ECE/MAT 211A winter 07

HWK#4 solutions

Proof of (5•3-29a,b)

(1) 5•3-29a

$$\kappa_0 = MS/\hbar^2 \tag{5•3-27}$$

$$K', \kappa \to \infty; \quad K'L, \kappa L \to 0, \tag{5•3-25a,b}$$

$$K'^2 L, \kappa^2 L \to 2M|S|/\hbar^2 = 2\kappa_0 \tag{5•3-26}$$

From (5•3-25a,b) we know

$$\cos K'L \approx 1 \quad \text{and} \quad \sin K'L \approx K'L \tag{1a,b}$$

Inserting (1a,b) into (5•3-5) yields

$$P_{11} = \cos K'L - i\frac{K^2 + K'^2}{2KK'}\sin K'L \approx 1 - i\frac{K^2 + K'^2}{2KK'}(K'L) = 1 - i\frac{K'^2\left(1 + \frac{K^2}{K'^2}\right)}{2K}L \tag{2}$$

Since $K' \to \infty$, $\frac{K^2}{K'^2} \to 0$, and (2) becomes

$$1 - i\frac{K'^2(1+0)}{2K}L = 1 - i\frac{K'^2 L}{2K} \approx 1 - i\frac{2\kappa_0}{2K} = 1 - i\frac{\kappa_0}{K} \tag{3}$$

So $\boxed{P_{11} = 1 - i\frac{\kappa_0}{K}}$ QED (4)

(2) 5•3-29b

Similarly,

$$P_{12} = +i\frac{K^2 - K'^2}{2KK'}\sin K'L \approx +i\frac{K^2 - K'^2}{2KK'}(K'L) = +i\frac{K'^2\left(-1 + \frac{K^2}{K'^2}\right)}{2K}L \approx -i\frac{K'^2 L}{2K} \approx -i\frac{2\kappa_0}{2K} \tag{5}$$

So $\boxed{P_{12} = -i\frac{\kappa_0}{K}}$ QED (6)

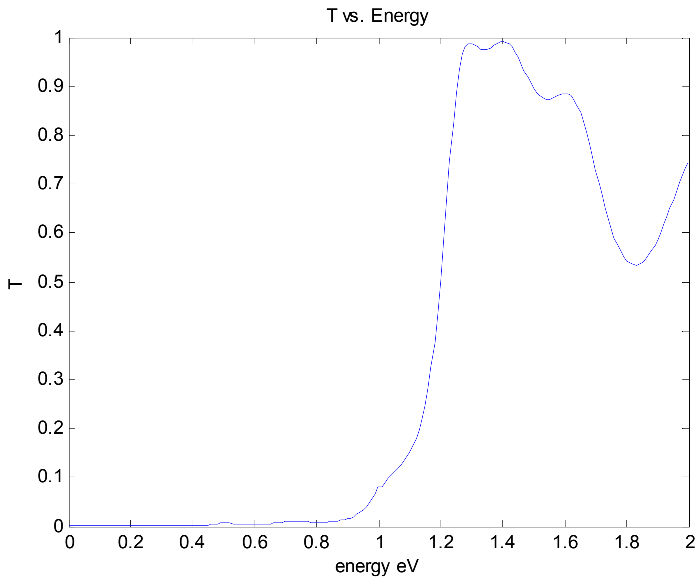
#5•5-1: Transmission Properties of a Composite Barrier

(a) complete code in Matlab:

