Homework 1

1. **Silicon capacitor**. Evaluate the maximum energy stored per unit volume and the maximum polarization (dipole moment per unit volume) in a parallel-plate capacitor filled with silicon, assuming $\varepsilon = 12.0$ and that the maximum (macroscopic) field sustainable in the solid is 3.0×10^5 V/cm.

2. **Polarizability of conducting sphere**.

   Show that the polarizability of a conducting metallic sphere of a radius α is $\alpha = a^3$. This result is most easily obtained by noting that E = 0 inside the sphere and then using the depolarization factor $4\pi/3$ for a sphere (Fig. 26). The result gives values of α of the order of magnitude observed polarizabilities of atoms. A lattice of N conducting spheres per unit volume has dielectric constant $\varepsilon = 1 + 4\pi Na^3$, for $Na << 1$. The suggested proportionality of α to the cube of the ionic radius is satisfied quite well for alkali and halogen ions. To do the problem in MKS units use ⅓ as the depolarization factor.

3. **Atomic polarizability**.
   (a) Diamond, silicon, and germanium are tetrahedrally-bonded semiconductors with a lot of covalent sharing of their valence electrons with the nearest neighbors, and surprisingly high relative dielectric constants ε_r. From Kittel or a more favored reference, look up the atomic density and ε_r for each of these.
   (b) Now use the Clausius Mosotti formula to determine the atomic polarizability from ε_r assuming just one dipole type in the solid, the induced atomic dipole.
   (c) Using (a) and (b) above with the result $\alpha \approx 4\pi\varepsilon_0 R^3$ derived in class from the spherical-electron-cloud model of the atom, estimate the radius of the diamond, Si, and Ge atoms.
   (d) Look up the distance between nearest neighbors in the three materials and compare half of this to the prediction of (c). For which of the three compound semiconductors is R closest to the half-distance between nearest neighbors ?

4. **Ionic polarizability**

   In the notes a model for the atomic polarizability of ionic solids is derived using fundamental principles of electrostatics, lattice waves, and crystal theory. The key result for an ionic crystal with a diatomic, dipolar basis is simple $\alpha = q^2/C$, where C is the interplanar, atomic spring constant – the same quantity used for lattice-wave theory. This problem will derive the dielectric properties of two more common ionic crystals other than NaCl: (1) MgO, and (2) KI (used worldwide as a source of iodine for humans in "iodized" table salt).

(a) Start by finding the room-temperature conventional cubic lattice constant, a, and the stiffness coefficient C_{11} for MgO and KI from any solid-state reference.

(b) Use (a) to find the speed of sound along the [100] axis, and the atomic polarizability [clue: carefully consider the ionic charge q]

(c) Use the polarizability to estimate the electric susceptibility and dielectric constant ε_r. Compare ε_r to accepted values obtained from books or the WWW.

5. **Ferroelectric criterion for atoms.**

Consider a system of three neutral atoms separated by a fixed separation *d*, each atom having a dipole moment p and a polarizability α. Find the relation between *d* and α for such a system to be ferroelectric. Consider two spatial configurations: (1) co-alignment, and (2) parallel alignment. Hint: use the expression for the vector electric field in the vicinity of a dipole.

6. **Temperature-dependence of ferroelectrics for $T < T_C$: Pyroelectricity**

One of the great applications of ferroelectricity today is in "uncooled" infrared focal plane arrays, one type having pyroelectric-type "thermal" detectors. An elegant application of thermodynamics is in predicting the behavior of macroscopic variables approaching phase transitions, of which the ferroelectric to paraelectric transition is a good example, and on which the pyroelectric detector derives its radiative sensitivity. Some of the displacement-type ferroelectrics, such as the Perovskite $LiTaO_3$, has a second-order transition for which the spontaneous polarization is given by $P_e = P_s\,(1\text{-}T/T_C)^{1/2}$.

(a) Use the second order critical expression to derive the pyroelectric coefficient, $|dP_e/dT|$. Evaluate this coefficient at T = 320 K for $LiTaO_3$, looking up the value for P_S and T_C for high-quality crystalline material. Make sure you answer is in MKSA units.

(b) Now consider an inferior ferroelectric material to $LiTaO_3$ from a P_S standpoint, triglycine sulfate (TGS). If TGS is used as the pyroelectric detector material at T = 320 K, what is the new pyroelectric coefficient assuming it also has the second-order critical behavior ? Compare this to the pyroelectric coefficient of $LiTaO_3$ and explain the difference.