Homework #7 Solutions

Problem 1

(a) We have $R_n = R_p$, and in the natural lifetime approximation, $R_n = n/\tau_e$, $R_p = p/\tau_h \Rightarrow \tau_e = n\tau_h / p$. So if we assume all donors are ionized, $n \approx 10^{16}/cm^3$, and $p = n_i^2/n \approx \left(1.14\times10^{10}\right)^2/10^{16}$ or $p = 1.3\times10^4/cm^3$. Thus, if $\tau_h = 200\,\mu s,\ \tau_e = n\tau_h/p = 1.5\times10^8\ s$.

(b) The equilibrium generation rate is determined by detailed balance whereby $G_n = R_n = n/\tau_e = 6.5\times10^7\ s^{-1} \Rightarrow G_p = R_p = p/\tau_h = 6.5\times10^7\ s^{-1}$.

(c) The recombination rate is simply $R_n = R_p = 6.5\times10^7\ s^{-1}$.

Problem 2.

(a) n-type Si, $N_D = 2\times10^{16}\ cm^{-3}$. $D^* \equiv \dfrac{\left(n_0 + p_0 + 2\delta p\right)D_n D_p}{\left(n_0 + \delta n\right)D_n + \left(p_0 + \delta p\right)D_p}$.

$n_0 p_0 = n_i^2 \approx \left(1.14\times10^{10}\right)^2$ for Si @ 300 K $\Rightarrow p_0 \simeq 6.6\times10^3$ for $n_0 = 2\times10^{16}$. Hence, $p_0 << n_0$ and $\delta p < n_0$ (low level injection) so that $D^* \approx n_0 D_n D_p / n_0 D_n \approx D_p$. From Einstein relation $D_p = \mu_e k_B T / e,\ \mu_h = 471 cm^2/V-s$ (intrinsic Si @ 300 K) so that $D^* \simeq D_p \cong 12.2\, cm^2/s$ @ 300 K. For the ambipolar mobility, we get

$$\mu^* = \frac{\left(p_0 - n_0\right)\mu_e\mu_h}{n\mu_e + p\mu_h} = \frac{\left(p_0 - n_0\right)\mu_e\mu_h}{\left(n_0 + \delta n\right)\mu_e + \left(p_0 + \delta p\right)\mu_h} \approx \frac{-n_0\mu_e\mu_h}{n\mu_e} \approx -\mu_h$$

So,

$$\mu^* \approx -\mu_h \cong -471 cm^2/V-s$$

(b) For pair-wise generation and recombination, the electron and hole lifetimes, are $\dfrac{1}{\tau^*} = \dfrac{1}{\tau_{p0}}\left[1 + \dfrac{p_0}{n_0} + \dfrac{\delta p}{n_0}\right] \approx \dfrac{1}{\tau_{p0}}$. Thus, $\tau^* \approx \tau_{p0} = 240\,\mu s$. We get τ_{n0} from equivalence of recombination rates in equilibrium:

$$n_0/\tau_{n0} = p_0/\tau_{p0} \text{ or } \tau_{n0} = n_0\tau_{p0}/p_0 = n_0^{\,2}\tau_{p0}/n_i^2 \approx 7.3\times10^8\ s$$

Problem 3.

Governing equation for minority holes is one dimensional drift-diffusion equation if both E and G' have no dependence on y or z. Away from g' region, we can write (in steady state)

This can be written $D^*\dfrac{d^2\delta p}{dx^2} + \mu^* E_0 \dfrac{d\delta p}{dx} - \dfrac{\delta p}{\tau^*} = 0$. This is a linear ordinary differential equation having the solution $\delta p = C_1 e^{\Gamma_1 x} + C_2 e^{\Gamma_2 x}$. To find Γ_1 and Γ_2 we substitute the form $C\exp(\Gamma x)$ into the above equation to get $C_1\left(D^*\Gamma^2 e^{\Gamma x} + \mu^* E_0 \Gamma e^{\Gamma x} - e^{\Gamma x}/\tau^*\right) = 0$ or $D^*\Gamma^2 + \mu^* E_0\Gamma - 1/\tau^* = 0$. This is just a quadratic equation with solution

$$\Gamma_{\pm} = \left[-\mu^* E_0 \pm \sqrt{\left(\mu^* E_0\right)^2 + 4D^*/\tau^*} \right] \left(\frac{1}{2D^*} \right)$$

$$= \frac{-\mu^* E_0 \tfrac{1}{2}\sqrt{\tau^*/D^*} \pm \sqrt{1 + \left(\mu^* E_0\right)^2 \tau^*/4D^*}}{\sqrt{D^* \tau^*}}$$

So if we define $\Rightarrow \gamma = -\mu^* E_0 (1/2)\sqrt{\tau^*/D^*}$, we get the elegant solution

$$\Gamma_{\pm} = \left(\gamma \mp \sqrt{1+\gamma^2}\right)/L$$

where $L = \sqrt{D^* \tau^*}$. For n-type material, we have

$$D^* \approx \frac{n_0 D_n D_p}{n_0 D_p} \approx D_p \qquad \mu^* \approx \frac{\left(p_0 - n_0\right)\mu_n \mu_p}{n_0 \mu_n} \approx -\frac{n_0 \mu_n \mu_p}{n_0 \mu_n} \approx -\mu_p$$

$$\frac{1}{\tau^*} \equiv \frac{1}{\tau_{p0}} \left[1 + \frac{p_0}{n_0} + \frac{\delta p}{n_0} \right] \approx \frac{1}{\tau_{p0}}$$

