Holonomic vs Nonholonomic Systems **Lecture 9**

Topics:

- Wheeled vehicle types: $\delta_M = \delta_m + \delta_s$
- Holonomic constraints
- Examples

Notices:

- HW 2 due date:
- Midterm date:
- Lab 4 due at your Lab 5 session.

Recall: Mobility, Steerability, and Maneuverability

- Degree of **mobility**, δ_m : **Instantaneous** DOF of robot due to **wheel velocities** (without turning wheels). Also called the differential degrees of freedom (DDOF).

- Degree of **steerability**, δ_s : **Instantaneous** DOF of robot due to **reorienting the wheel**. (DOF of ICR on the plane.)

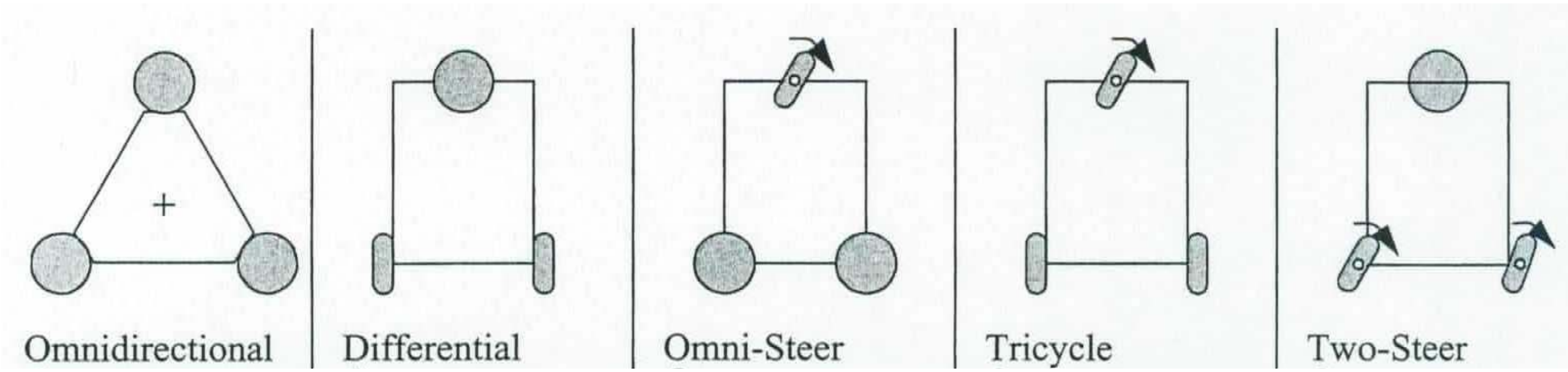
- Degree of **maneuverability**, δ_M : All instantaneous DOF.

$$\delta_M = \delta_m + \delta_s$$

- DOF: The **long-term** DOF. For a wheeled vehicle on a plane, this is at most 3. (x, y, and theta)

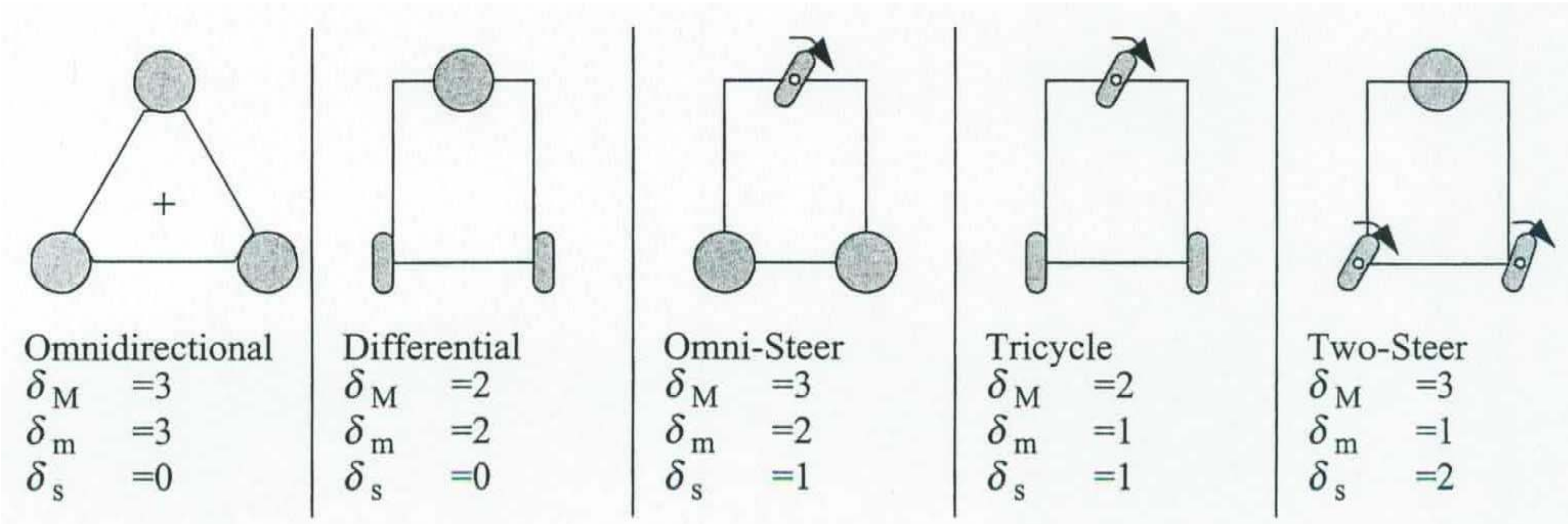
$$DDOF = \delta_m \leq \delta_M \leq DOF$$