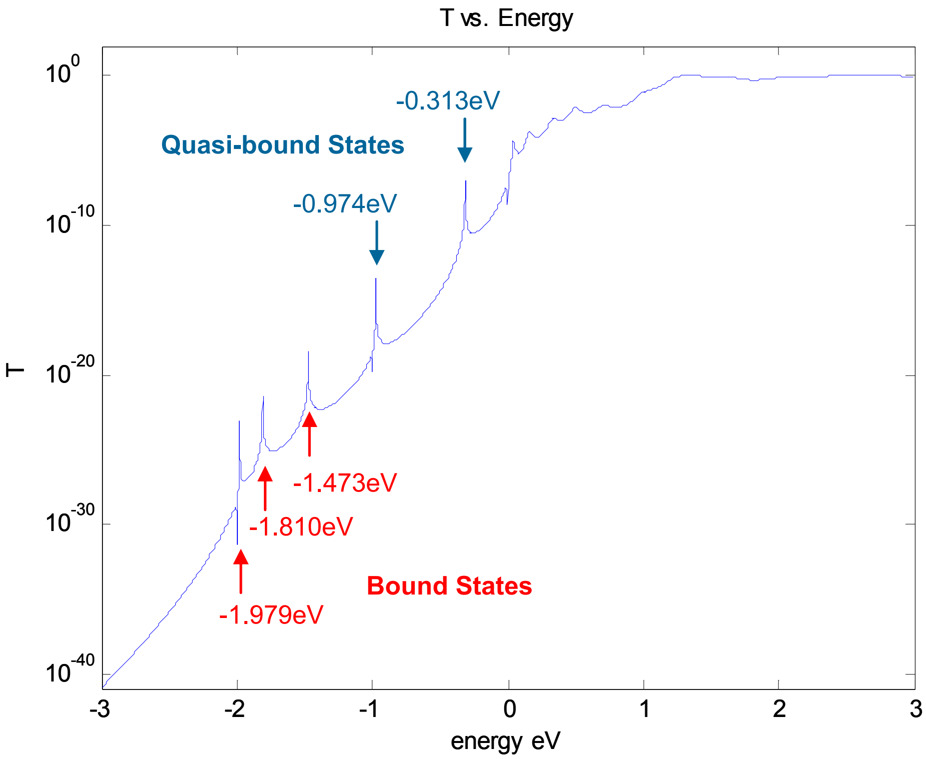
```
% ECE211A HW4
% 5.5-1
%      _ _ _ _  V=1
%     |       |
%     |       |
% V=0 _|       | x=1        x=3  _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ x=6
%    x=0      |              |                        |
%             |              |                        |
%             |              |                        |_ V=-1
%             |              |
%             |              |
%             |_ _ _ _ _ _ _ _ _ | V=-2

clc;clear all;close all;
M = 9.1e-31; %kg
h_bar = 1.054e-34; %J*s
q = 1.6e-19;
E = 0:1e-2:2; %(b) plot
%E = -2:1e-3:0; %(c) look for bound states
E = E-1e-5;

discontinuities = input('number of discontinuities = ');
n = discontinuities+1;
if n>=0
   L = 1:1:n; %nm
   V = 1:1:n; %eV
   s=1;
   while s<n+1
      length = input('length of flat potential = ');
      L(s) = length;
      potential = input('height of potential = ');
      V(s) = potential;
      s=s+1;
   end
end
L = L*1e-9;
x = 1;
temp = size(E);
while x < temp(2)+1;
   s=1;
   K = 1:1:n;
   while s<n+1
      K(s) = sqrt(2*M/(h_bar)^2*q*(E(x)-V(s)));
      s=s+1;
   end

   i=1;
   aP_matrix = eye(2);
   while i<n-1
      aP_11 = K(i)+K(i+1);
      aP_12 = K(i)-K(i+1);
```

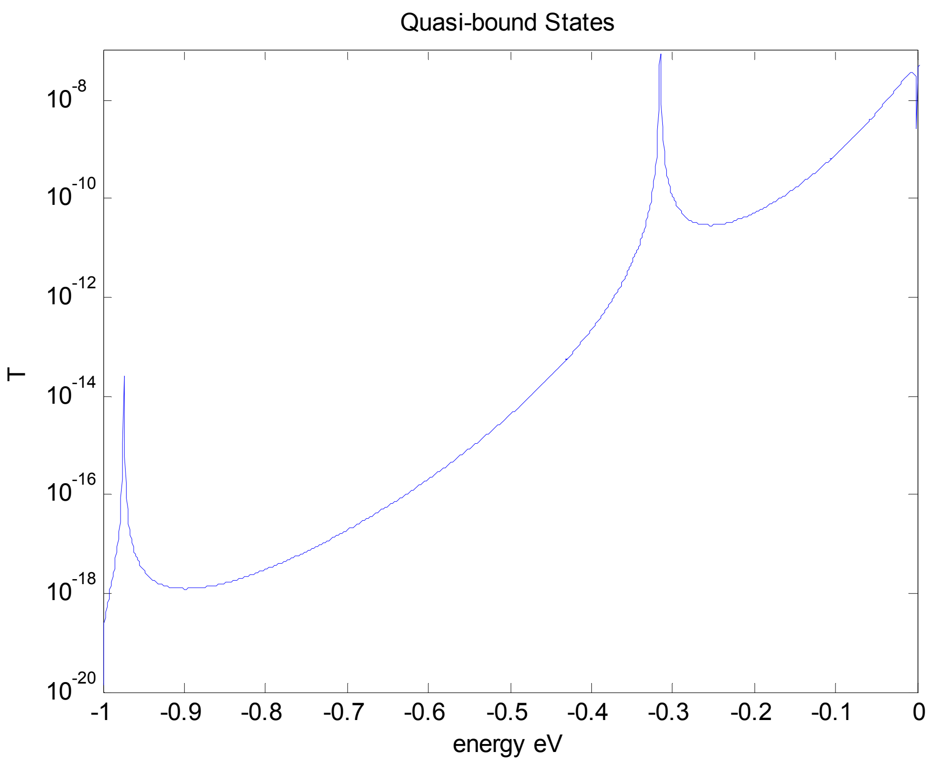
```
        aP_22 = conj(aP_11);
        aP_21 = conj(aP_12);
        aP_pot = 1/(2*sqrt(K(i)*K(i+1))).*[aP_11 aP_12; aP_21 aP_22];

        aP_TL_11 = exp(-j*K(i+1).*L(i+1));
        aP_TL_12 = 0;
        aP_TL_22 = exp(j*K(i+1).*L(i+1));
        aP_TL_21 = 0;
        aP_TL = [aP_TL_11 aP_TL_12; aP_TL_21 aP_TL_22];

        aP_matrix = aP_matrix*aP_pot*aP_TL;
        i=i+1;
    end
    bP_11 = K(n-1)+K(n);
    bP_12 = K(n-1)-K(n);
    bP_22 = conj(bP_11);
    bP_21 = conj(bP_12);
    bP_matrix = 1/(2*sqrt(K(n-1)*K(n))).*[bP_11 bP_12; bP_21 bP_22];

    abP_matrix = aP_matrix*bP_matrix;

    P_11 = abP_matrix(1);
    T(x) = 1./(P_11.*conj(P_11));
    x=x+1;
end
plot(E,T),ylabel('T'),xlabel('energy eV'),axis([0 2 0 1])
%semilogy(E,T),ylabel('T'),xlabel('energy eV'),axis([0 2 0 1])
%plot(E,T),ylabel('T'),xlabel('energy eV'),axis([-3 3 0 1])
%semilogy(E,T),ylabel('T'),xlabel('energy eV'),axis([-3 3 0 1e2])
%semilogy(E,T),ylabel('T'),xlabel('energy eV'),axis([-2.2 0.5 1e-35 1e-3])
%semilogy(E,T),ylabel('T'),xlabel('energy eV'),axis([-1 0 1e-20 1e-7])
%semilogy(E,T),ylabel('T'),xlabel('energy eV')
```

