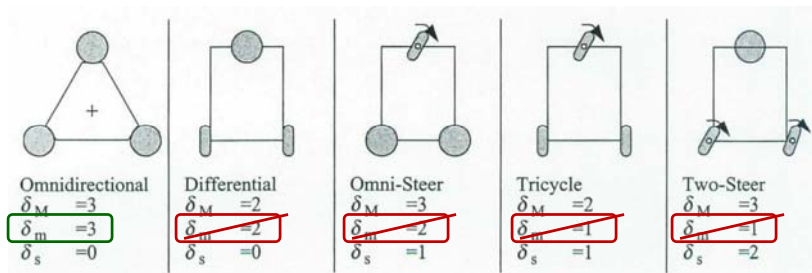
Siegwart and Nourbakhsh define a holonomic wheeled robot in terms of the posture kinematics. i.e., only 3 DOF are required to define a “pose” of a mobile robot. They state (p.77):

“...a robot is holonomic if and only iff DDOF=DOF.”

Recall DDOF is just mobility, δ_m . Which robot(s) above are holonomic, through this (posture kinematics) definition?

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Wheeled Vehicle Types: $\delta_M = \delta_m + \delta_s$



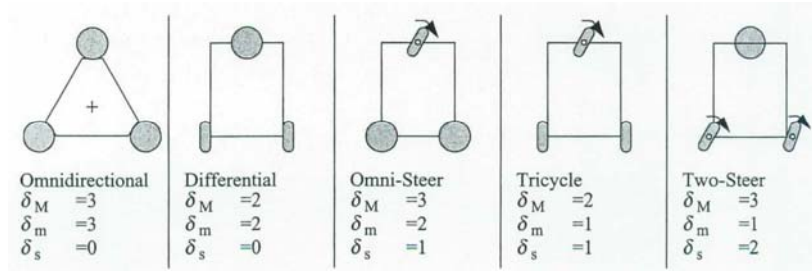
Siegwart and Nourbakhsh define a holonomic wheeled robot in terms of the posture kinematics. i.e., only 3 DOF are required to define a “pose” of a mobile robot. They state (p.77):

“...a robot is holonomic if and only iff DDOF=DOF.”

Recall DDOF is just mobility, δ_m . Which robot(s) above are holonomic, through this (posture kinematics) definition? **Omnidirection, only.**

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Wheeled Vehicle Types: $\delta_M = \delta_m + \delta_s$



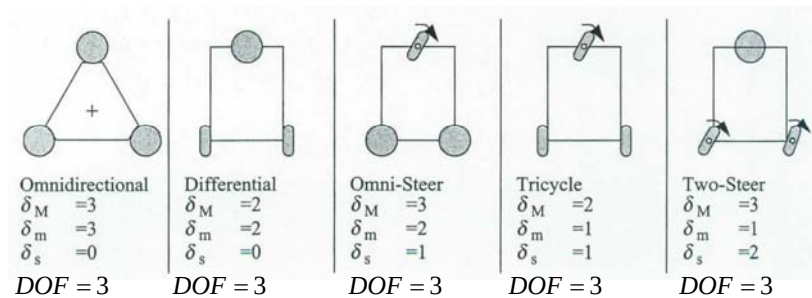
What are the DOF for each?

For what vehicles is $\delta_M < DOF$?

What does this mean?

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Wheeled Vehicle Types: $\delta_M = \delta_m + \delta_s$



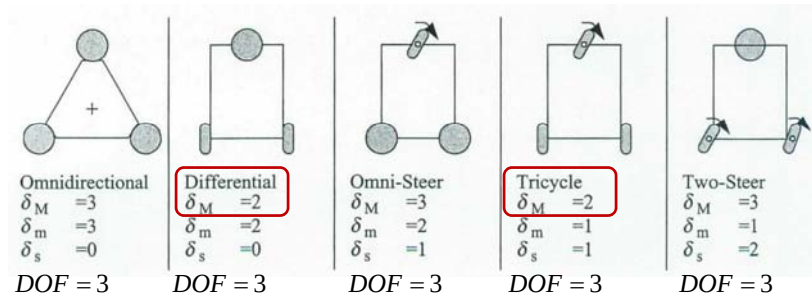
What are the DOF for each? **DOF=3 for every vehicle here, meaning you can "eventually" position into any x,y, theta pose.**

For what vehicles is $\delta_M < DOF$?

What does this mean?

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Wheeled Vehicle Types: $\delta_M = \delta_m + \delta_s$



What are the DOF for each? $DOF=3$ for every vehicle here, meaning you can “eventually” position into any x,y, theta pose.

For what vehicles is $\delta_M < DOF$? Differential drive car and Tricycle

What does this mean? They cannot follow an arbitrary path to a desired final pose. Also, omni-steer and two-steer are not holonomic (kinematic posture sense), since internal DOF(s) must change to follow a path.

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Recall: Holonomic and Nonholonomic Constraints

- A **holonomic** constraint can be expressed as an explicit function of **position variables, only**. (i.e., not requiring velocity variables.)
- A **nonholonomic** constraint requires a differential relationship. (i.e., velocity terms like $\dot{x}_w$ or $\dot{\xi}_w$ *cannot be avoided* when describing how motion is constrained.)