Wheeled Vehicle Types: $\delta_M = \delta_m + \delta_s$

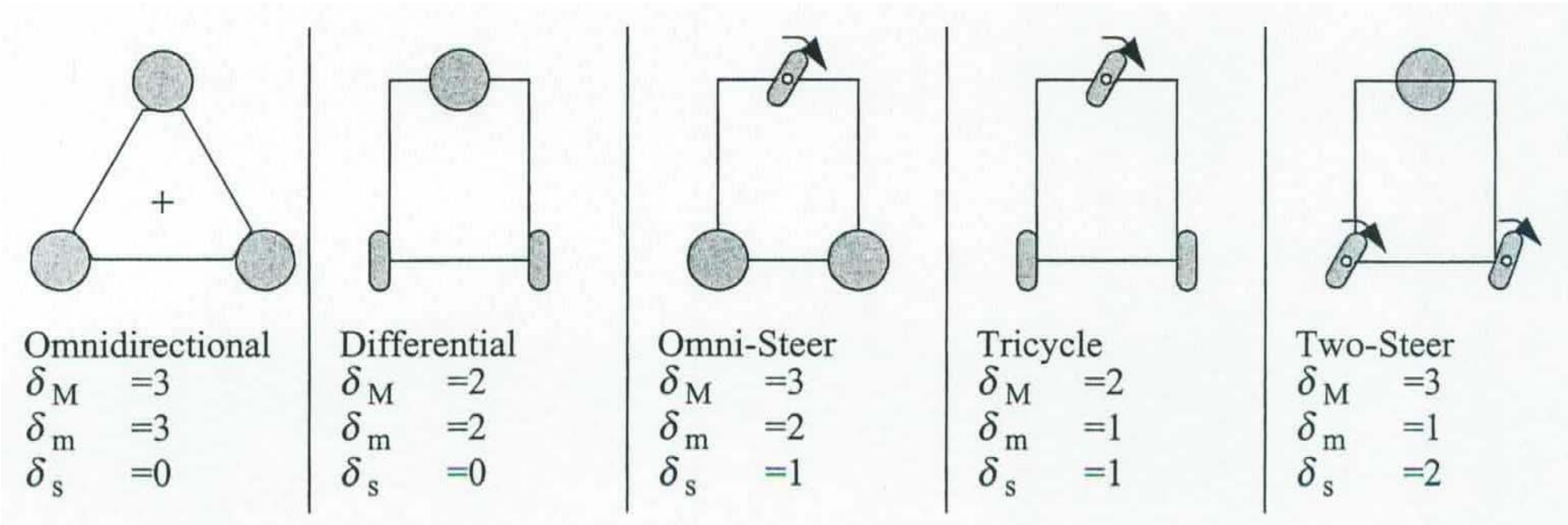


3. δ_M				
1. δ_m				
2. δ_s				

Wheeled Vehicle Types: $\delta_M = \delta_m + \delta_s$



Wheeled Vehicle Types: $\delta_M = \delta_m + \delta_s$

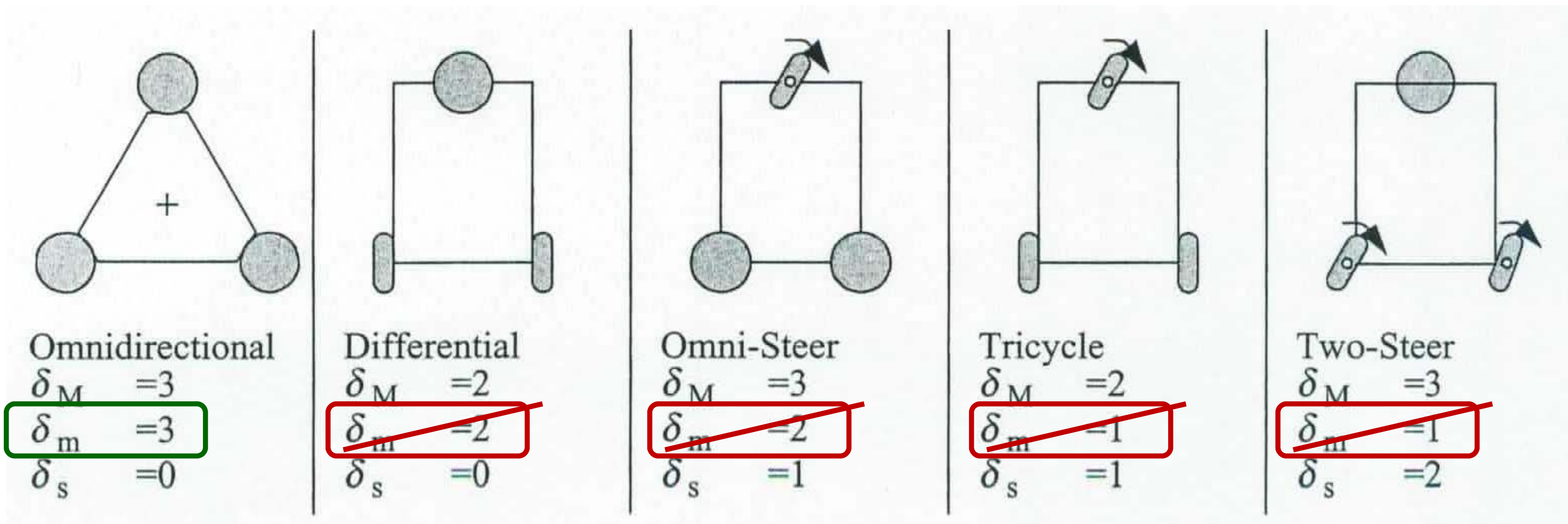


Siegwart and Nourbakhsh define a holonomic wheeled robot in terms of the posture kinematics. i.e., only 3 DOF are required to define a "pose" of a mobile robot. They state (p.77):

"…a robot is holonomic if and only iff DDOF=DOF."

Recall DDOF is just mobility, δ_m . Which robot(s) above are holonomic, through this (posture kinematics) definition?

