ECE215A, Winter 2010, E.R. Brown

HW#2 Problems, Elasticity and Piezoelectricity of Solids

1. Assume that a cubic solid having a cubic lattice is stressed in *compression* by a uniaxial piston along a [100] axis (assume to be the x axis) of the crystal, but is "clamped" along the y and z axes. Use the notation e_{xx} for the strain; P_{xx}, P_{yy}, and P_{zz} for the stresses.

(a) Find the values of the strain terms e_{yy} and e_{zz} .
(b) Derive expressions for the stress terms P_{XX} and P_{ZZ} in terms of e_{xx} and stiffness coefficients.
(c) Evaluate the stress terms P_{XX} and P_{ZZ} assuming $e_{xx} = 0.01$ and that the cubic material is Si for which $C_{11} = 1.66\text{x}10^{11}$ N/m^2, $C_{12} = 0.64\text{x}10^{11}$ N/m^2 , $C_{44} = 0.80\text{x}10^{11}$ N/m^2.
(d) Now assume the cubic crystal is replaced by an *isotropic, homogeneous* solid. How many nonzero elements of stiffness matrix are there ? How many independent stiffness coefficient are there ?
(e) Now suppose the clamps are removed and the sample is "free" along y and z. Write the relationship between e_{xx} and P_{xx} , and between e_{yy} and P_{xx} in terms of Young's modulus and Poisson's ratio.

2. It is often easier to measure the compliance coefficients than the stiffness coefficients because of the relative ease of applying a known uniaxial pressure and measuring the displacement. Suppose we have a solid whose compliance matrix is given as follows [in units of 10^{-11} m^2/N]

0.7664	-0.2130	-0.2130	0	0	0
-0.2130	0.7664	-0.2130	0	0	0
-0.2130	-0.2130	0.7664	0	0	0
0	0	0	1.2563	0	0
0	0	0	0	1.2563	0
0	0	0	0	0	1.2563

(a) What is the form of the stiffness matrix ?
(a) What are the stiffness coefficients (accurate to 3 decimal points) ?
(clue: this problem is very easy with a matrix math tool, such as MATLAB).

3. Assume that a 1 cm^3 Si cube is pulled in tension by a uniaxial force of 10 N along the cubic **x** axis, but is free along the **y** and **z** axes.

(a) Evaluate the stress along the **x** axis (including sign). Write down an analytic expression for the strain along the **x** axis in terms of the Young's modulus. Evaluate this strain assuming Y = $1.3\text{x}10^{11}$ N/m^2.
(a) Write down expressions for the lateral strains (**y** and **z** axes) and evaluate them as in (a) using σ = 0.28.

4. An i*sotropic* solid is subject to tension along the x axis

(a) Find expressions for the stiffness coefficients in terms of Young's modulus and Poisson's ratio.
(b) Now find the Lame coefficients λ_L and μ_L in terms of Young's modulus and Poisson's ratio.

5. It is well known that purified aluminum – the metal of choice in silicon VLSI - has a Young's modulus of 73 GPa and a Poisson ratio of 0.33. Another very important solid in VLSI is amorphous SiO_2 (glass). Although the glasses vary between types slightly, good "ballpark" numbers are: Young's modulus = 85 GPa, Poisson ratio = 0.25. To first order, both of these solids can be assumed isotropic.

(a) From these values, calculate the Lame coefficients λ_L and μ_L.
(b) From the Lame coefficients, calculate the stiffness coefficients: C_{11}, C_{12}, and C_{44}.

6. This problem solves the generic wave equation $\frac{\partial^2 F}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 F}{\partial t^2} = 0$ in a manner first carried out by the great mathematician L. Euler.

(a). First make a change of variables u = x +vt and w = x –vt and transform the term $\partial^2 F / \partial x^2$ into dependence on u and w.

(b) Using the same change of variables, transform the term $\partial^2 F / \partial t^2$ into dependence on u and w.

(c) Write down the new wave equation for F in terms of the variables u and w.

(d)Write down the general solution to the transformed wave equation (hint second-order equations generally have two linearly independent solutions).

7. Speed of elastic waves in an isotropic solid.

(a)Find a formula for the velocity of a longitudinal acoustic wave in an isotropic solid along the **x**, **y**, or **z** directions in terms of the stiffness and Lame coefficients.

(b) Same as (a) but for a shear acoustic wave.

(c) Now work out the formula for the velocity along the **x + y + z** direction.

(d) Evaluate the longitudinal and shear velocities in isotropic aluminum and glass using the data from Problem from 5.

8. AlN as a Piezoelectric Material. AlN is an emerging piezoelectric material with the advantage of being producible in thin-film form by epitaxial techniques on substrates (e.g., SiC) of very good mechanical (i.e., high stiffness) and electrical (i.e., high resistivity) properties. Suppose you have an AlN film in the x-y plane in a completely "clamped" state (e.g., sandwiched between two stiffer materials) and subjected to a uniform external electric field of E_0 = 10 KV/cm along the z axis.

(a). Write the appropriate connection equations between the stress and E_0 and evaluate all nonzero stress components.

(b) Write and evaluate an expression between the surface charge density in the two interface places and E_0.

(c) Now suppose you wish to use the AlN film as a MEMS cantilever by unclamping it completely. For the same application of E field, write down the connection equations between the strain and E_0.

(d) Use Matlab or your favorite computational tool to invert the relations and find the values of the first three strain components, S_1, S_2, and S_3 if E_0 = 10 KV/cm.

(clues: "clamped" means all the strain components are zero, "unclamped" means that all stress components are zero; you will want to use the 6x3 form of the piezoelectric stress matrix for this problem; the non-zero stiffness coefficients for AlN are given below).

	Piezoelectric Constants (Coulomb/m^2)						$\boldsymbol{\varepsilon_r}$
Material	**e11**	**e14**	**e15**	**e22**	**e31**	**e33**	$\boldsymbol{\varepsilon_r}$
AlN			-0.48		-0.58	1.55	8.5

AlN Stiffness Coefficients (x10^{10} Newton/m^2)						
C11	**C12**	**C13**	**C14**	**C33**	**C44**	**C66**
41	14	10		39	12	13.5