(b)



(c)


T vs. Energy
-0.313eV
Quasi-bound States
-0.974eV
-1.473eV
-1.810eV
Bound States
-1.979eV
T
energy eV


Quasi-bound States
T
energy eV

Exercise on page 163

(1) For r>0, $\delta(r)=0$

$$\frac{d}{d\theta}=\frac{d}{d\theta}=0 \text{, since } G(r)=\frac{e^{ikr}}{4\pi r} \text{is only r – dependent}$$

$$\nabla^2 G(r)+k^2 G(r)$$

$$= \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dG(r)}{dr}\right)+k^2G(r) = \frac{1}{r^2}\frac{d}{dr}\left(r^2\left(\frac{ik\cdot e^{ikr}}{4\pi r}-\frac{e^{ikr}}{4\pi r^2}\right)\right)+k^2\frac{e^{ikr}}{4\pi r}$$

$$= \frac{1}{r^2}\frac{d}{dr}\left(\frac{ikr\cdot e^{ikr}}{4\pi}-\frac{e^{ikr}}{4\pi}\right)+k^2\frac{e^{ikr}}{4\pi r} = \frac{1}{r^2}\left(\frac{ik\cdot e^{ikr}}{4\pi}-\frac{k^2r\cdot e^{ikr}}{4\pi}-\frac{ik\cdot e^{ikr}}{4\pi}\right)+k^2\frac{e^{ikr}}{4\pi r}$$

$$= -k^2\frac{e^{ikr}}{4\pi r}+k^2\frac{e^{ikr}}{4\pi r} = 0 = -\delta(r) \quad \text{QED} \tag{1}$$

(2) For $r\to 0$

$$-\int_V \delta(r)dV=-1 \tag{2}$$

$$\int_V \nabla^2 G dV=\oint_S(\nabla G)\cdot dS=\left(\int_0^{2\pi}d\phi\int_0^{\pi}\sin\theta\cdot d\theta\right)r^2\frac{dG}{dr}=4\pi r^2\left(\frac{ik\cdot e^{ikr}}{4\pi r}-\frac{e^{ikr}}{4\pi r^2}\right)$$

$$=ikr\cdot e^{ikr}-e^{ikr}\approx 0(1)-1=-1 \tag{3}$$

$$k^2\int_V GdV=k^2\int_0^{2\pi}d\phi\int_0^{\pi}\sin\theta\cdot d\theta\int_0^r r^2\frac{e^{ikr}}{4\pi r}dr=\frac{k^2\cdot 2\pi\cdot 2}{4\pi}\int_0^r re^{ikr}dr=k^2\left[\frac{re^{ikr}}{ik}+\frac{e^{ikr}}{k^2}\right]_0^r$$

$$=k^2\left[\frac{re^{ikr}}{ik}+\frac{e^{ikr}}{k^2}-\frac{1}{k^2}\right]\xrightarrow{r\to 0}k^2\left[0+1-1\right]=0 \tag{4}$$

so (3) + (4) = (2) QED