Wheeled Vehicle Types: $\delta_M = \delta_m + \delta_s$

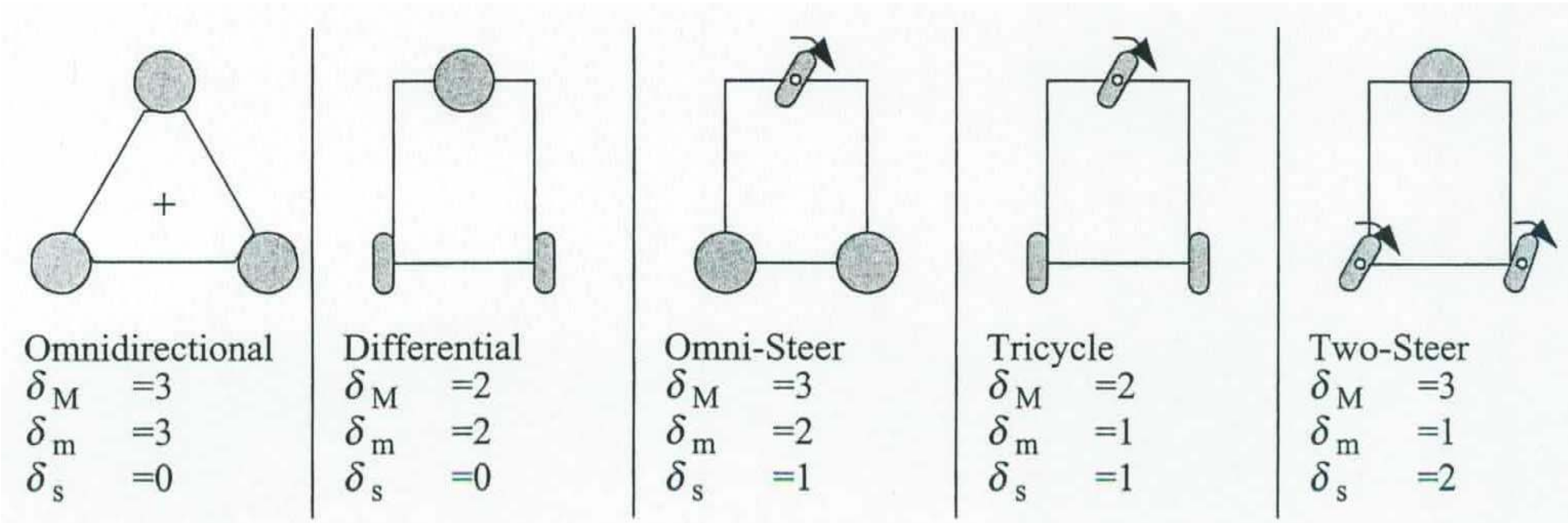


Siegwart and Nourbakhsh define a holonomic wheeled robot in terms of the posture kinematics. i.e., only 3 DOF are required to define a "pose" of a mobile robot. They state (p.77):

"…a robot is holonomic if and only iff DDOF=DOF."

Recall DDOF is just mobility, δ_m . Which robot(s) above are holonomic, through this (posture kinematics) definition? **Omnidirection, only.**

Wheeled Vehicle Types: $\delta_M = \delta_m + \delta_s$

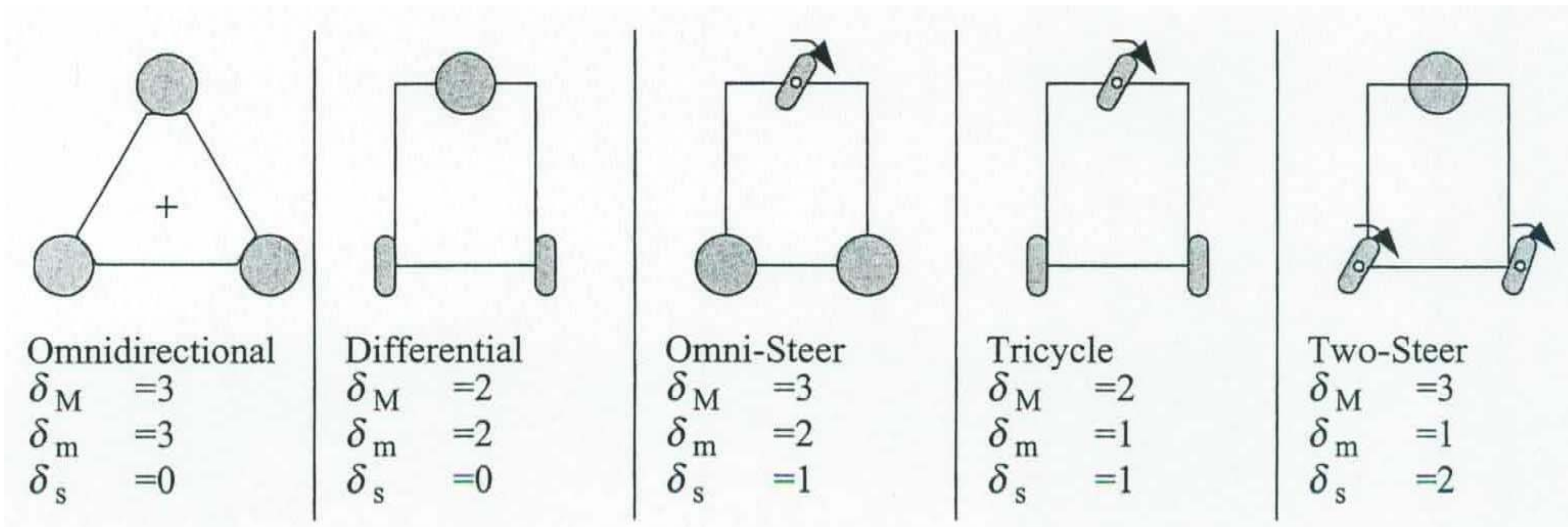


What are the DOF for each?