So that,

$$\gamma \approx \mu_p E_0 \tfrac{1}{2} \sqrt{\tau_{p0}/D_p} = \mu_p E_0 L_p / (2D_p);\ L = \sqrt{D_p \tau_{p0}}\ .$$

Thus we have,

$$\Gamma_+ = \frac{\gamma + \sqrt{1+\gamma^2}}{L} > 0 \quad \text{and} \quad \Gamma_- = \frac{\gamma - \sqrt{1+\gamma^2}}{L} > 0 \qquad .$$

The final solution is dictated by boundary conditions on the solution: $\delta p = C_1 e^{\Gamma_1 x} + C_2 e^{\Gamma_2 x}$.

One boundary conditions is: $\delta p \rightarrow 0 \ as\ x \rightarrow \pm\infty$, which requires that $C_1 = 0$ for x > 0, $C_2 = 0$ for x < 0, and thus

$$\delta p = C_2 e^{\Gamma_- x} \quad \text{for } x > 0 \qquad , \qquad \delta p = C_1 e^{\Gamma_+ x} \quad \text{for } x < 0$$

A second condition is continuity of δp at x = 0 $\Rightarrow C_1 = C_2 = C$, and we get finally

$$\delta p = C e^{\Gamma_- x} \quad \text{for } x > 0 \qquad , \qquad \delta p = C e^{\Gamma_+ x} \quad \text{for } x < 0$$

(b) If know δp at x = a and x = b, then we can define $K \equiv \delta p(\mathrm{a})/\delta p(b) = e^{\Gamma_-(\mathrm{a}-\mathrm{b})} = e^{-\Gamma_- d}$

And we can re-write $\gamma = \dfrac{\mu_p E_o L_p}{2D_p} \approx \dfrac{\mu_p E_o L_p e}{2k_B T \mu_p} = \dfrac{eE_o L_p}{2k_B T}$ where the second step used the Einstein relation. This allows us to state

$$\ln K = -\Gamma_- d = \left(-d\left[\gamma - \sqrt{1+\gamma^2}\right]\right)/L_p \quad \text{or} \quad \ln K + d\gamma/L_p = d\sqrt{1+\gamma^2}/L_p$$

$$\text{or} \quad \left(\ln K\right)^2 + (2d\gamma/L_p)\ln K + d^2\gamma^2/L_p^2 = (d^2/L_p^2)(1+\gamma^2)$$

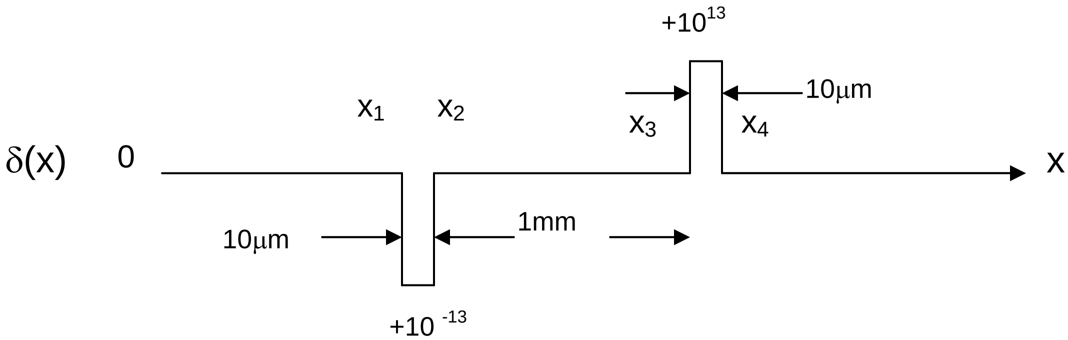
$$\text{or} \quad L_p^2\left(\ln K\right)^2 + 2d\gamma(\ln K)L_P - d^2 = 0 \quad \text{or} \quad L_p^2\left[\left(\ln K\right)^2 + d\left(\ln K\right)\frac{eE_o}{k_B T}\right] = d^2$$

which finally yields $$L_p = \frac{d}{\sqrt{\ln K\left(\ln K + deE_0 / k_B T\right)}}$$

Problem 4.