Also:

- For a holonomic system:
 - All constraints are holonomic. The total number of generalized coordinates (complete and independent set) required to describe system configurations is equal to the instantaneous degrees of freedom
- A holonomic constraint:
 - imposes a one-to-one restriction on the total number of **generalized coordinates** ξ_j (required to describe the system configuration) and a restriction of the **instantaneous independent degrees of freedom** $\delta \xi_k$ (aka admissible variations).

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Nonholonomic systems

- **For a nonholonomic system:**
 - The accessible configuration space has a higher dimension than the accessible velocity space.
 - More generalized variables are required to describe the long-term configurations that are achievable than there are DOF for local motion, instantaneously.

$$GC = \xi_j > \delta \xi_k = DOF \longleftrightarrow \text{Nonholonomic}$$

$$GC = \xi_j = \delta \xi_k = DOF \longleftrightarrow \text{Holonomic}$$

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Why care?

- Why care if a system is subject only to holonomic constraints or not?

Answers:

1. Holonomic systems can be much easier to understand, intuitively, and to plan motions for.
2. **Our upcoming development of “Euler-Lagrange” equations of motion requires that the system is holonomic.**

(Note: For lumped-parameter systems, Lagrange’s equations typically provide the most straight-forward method for obtaining equations of motion. E.g., MATLAB’s symbolic toolbox can be used to help derive these equations, once the “Lagrangian” is properly defined.)

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Generalized Variables

- **“Generalized coordinates** are variables that locate a dynamic system wrt a reference frame.” (Williams, JH. “Fundamentals of Applied Dynamics”, 1996.)
- **Completeness:** “A set of generalized coordinates is complete if it is capable of locating all parts of the system at all times.”
- **Independence:** “A set of generalized coordinates is independent if, when all but one of the generalized coordinates are fixed, there remains a continuous range of values for that one coordinate.”

Examples:

1. Point on a plane.

Cartesian coords.

polar coords.

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Generalized Variables

Examples:

2. Rod in a plane.

$\xi_j : x_1, y_1, x_2, y_2$

$\xi_j : x_1, y_1, \theta$

Complete?

Independent?

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Generalized Variables

Examples:

2. Rod in a plane.

$$\xi_j : x_1, y_1, x_2, y_2$$

$$\xi_j : x_1, y_1, \theta$$

Complete? **Both are complete.**

Independent? **But only the righthand case (3 GC's) is independent.**

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Admissible Variations

- An **admissible variation** of a generalized coordinate is a hypothetical instantaneous change from one geometrically allowable state to a “neighboring” geometrically allowable state.
- **Completeness:** A set of admissible variations, ξ_k , is complete if it is capable of representing all geometric variations of the system at all time (over arbitrary, long paths)
- **Independent:** A set of admissible variations, ξ_k , is independent if when all but one of the admissible variations are fixed, there remains a continuous range of (local) values for that one admissible variation. (i.e., if it remains a moveable degree of freedom)
- **Degrees of Freedom:** The number of admissible variations in a complete and independent set is the number of degrees of freedom of the system.
- **Geometric constraint:** any requirement that reduces the # of DOF.

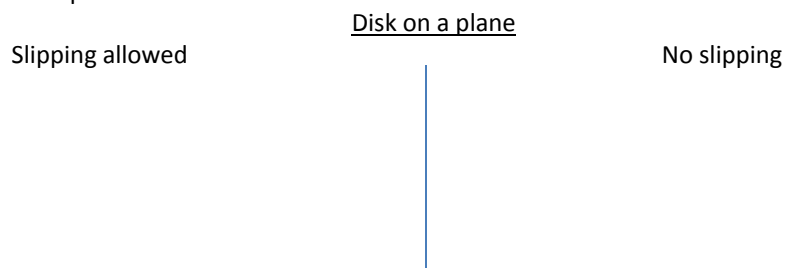
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Complete and independent GC's and DOFs

- GC's are denoted by: ξ_k
- DOFs are denoted by: $\delta \xi_k$

Determining if a system is holonomic.

Examples:



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Review

Holonomic iff: $GC = \xi_j = \delta \xi_k = DOF$

- That is, if the number of generalized coordinates in a complete and independent set equals the number of immediate admissible variations in coordinates.
- Otherwise, “nonholonomic”.
- Iff holonomic, all constraints can be written as:

$$fn(\xi_1, \xi_2, \dots, \xi_n, t) = 0$$

(i.e., a function of geometry, but not of velocity)

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Omnibot revisited...

- Posture kinematics: Only the chassis x, y , and ϕ are considered (to define body pose, only).
- Configuration kinematics: Internal degrees of freedom (wheel rotation angles) are also included. A complete set of GC's would now include x, y, ϕ, q_1, q_2 , and q_3 .

The internal variables have not really “gone away” if we consider only “kinematic posture” of the omnibot. They may be “hidden” within a set of feedback laws for control.