For what vehicles is $\delta_M < DOF$?

What does this mean?

Wheeled Vehicle Types: $\delta_M = \delta_m + \delta_s$



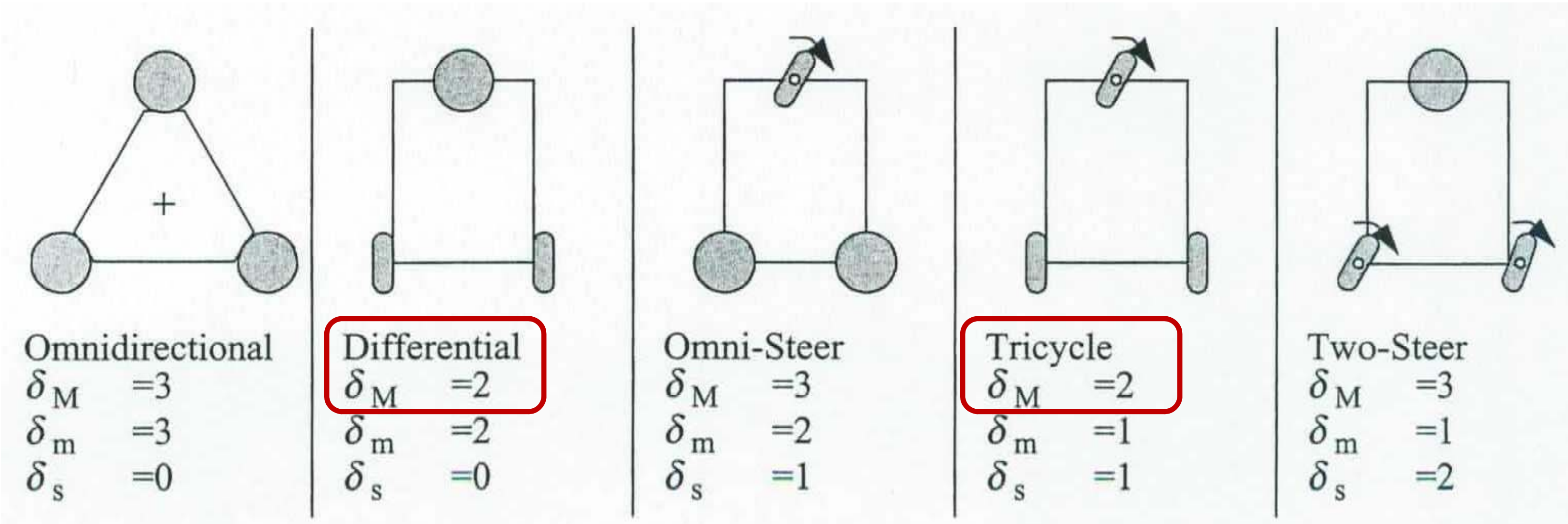
$DOF = 3$ $\quad$ $DOF = 3$ $\quad$ $DOF = 3$ $\quad$ $DOF = 3$ $\quad$ $DOF = 3$

What are the DOF for each? DOF=3 for every vehicle here, meaning you can "eventually" position into any x,y, theta pose.

For what vehicles is $\delta_M < DOF$?

What does this mean?

Wheeled Vehicle Types: $\delta_M = \delta_m + \delta_s$



$DOF = 3$ $\quad DOF = 3$ $\quad DOF = 3$ $\quad DOF = 3$ $\quad DOF = 3$

What are the DOF for each? DOF=3 for every vehicle here, meaning you can "eventually" position into any x,y, theta pose.

For what vehicles is $\delta_M < DOF$? Differential drive car and Tricycle

What does this mean? They cannot follow an arbitrary path to a desired final pose. Also, omni-steer and two-steer are not holonomic (*kinematic posture sense*), since internal DOF(s) must change to follow a path.

Recall: Holonomic and Nonholonomic Constraints

- A **holonomic** constraint can be expressed as an explicit function of **position variables, only**. (i.e., not requiring velocity variables.)
- A **nonholonomic** constraint requires a differential relationship. (i.e., velocity terms like $\dot{x}_w$ or $\dot{\xi}_w$ *cannot be avoided* when describing how motion is constrained.)