(1) InGaAs sample, $n_o = 1\times10^{16}$, $p_o = \frac{n_i^2}{n_o} = 5.3\times10^{10}$

The sample is perturbed by two excess charge regions: $\delta n = \delta p = 1\times10^{13}/cm^3$



a) Electric field follows from Poisson equation

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon} = \frac{e\left(\delta p - \delta n\right)}{\varepsilon_r \varepsilon_o} \text{ or } E = \int \frac{\rho}{\varepsilon_r \varepsilon_o} dx = \frac{\rho}{\varepsilon} + C \text{ in excess charge regions}$$

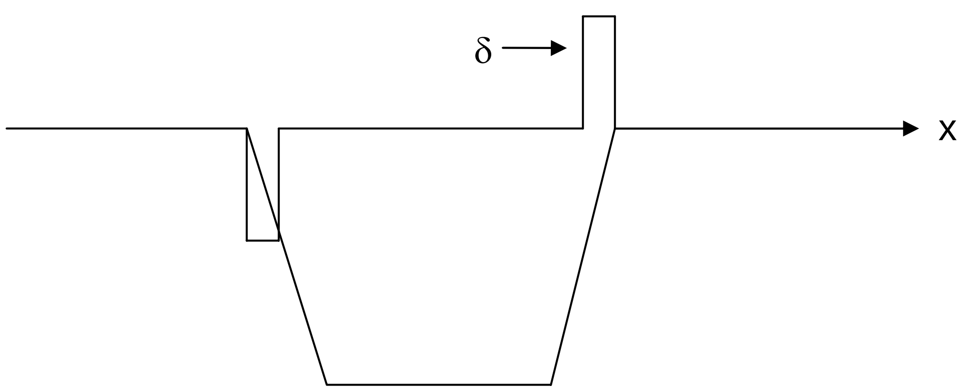
E = constant everywhere else

Coming from x = -∞ or x = +∞, we must have E = 0.

In left-hand charge region: $E\left(x\right) = \frac{-e\delta n}{\varepsilon_r \varepsilon_o}\left(x - x_1\right)$

In right-hand charge region: $E\left(x\right) = \frac{+e\delta p}{\varepsilon_r \varepsilon_o}\left(x - x_4\right)$

So in region between them $E = \frac{-e\delta n\left(x_2 - x_1\right)}{\varepsilon_r \varepsilon_o} = \frac{e\delta p\left(x_3 - x_4\right)}{\varepsilon_r \varepsilon_o}$



Since ε_r (InGaAs) = 16, E = -1.13×10^5 V/m

Note: If position of δn and δp were exchanged, we would get E = $+1.13\times10^5$ V/m

b) Rate of charge depletion: Outside of each excess chargeb region, $J = 0$ because E = 0
Inside $\vec{J} = \sigma \vec{E} = \left(n_o e \mu_e + p_o e \mu_h \right) \vec{E}$
Assume for InGaAs $\mu_e \approx 3900 \, {}^{cm^2}\!/\!{}_{V-s}$, $\mu_h = 1900 \, {}^{cm^2}\!/\!{}_{V-s}$

Inside left-hand charge region, from continuity equation

$$\frac{dQ}{dt} = \frac{-dJ}{dx} \approx \frac{-\left(J\left(x_2\right) - J\left(x_1\right) \right)}{x_2 - x_1} = \frac{+7.05\times 10^3 \, {}^{A}\!/\!{}_{cm^2}}{10\mu m} = \frac{+7.05\times 10^6 A}{cm^3}$$

or

$$\frac{dQ}{dt} = \frac{+7.05\times 10^6 c}{cm^3 - s} = \frac{+7.05\times 10^{12} c}{m^3 - s}$$

Similarly in the right-hand charge region,

$$\frac{dQ}{dt} = \frac{-dJ}{dx} = \frac{-\left(J\left(x_4\right) - J\left(x_3\right) \right)}{x_4 - x_3} = \frac{-7.05\times 10^3 \, {}^{A}\!/\!{}_{cm^2}}{10\mu m} = \frac{-7.05\times 10^6 A}{cm^3}$$

or

$$\frac{dQ}{dt} = \frac{-7.05\times 10^6 c}{cm^3 - s} = \frac{-7.05\times 10^6 c}{m^3 - s}$$

c) Time required for neutralization: This is a bit tricky because rate of depletion depends onJ in between, which depends on E in between, which depends on δn or δp. To find this, we note that $dQ = -d\delta n \cdot e$. So that,

$$\frac{-ed\delta n}{dt} = \frac{dJ}{dx} \approx \frac{-J\left(x_2\right)}{\Delta x} = \frac{-n_o e \mu_e}{\Delta x} \underbrace{\left(\frac{-e\delta n \Delta x}{\varepsilon_r \varepsilon_o} \right)}_{\uparrow E \text{ from } part(a)}$$

or

$$\frac{d\delta n}{dt} = \frac{-n_o e \mu_e}{\varepsilon_r \varepsilon_o} \delta n$$

This is a linear 1st order differential eqn having the solution $\delta n = ce^{-t/\tau}$. Substitution yields

$$-\frac{1}{\tau} = \frac{-n_o e \mu_e}{\varepsilon_r \varepsilon_o}$$

or

$$\tau = \frac{\varepsilon_r \varepsilon_o}{n_o e \mu_e} = \frac{\varepsilon}{\sigma} \quad \text{dielectric relation time}$$

For the sample at hand $\tau = 624\left(\Omega - m\right)^{-1}$ *and* $\tau = 2.27\times 10^{-13} s$. So complete neutralization takes infinite time; $\frac{1}{e}$ neutralization takes $\approx 2.27\times 10^{-13} s = 0.23 ps$