Also:

- For a holonomic system:
  - All constraints are holonomic. The total number of generalized coordinates (complete and independent set) required to describe system configurations is equal to the instantaneous degrees of freedom
- A holonomic constraint:
  - imposes a one-to-one restriction on the total number of **generalized coordinates** ξ_j (required to describe the system configuration) and a restriction of the **instantaneous independent degrees of freedom** $\delta\xi_k$ (aka admissible variations).

Nonholonomic systems

- For a nonholonomic system:
  - The accessible configuration space has a higher dimension that the accessible velocity space.
  - More generalized variables are required to describe the long-term configurations that are achievable than there are DOF for local motion, instantaneously.

$$\boxed{GC = \xi_j > \delta\xi_k = DOF \quad \longleftrightarrow \quad \text{Nonholomic}}$$

$$\boxed{GC = \xi_j = \delta\xi_k = DOF \quad \longleftrightarrow \quad \text{Holonomic}}$$

Why care?

- Why care if a system is subject only to holonomic constraints or not?

Answers:

1. Holonomic systems can be much easier to understand, intuitively, and to plan motions for.
2. Our upcoming development of “Euler-Lagrange” equations of motion requires that the system is holonomic.

   (Note: For lumped-parameter systems, Lagrange’s equations typically provide the most straight-forward method for obtaining equations of motion. E.g., MATLAB’s symbolic toolbox can be used to help derive these equations, once the “Lagrangian” is properly defined.)

Generalized Variables

- *"**Generalized coordinates***are variables that locate a dynamic system wrt a reference frame." (Williams, JH. "Fundamentals of Applied Dynamics", 1996.)
- **Completeness**: "A set of generalized coordinates is complete if it is capable of locating all parts of the system at all times."
- **Independence**: "A set of generalized coordinates is independent if, when all but one of the generalized coordinates are fixed, there remains a continuous range of values for that one coordinate."

<u>Examples:</u>

1. Point on a plane.

Cartesian coords.

polar coords.

Generalized Variables

Examples:

2. Rod in a plane.

$$\xi_j : x_1, y_1, x_2, y_2$$

$$\xi_j : x_1, y_1, \theta$$

Complete?

Independent?

Generalized Variables

Examples:

2. Rod in a plane.

$\xi_j : x_1, y_1, x_2, y_2$

$\xi_j : x_1, y_1, \theta$

Complete? Both are complete.

Independent? But only the righthand case (3 GC's) is independent.

Admissible Variations

- An ***admissible variation*** of a generalized coordinate is a hypothetical instantaneous change from one geometrically allowable state to a "neighboring" geometrically allowable state.

- **Completeness**: A set of admissible variations, ξ_k , is complete if it is capable of representing all geometric variations of the system at all time (over arbitrary, long paths)

- **Independent**: A set of admissible variations, ξ_k , is independent if when all but one of the admissible variations are fixed, there remains a continuous range of (local) values for that one admissible variation. (i.e., if it remains a moveable degree of freedom)

- **Degrees of Freedom**: The number of admissible variations in a complete and independent set is the number of degrees of freedom of the system.

- **Geometric constraint**: any requirement that reduces the # of DOF.

Complete and independent GC's and DOFs

- GC's are denoted by: ξ_k
- DOFs are denoted by: $\delta\xi_k$

Determining if a system is holonomic.

Examples:

<u>Disk on a plane</u>

Slipping allowed

No slipping

Review

Holonomic iff: $GC = \xi_j = \delta\xi_k = DOF$

- That is, if the number of generalized coordinates in a complete and independent set equals the number of immediate admissible variations in coordinates.
- Otherwise, "nonholonomic".
- Iff holonomic, all constraints can be written as:

$$fn(\xi_1, \xi_2, \ldots, \xi_n, t) = 0$$

(i.e., a function of geometry, but not of velocity)

Omnibot revisited…

- Posture kinematics: Only the chassis x,y, and phi are considered (to define body pose, only).

- Configuration kinematics: Internal degrees of freedom (wheel rotation angles) are also included. A complete set of GC's would now include x,y,phi,q1,q2, and q3.

  The internal variables have not really "gone away" if we consider only "kinematic posture" of the omnibot. They may be "hidden" within a set of feedback laws for control.